A Landscape Analysis of Constraint Satisfaction Problems

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Constraint Satisfaction problems

We are asked to find configurations of a system that do not violate a constraint: no particles overlap, no logical statements contradict, no adjacent nodes of a graph have the same color ...

We have a parameter that sets the difficulty: particle size, number of logical statements, number of graph links ...

We try to 'optimize': i.e. find solutions in the hardest conditions.

Usually:

Introduce an energy= number of errors (overlaps, contradictions, miscolorings) and study the zero-energy configurations.
e.g. soft particles in the zero-temperature limit

Alternative strategy: we work in a space without allowing errors.
We introduce a pseudo-energy landscape and its conjugate variable (a pressure)

and make the assumption that complex pseudo-energy landscapes are qualitatively similar to complex energy landscapes.

Some puzzling questions become clarified (even trivialized).

Two examples:

The 'unreasonable' effectiveness of unsophisticated optimization algorithms.

Why do algorithms like Walk-SAT find solutions well beyond the putative 'hard' level?

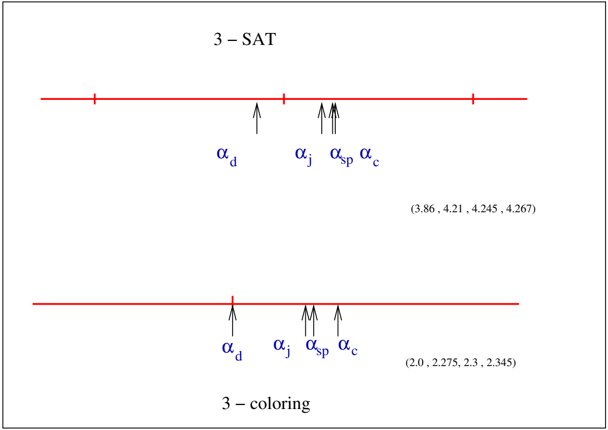
Beyond which level is an algorithm truly admirable?

The J-Point

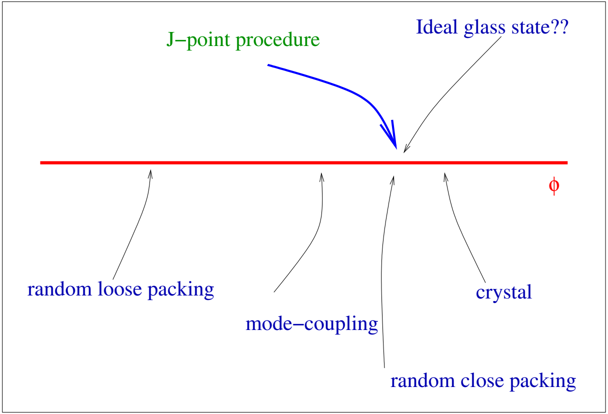
How do we place it in the context of the rest of glass theory?

Performances in terms of α
(average number of clauses or graph connectivity)

Contrary to expectations, α_d does not seem to play a role



Hard sphere packings



J-Point

J

Procedure: increase the radius of the spheres gradually, infinitesimal overlaps are removed through repulsion. Continue until the pressure is infinite: this is the J-Point.
(O'Hern et al., Lubachevsky-Stillinger,...)

The actual volume fraction reached is very close to the one quoted as Random Close Packing

The J-Point so defined has criticality properties (soft modes, diverging lengths and susceptibilities, isostaticity).

