

Stick-slip transition in the Scalar Arching Model

P. Claudin, J.-P. Bouchaud

Abstract When some granular material contained into a silo is pushed upwards with a piston, an irregular stick-slip motion of the system of grains is observed. We show how one can adapt the ‘Scalar Arching Model’ (SAM) – proposed as a model for giant stress fluctuations in silos – in order to describe this stick-slip phenomenon. As a function of the sensitivity of the system to mechanical noise, the system exhibit two different phases: a ‘jammed’ phase, and a ‘sliding’ phase where irregular stick-slip is observed. We analyze the transition, which is found to be of mean-field type, and study the statistical properties of the intermittent stick-slip motion.

Stick-slip motion is a very common phenomenon which occurs when two solids slide on each other, and has been much studied in the recent years in connection with solid friction [1, 2]. Stick-slip motion also occurs in granular materials [3–5]. For example, the Jussieu group has performed the following experiment [4, 5]: a 2D vertical cell containing aluminium beads is pushed upwards by a piston through to a spring tightened at a constant speed. A very irregular stick-slip motion is observed, which can be attributed to the fact that force propagation in granular media is strongly inhomogeneous: most of the force is concentrated on stress paths (or arches). Some configurations of these paths propagate all the external pushing force F to the walls, and jamming occurs. Other configurations propagate the pushing force right up to the free surface, and the beads move upwards (slip). The temporal fluctuations of these stress paths then lead to an intermittent stick-slip motion, which we attempt to model here using the ‘Scalar Arching Model’ (SAM). This model was proposed to account for giant stress fluctuations in silos [6]. The SAM is an extension of the Chicago group’s stochastic model [7, 8] which allows for the formation of arches. We

have shown how small perturbations could lead to sudden transformations of these arches, and thus to large fluctuations of the weight on the bottom plate of a silo. The idea is to adapt the same model to the experimental situation described above, and to quantify how the formation of arches can indeed lead to very irregular stick-slip motions.

Let us briefly recall the main features of the SAM. The granular packing is represented in 2D by a regular lattice: each site is a ‘grain’ labelled by two integers (i, n) giving its horizontal and vertical coordinate. We neglect the weight of the grains compared with the applied force F , and focus on the transmission of F through the grains from the bottom of the cell ($n = 0$) to the walls ($i = \pm L$) and the free surface ($n = H$), see figure 1. Each grain then supports the force w of its two downstairs neighbours, and shares its own load randomly between its two upstairs neighbours.

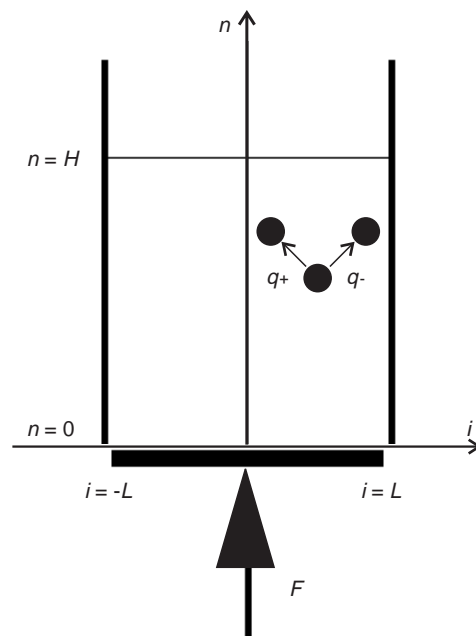


Fig. 1. Beads are confined into a 2D silo. An upwards force F is applied on a piston at the bottom of the silo. Such a system of beads gives rise to an irregular stick-slip motion. In the SAM model, beads propagate forces according to rules which combines random propagation (encoded by the random numbers q_{\pm}) and arch formation. In all the present paper, simulations have been performed with $L = 30$ (bead radii), which is close to the experimental system of [4, 5]

