Glassy Landscapes and models

A. States

What is the role of metastable states in glassy systems? This seems a very easy question to answer: the lanscape is rugged and the system gets trapped in them. And yet, the closer one looks at the problem, the more elusive it becomes.

To begin with, beyond mean-field, all metastable states with energy density different from the ground state will eventually decay through nucleation. Hence, what we call a 'state' depends unavoidably on a timescale: two different states at a time scale become fused into one at a larger one.

Next, there is the problem of the basins of attraction. The Sherrington-Kirkpatrick model, for example, has an exponential number of states with higher free energy density, but all of them are completely avoided by the dynamics, which relaxes to the ground state free energy density. Even at the level of mean-field glasses, where all the questions seem straightforward, the literature on metastable states is enormous and frequently contradictory.

As mentioned above, in my work [11], I discussed the evaluation of the number of states at the mean-field level, the sign problem, and a method to circumvent it.

Later, in a paper with Parisi and Virasoro [15], we computed the barriers in 'Random First Order' models – those that are supposed to represent structural glasses. This paper was followed by the solution of the dynamics of such models [16], where the role of the 'threshold level' became manifest. The stability properties of states was discussed in the paper in collaboration with L. Laloux [26]: threshold states dominating the out of equilibrium dynamics are marginal, while deeper ones are not. There have been many subsequent articles studying different aspects of the organisation and structure of metastable states, and it impossible to outline them here - for a review see Cavagna's review cited above.

In collaboration with G. Biroli we addressed directly the problem of timescales [43]. We followed an approach that had been developed in the mathematical literature by Gaveau, Schulman, Bovier, Gayrard, Klein, and others. In the presence of a gap in timescales, one can make a *dynamic* definition of states as those that are diconnected at times $t < t_{gap}$ smaller than the gap. It turns out that one can calculate the number of such states from first principles by computing the number of *periodic trajectories* of period $<< t_{gap}$. In [43] we did this for mean field glasses, and reobtained the known results of the free-energy landscape description, showing the viability of the approach.

The approach of [43] involves computing periodic orbits, which are trajectories that break causality, because one demands a condition on the endpoint. Such solutions give us a first glimpse of how the *instanton* solutions will look like, and hence a beginning for a theory of activated processes.

Another interesting point that has not been persued is to use this method to compute the number of *stable* metastable states in spin-glass like models, avoiding all the uncertainties of the alternative approches.

B. Models without disorder

By the mid-nineties, the idea became accepted that random systems with $p > 2$ spin interactions are a mean-field caricature of structural glasses. Once one accepts this, a question immediately arises: why should quenched disorder be necessary? After all, we know that real structural glasses do not have random interactions.

There followed a enormous amount of mean-field models without quenched disorder having the Random First Order scenario, starting from the early work of Kirkpatrick and Thirumalai, and in more modern version by Parisi, Migliorini, Bouchaud and Mézard.

Together with L. Cugliandolo, F. Ritort and G. Parisi we introduced a model [21] whithout quenched disorder which was neither the first nor the last of its class, but it has the intersting property of having the form of a many-particle system [1]. This we later generalised in collaboration with L. Cugliandolo, R. Monasson and G. Parisi.

The role of quenched disorder in mean-field models is by now well-established, and there is no need for further models of the kind. What remains open is the question of whether crystallisation can be neglected – at least conceptually – in finite dimensions, or whether it is an essential component.

[1] A similar class was later introduced by M. Soljacic and F. Wilczek Phys.Rev.Lett. 84 (2000) 2285