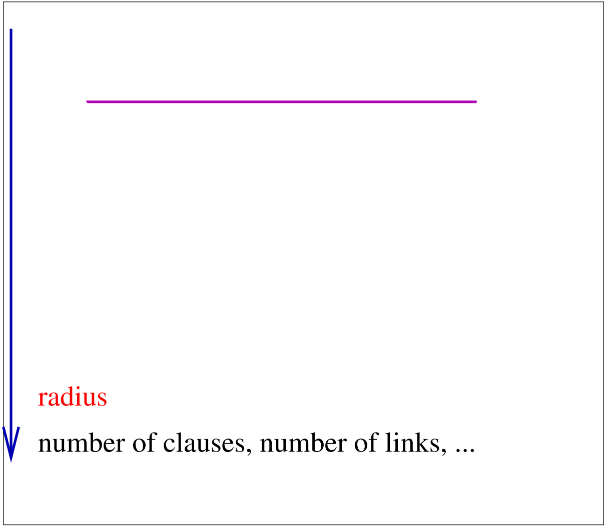
Sequential Incremental

J

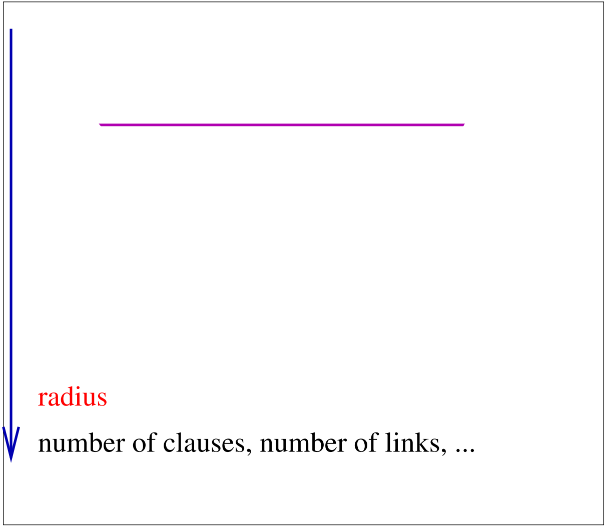
We can by analogy follow a similar strategy for Coloring or SAT problems, and define a J-point for them.

For graph coloring: we write a list of the links. Start from no links, and add them one by one. At each step correct to avoid miscolorings.