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The corresponding scalar equation for force propagation is thus:

$$w(i, n) = q_+(i+1, n-1)w(i+1, n-1) + q_-(i-1, n-1)w(i-1, n-1) \quad (1)$$

$q_{\pm}(i, n)$ is the fraction of the force transmitted to the grain ($i \mp 1, n+1$), and is a random variable between 0 and 1, subject to the conservation constraint $q_+(i, n) + q_-(i, n) = 1$. These random coefficients model the randomness of the local packing, size and shape of the grains, etc. At this stage, the model is the one considered in [7], [8], albeit upside down. We now include a ‘local slip condition’: when the shear on a given grain is too strong, the grain can slip and lose its contact with its neighbours opposite to the direction of the shear. More precisely, we introduce a threshold R_c such that

$$q_+(i, n) = 1 - q_-(i, n) = 0 \quad \text{if} \quad \frac{w_- - w_+}{w(i, n)} \geq R_c \quad (2)$$

$$q_-(i, n) = 1 - q_+(i, n) = 0 \quad \text{if} \quad \frac{w_+ - w_-}{w(i, n)} \geq R_c \quad (3)$$

where $w_{\pm} = q_{\pm}(i \pm 1, n-1)w(i \pm 1, n-1)$. These rules lead to arch formation [6].

The walls play a crucial rôle in the stick-slip process, since the friction forces there can balance the external force F and allow the system to jam. We thus introduce a new parameter, the ‘jamming ability’ α , such that with probability α the load $w(\pm L, n)$ is completely ‘absorbed’ by the wall which balances all the force carried by the grain. With probability $1 - \alpha$, we apply the same rule as in the bulk, i.e. for $i = \pm L$, the fraction $q_{\mp}(\pm L, n)$ of the load $w(\pm L, n)$ hits the wall. In addition to the random $q_{\pm}(i, n)$, we thus define $2(H+1)$ uniform random numbers $\alpha_{\pm}(n)$ which, compared to α , decide whether the site ($i = \pm L, n$) is ‘absorbing’ (i.e. if the grain is supported only by the wall) or not. In the following, the $q_{\pm}(i, n)$ ’s are also chosen uniformly between 0 and 1, although other choices lead to the same qualitative conclusions.

In order to reproduce the overall stick-slip motion of the assembly of grains, we propose to capture the different physical phenomena which occur as follows. For a given

applied external force F , and a given set of random numbers $q_{\pm}(i, n)$ and $\alpha_{\pm}(n)$,

- we calculate the total forces on the walls F_w and on the free surface of the silo F_{fs} . Obviously, $F = F_w + F_{fs}$.
- if $F_{fs} = 0$, the grains do not move, corresponding to a *stick* situation. We then increase the applied force F by some fixed amount ΔF and the time t by Δt . In order to mimic the mechanical noise which necessarily occurs when the external load is increased and might trigger some local rearrangements, we also change a fraction p of all the random numbers and recalculate F_{fs} .
- if $F_{fs} > 0$, the equilibrium condition for the top grains is not satisfied, which means that grains are moving. It is a *slip* situation. Correspondingly, the spring loosens and the applied force F is decreased by ΔF . We also change *all* random numbers (because the flow motion completely rearranges the packing). The flow stops when a randomly chosen configuration has enough ‘anchoring’ sites at the walls to yield $F_{fs} = 0$.

The simulation starts at $t = 0$ with $F = 0$ and lets F increase progressively. It is important to note that our model is actually purely static: no dynamics is explicitly included. Therefore, the motion of the grains during a slipping event is assumed to be infinitely quick on the scale of sticking events (t is thus kept constant during slipping events). We thus actually describe only sticking situations, separated by slipping events which have two effects: untighten the spring governing the external force F , and reinitialize the structure of the packing (i.e. the random numbers). As seen on figure 2, such an ‘algorithm’ indeed leads to an irregular stick-slip motion. Note that for $\alpha = 0$ the probability that F_{fs} vanishes is exponentially small in H . Physically, this means that, in order to resist to the external force F , the system of beads must generate arches which are strongly ‘anchored’ by the walls.

The model is controlled by four parameters. The first two – namely R_c , the threshold of the SAM, and b the aspect ratio of the cell – are of secondary importance: they do not affect the general features of our results. More important are the jamming ability α of the walls and the

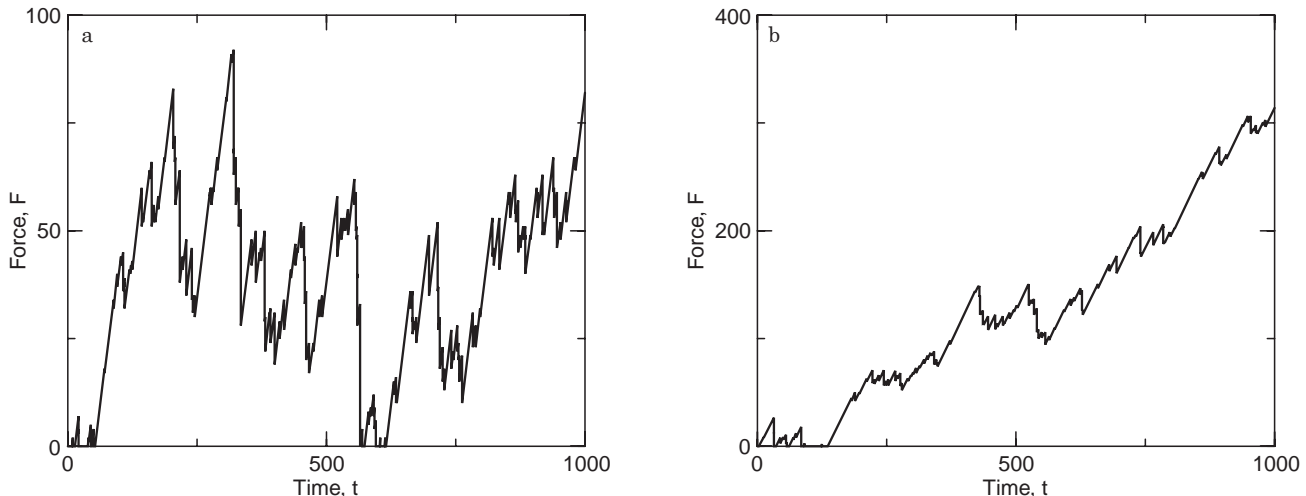


Fig. 2a, b These plots show the temporal evolution of the applied force F for (a) $\alpha = 0.83 < \alpha_c$ (sliding phase) and (b) $\alpha = 0.85 > \alpha_c$ (jammed phase). Both plots have been obtained with $R_c = 0.5$, $p = 0.01$ and $b = 1$ for which $\alpha_c \sim 0.838$

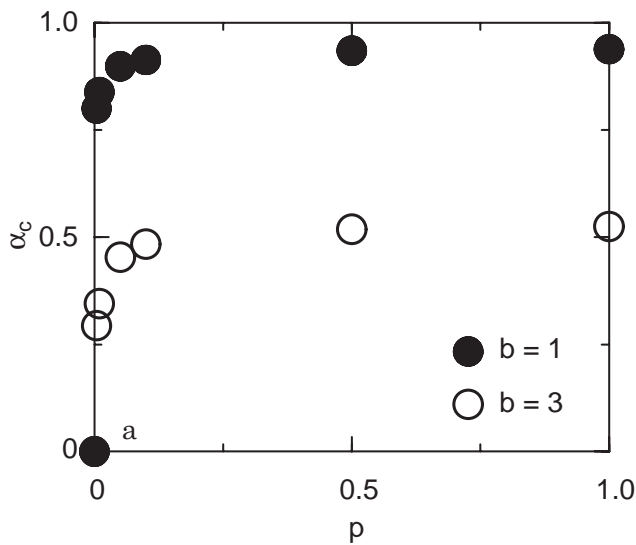
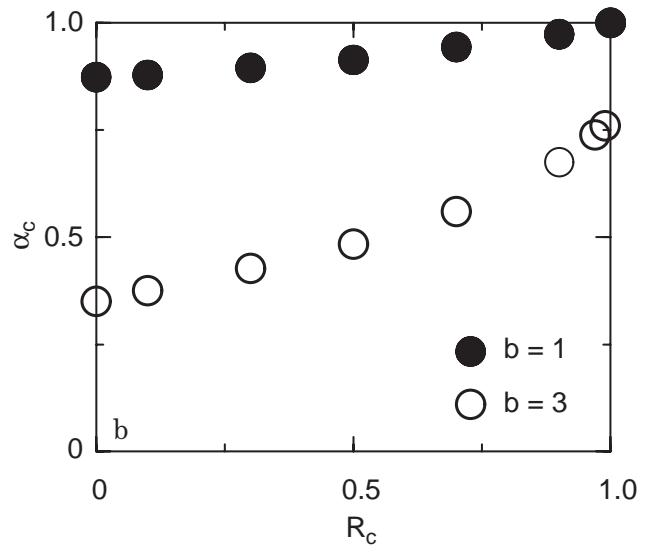