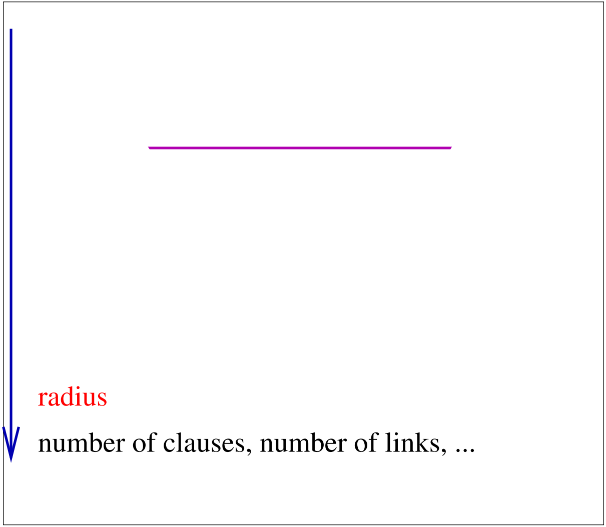
Landscape conjugated to pressure



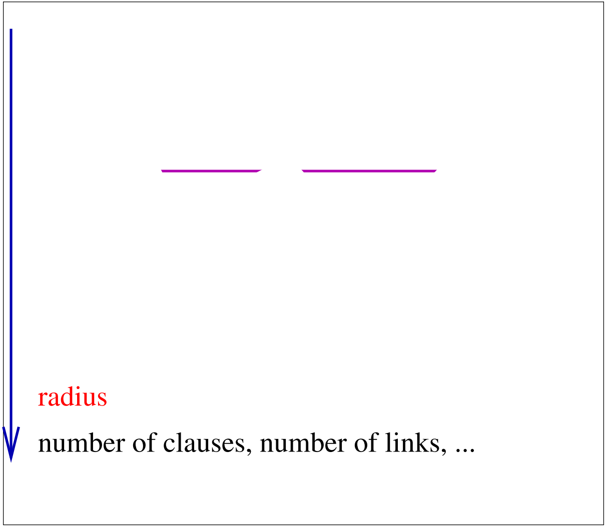
Landscape conjugated to pressure



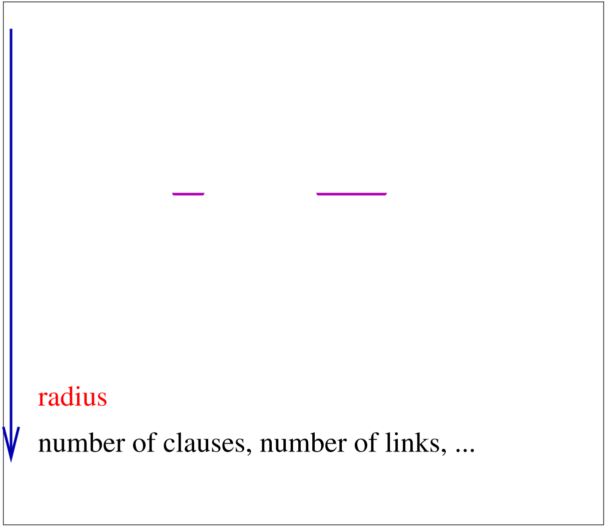
Landscape conjugated to pressure



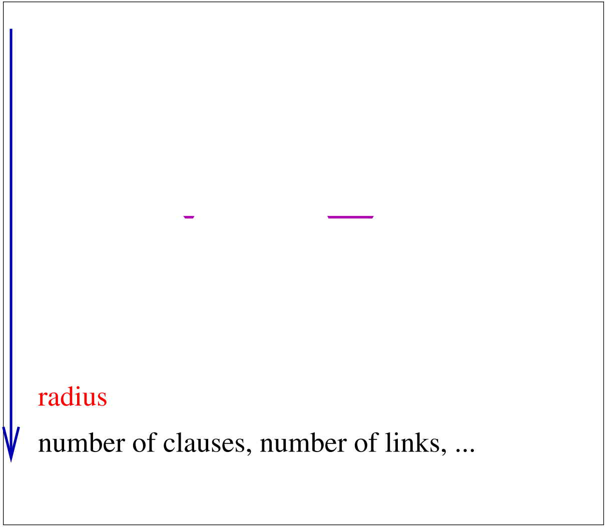
Landscape conjugated to pressure



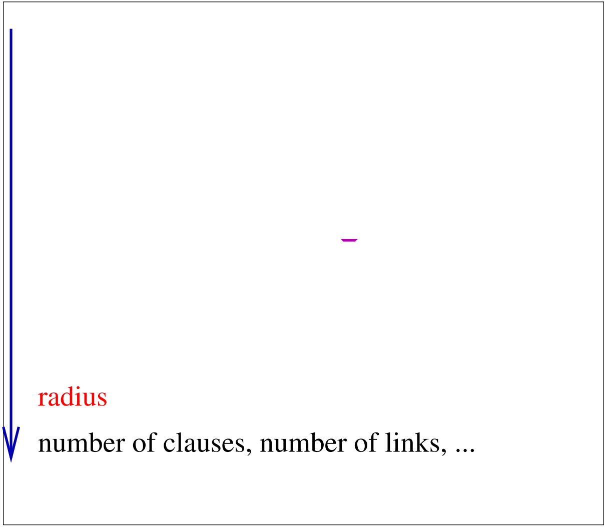
Landscape conjugated to pressure



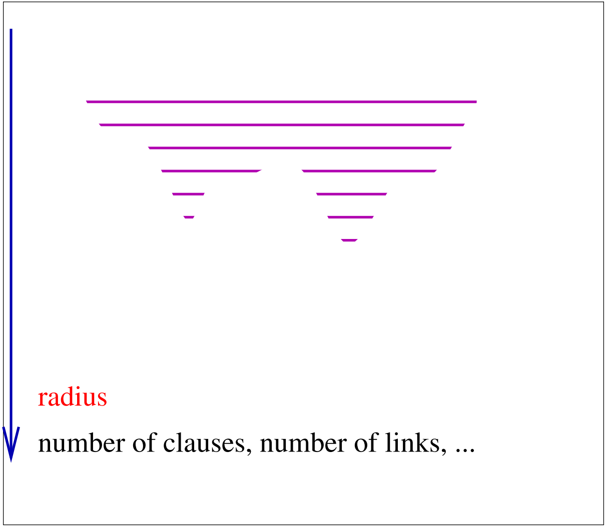
Landscape conjugated to pressure



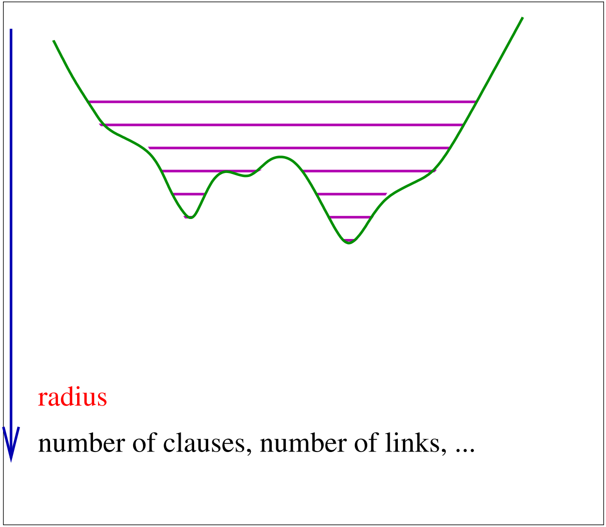
Landscape conjugated to pressure



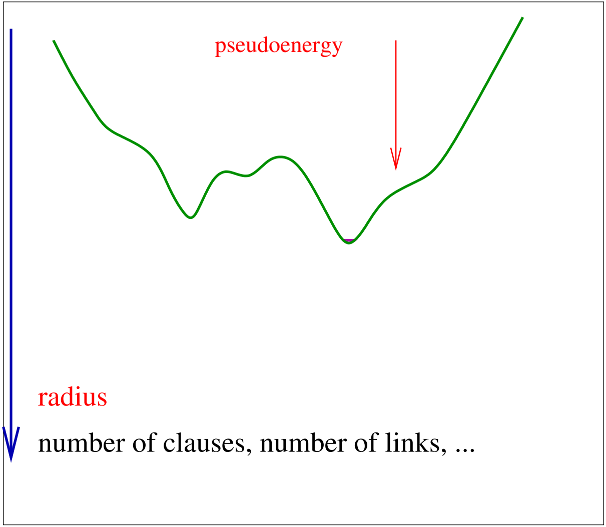
Landscape conjugated to pressure



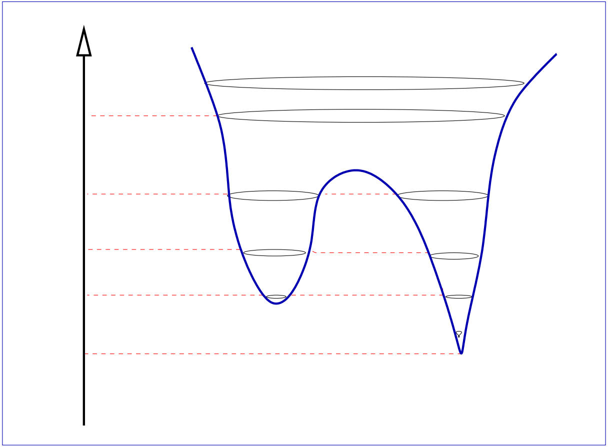
Landscape conjugated to pressure



Landscape conjugated to pressure



Conversely, cutting microcanonical slices of an energy landscape we obtain a constraint optimization problem (and benefit from a long experience).

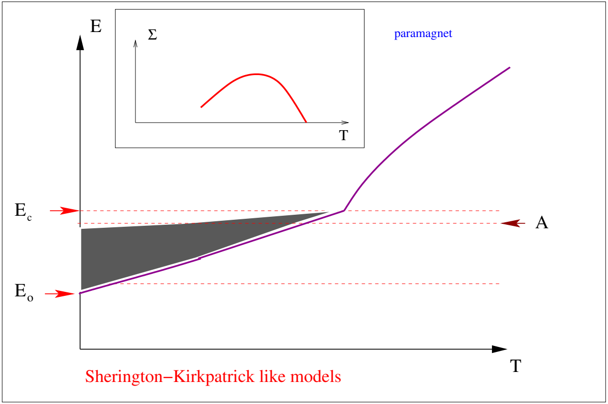


The J-Point and Sequential Incremental procedures are zero-temperature descents in pseudo-energy, starting from a random configuration

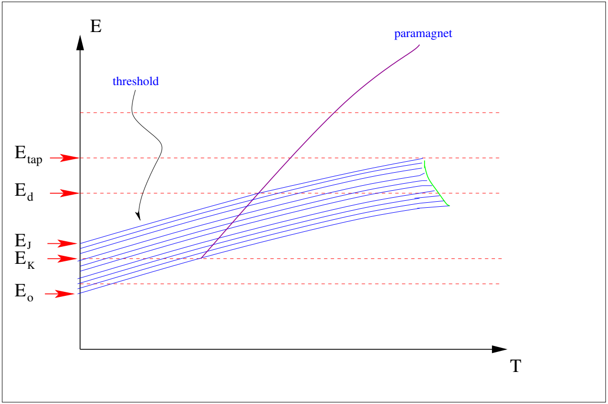
(i.e. analogous to an infinite temperature inherent structure).