Fig. 3a, b The curve $\alpha = \alpha_c(p)$ separates the sliding phase (below) from the jammed phase (above). The phase diagram (a) has been plotted for $R_c = 0.5$ and two aspect ratios, $b = 1$



and $b = 3$. Alternatively, we can represent the phase diagram by the curve $\alpha = \alpha_c(R_c)$. The parameters chosen for the figure (b) are $p = 0.1$ and again, $b = 1$ and $b = 3$

rôle of the mechanical noise which modifies the local structure of the packing, measured by p . For R_c and b fixed, depending on the values of α and p , two distinct phases are found. For small p or large α , the system is jammed in the sense that although the grains move from time to time, F goes to infinity as time increases. This phase can be described by the average rate of increase of F , $s = F(t)/t$. On the other hand, for large p or small α , the system slips very easily. This sliding phase is characterized by the fact that $F(t)$ always goes back to 0. The relevant quantity in this phase is the delay τ between two consecutive times where F vanishes. Figure 2 shows some typical plots of $F(t)$ in these two phases. The transition between the

two phases occurs for a critical value of $\alpha = \alpha_c(p)$, which allows us to obtain the phase boundaries, as plotted on figure 3. Experimentally, these two regimes could be reached by preparing the system in different ways. A compact system would provide a rigid structure (small p). By contrast, a loose packing would be subject to large rearrangements (large p). Similarly, the state of the walls allows one to change the value of α . An experimental recording of $F(t)$ actually show parts of the two different regimes [4, 5].

We studied how the system behaves near criticality. On figure 4, we show the integrated histogram of τ for R_c, b and p fixed, and for different values of α . We see that τ

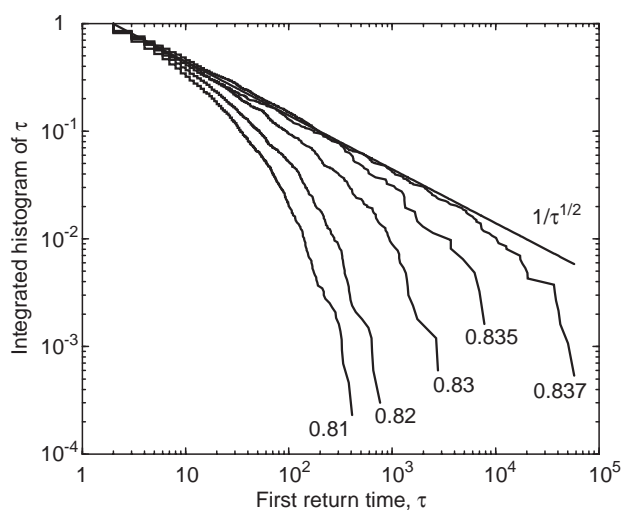


Fig. 4. These curves represent integrated histograms of the first return time τ , i.e. the interval of time between two times where F vanishes. They have been computed with $R_c = 0.5$, $p = 0.01$, $b = 1$, and with different values of α indicated on the plot. As $\alpha \rightarrow \alpha_c$, this histogram gets broader and broader, and tends to the power law $\tau^{-1/2}$ which is characteristic of the return time of simple random walks

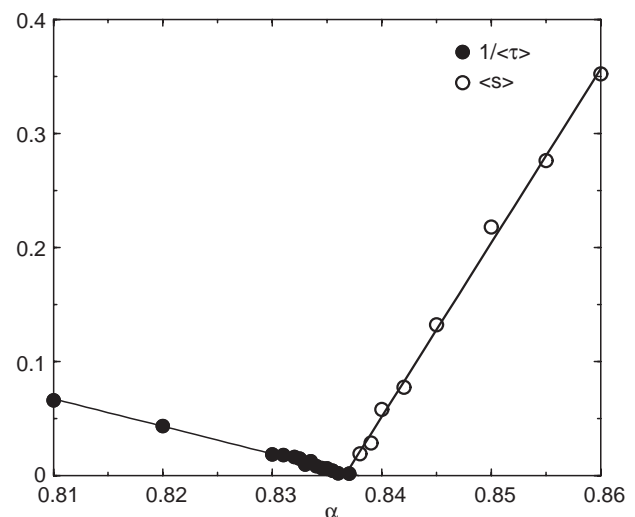


Fig. 5. Below the transition, the system is characterized by the averaged first return time $\langle \tau \rangle$, and above it by the averaged slope $\langle s \rangle$ of the applied force F versus time. Near the transition, we find that $\langle \tau \rangle$ diverges like $1/(\alpha_c - \alpha)$ and that $\langle s \rangle$ grows like $\alpha - \alpha_c$. This plot has been computed with $R_c = 0.5$, $p = 0.01$ and $b = 1$. Linear regressions for $1/\langle \tau \rangle$ and $\langle s \rangle$ give respectively $\alpha_c = 0.839 \pm 0.004$ and $\alpha_c = 0.837 \pm 0.004$