The Spin-Glass landscape

(just for reference)



Random First-Order Landscapes



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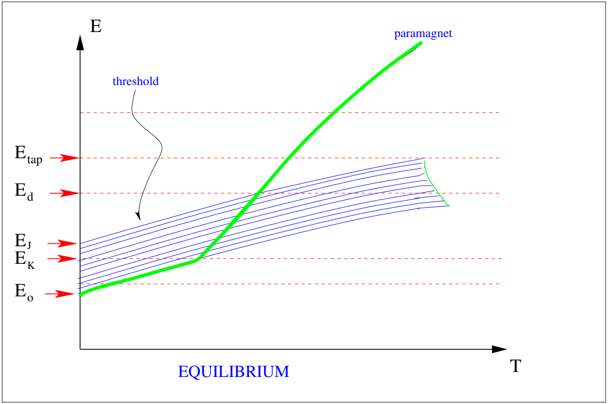

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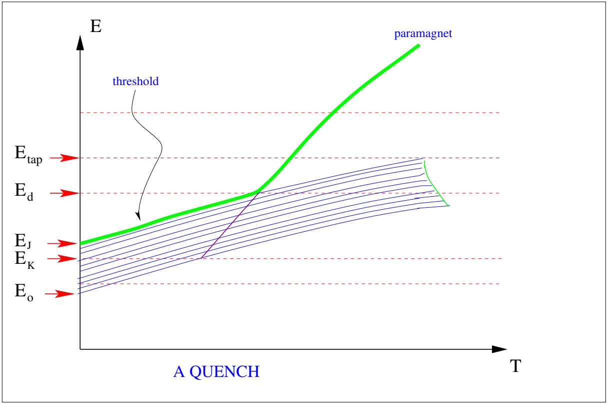

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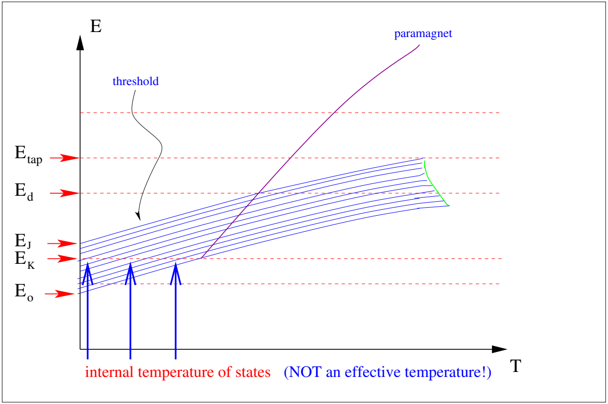

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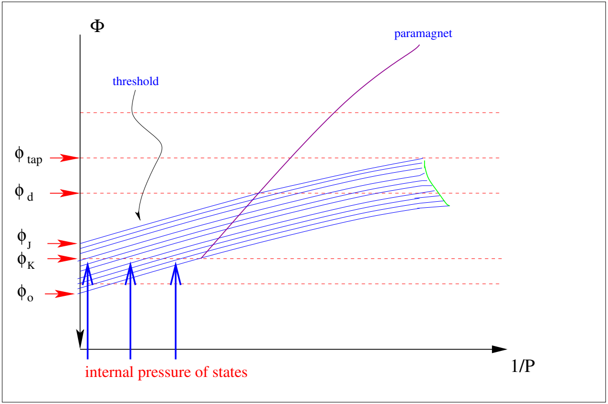

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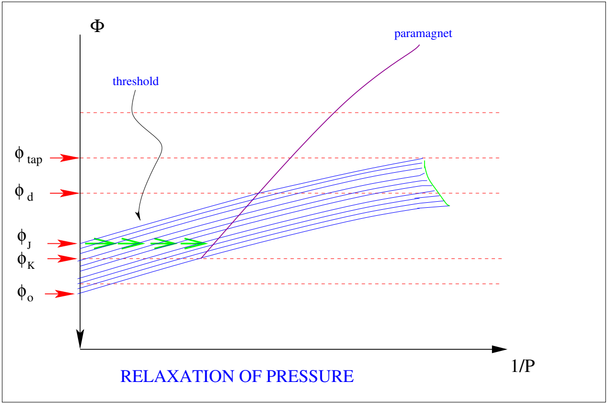
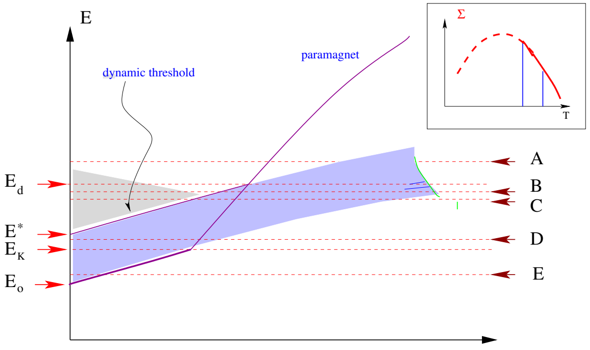
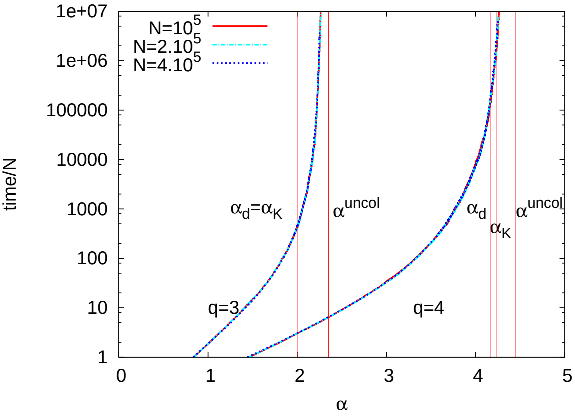


Random First-Order Landscapes

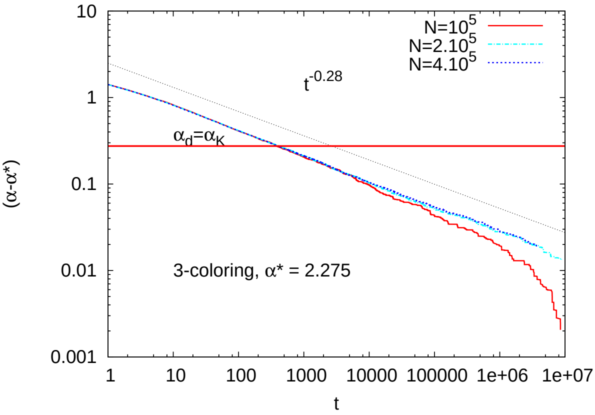


in general many 'transparent states' (cf. an old bet)

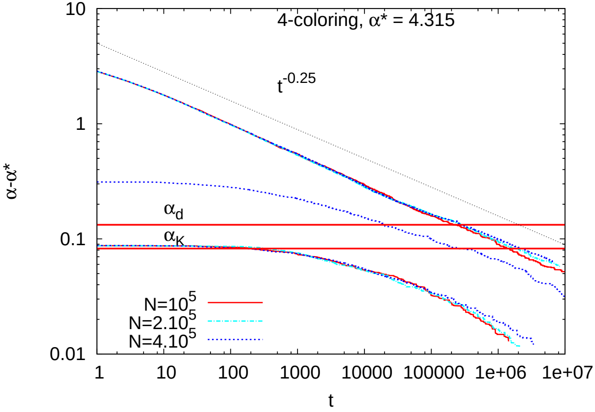
COLORING: ‘time’ versus difficulty.



3 Coloring


10
1
0.1
0.01
0.001
1
10
100
1000
10000
100000
1e+06
1e+07
t
(α-α*)
N=10^5
N=2.10^5
N=4.10^5
t^-0.28
α_d=α_K
3-coloring, α* = 2.275

4 Coloring

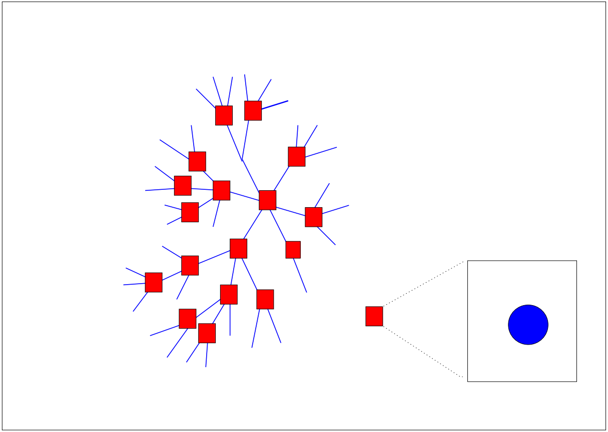

4-coloring, α* = 4.315
t^-0.25
α_d
α_K
N=10^5
N=2.10^5
N=4.10^5
10
1
0.1
0.01
1
10
100
1000
10000
100000
1e+06
1e+07
t
α-α*

CONCLUSION

Thinking in terms of pseudo-energy landscapes one can make use of the intuition developed for the energy landscapes:

- Are the configurations reached after a pressure quench the most abundant?
- J-point as an inherent structure: relation to the ideal glass state.
- Effect of relaxation on an infinite-pressure state
- ... and also understand the unreasonable effectiveness of unsophisticated constraint optimization algorithms.