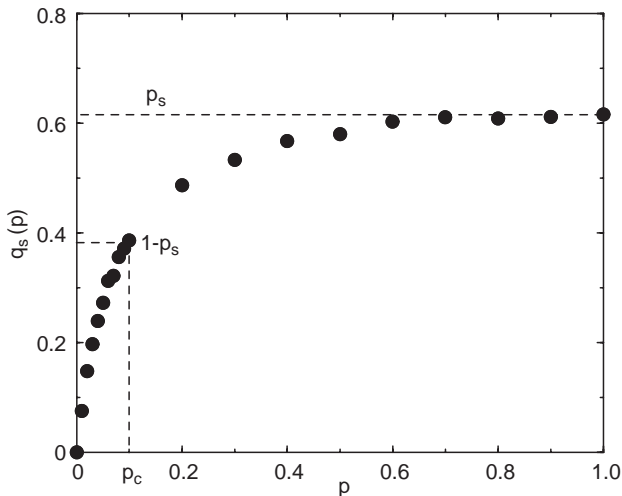


Fig. 6. The probability q_s that a system in a jamming configuration at time t becomes sliding at time $t + \Delta t$, depends on p . For fixed R_c , b and α , we call p_c the critical value of p such that $\alpha = \alpha_c$. Three points of the curve $q_s(p)$ are analytically known: $q_s(0) = 0$, $q_s(1) = p_s$ and $q_s(p_c) = 1 - p_s$. The whole curve has been obtained numerically for $R_c = 0.5$, $\alpha = \alpha_c(p = 0.1) = 0.914$ and $b = 1$

tends to be power-law distributed as $\alpha \rightarrow \alpha_c$, with an exponent $-1/2$. Note that, as argued below, this power-law corresponds to the first return probability of a one dimensional random walk. $\langle \tau \rangle$ diverges for $\alpha = \alpha_c$; we found numerically that $\langle \tau \rangle \propto 1/(\alpha_c - \alpha)$ for $\alpha < \alpha_c$. In the same way, we found that the average slope of F versus time behaves like $\langle s \rangle \propto (\alpha - \alpha_c)$ for $\alpha > \alpha_c$ (see figure 5).

These critical laws can be understood within a simple mean field analysis. Neglecting the correlations, the temporal evolution of F can be approximated as a Markovian two-state process. Suppose the system is sliding at time t . We call p_s the probability that it is still sliding at time $t + \Delta t$. Similarly, we call q_s the probability of sliding at time $t + \Delta t$ knowing that the system is in a jamming configuration at time t . Obviously, p_s depends on α and q_s on p and α . For example, one has $q_s(p = 0) = 0$ and $q_s(p = 1) = p_s$. Figure 6 shows $q_s(p)$, as determined numerically. This simple two-state model can be explicitly solved. The critical point is found to be when the probability of sliding after jamming is equal to the probability of jamming after sliding, i.e. $q_s(p) = 1 - p_s$. At this point, the probability that F increases is equal to the probability that F decreases, which implies indeed that F behaves as a random walk. Provided that the functions p_s and q_s are regular near the critical line, one also finds the observed linear behaviour of s and $1/\langle \tau \rangle$. The fact that the temporal correlations are found to be small however means that our model fails to capture ‘precursor’ effects before the slip, which have been observed experimentally in [3]. This is related to our simple rule where each grain can ‘move’ under the influence of the external noise with equal probability. Finally, we also looked at the two following quantities: the distribution of the heights of the slips, and the distribution of the intervals of time between two slips. The tails of these distributions are found to be exponentially decaying.

In conclusion, we discussed how the Scalar Arching Model can be modified to describe granular systems undergoing intermittent stick-slip motion. We showed that such a system can present two different phases: a ‘slipping’ stick-slip (or sliding) phase, where the external pushing force remains finite, and a ‘sticking’ stick-slip (or jammed) phase. The transition between these two regimes is found to be of mean-field type.

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