A Model (hopeful ‘Rosetta stone’ to make contact with granular and *two level system* literature)



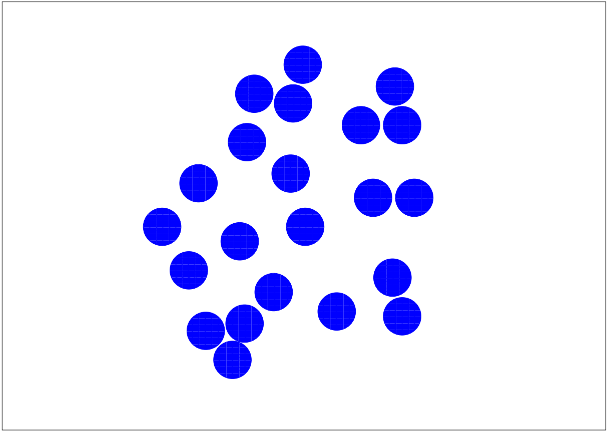
The unreasonable effectiveness of unsophisticated algorithms in constraint optimization problems

why is it so easy to go beyond the dynamic transition point α_d and even α_k

The relation between the ‘J-point’, Random Close Packing, the Mode-Coupling and the Kauzmann point

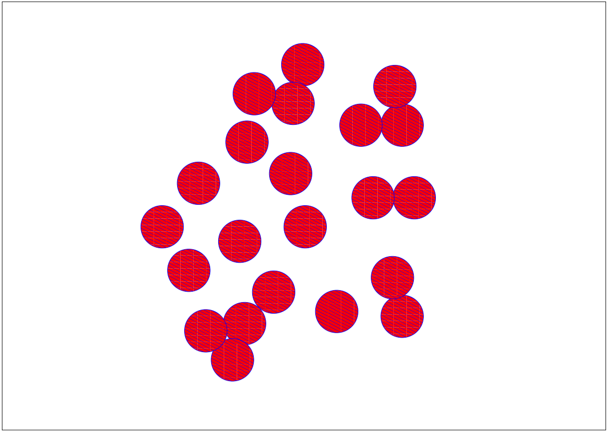
PACKING

A given configuration

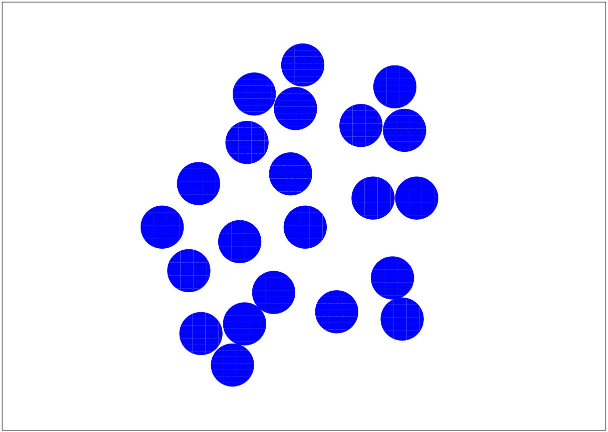


PACKING

Inflate slightly

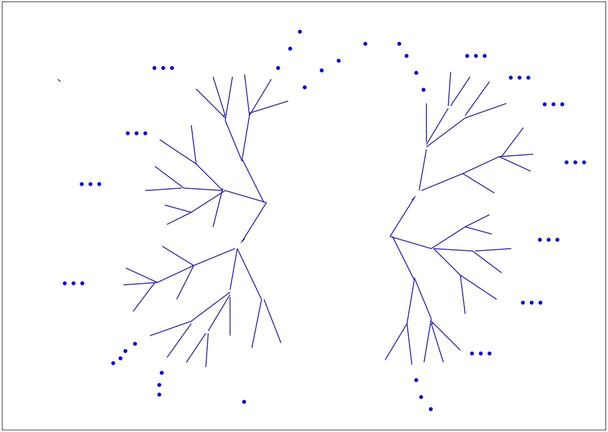


Displace particles to resatisfy



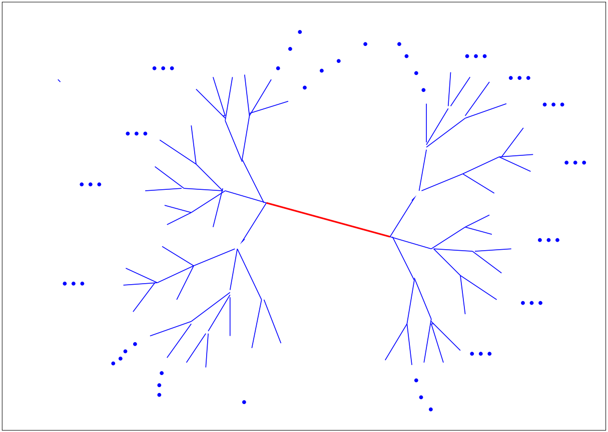
GRAPH COLORING

A well colored graph

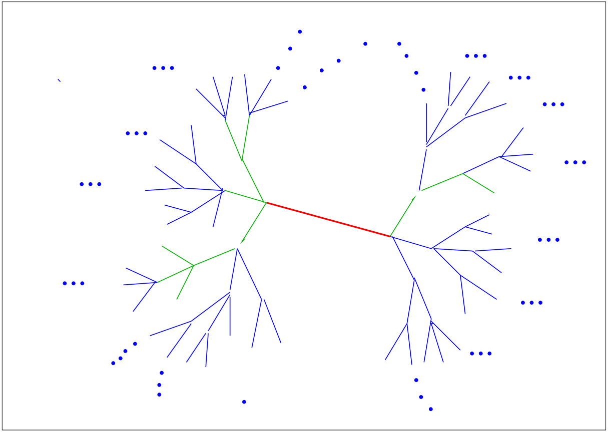


GRAPH COLORING

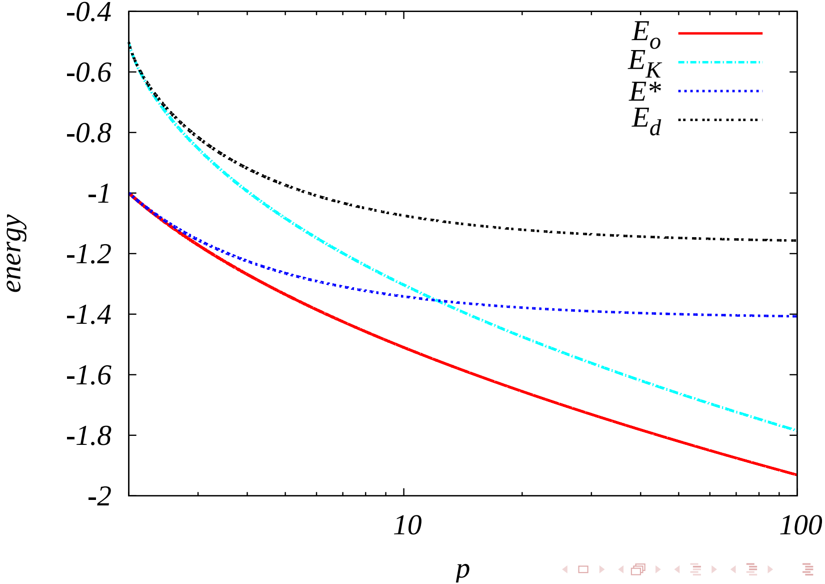
add one link



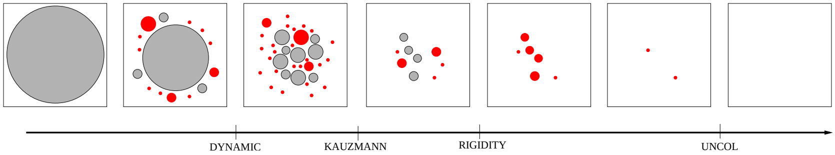
Modify nearby configurations to resatisfy



Energies as a function of p

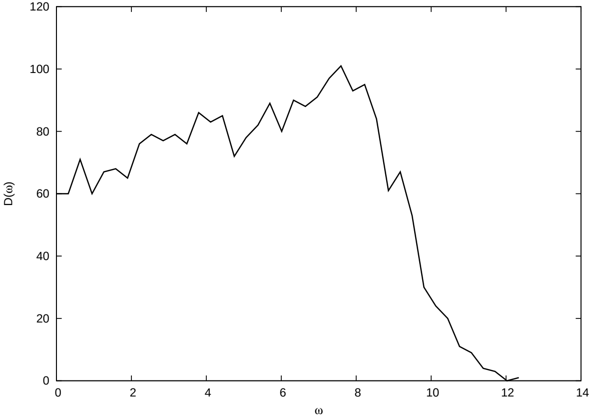


A Sketch



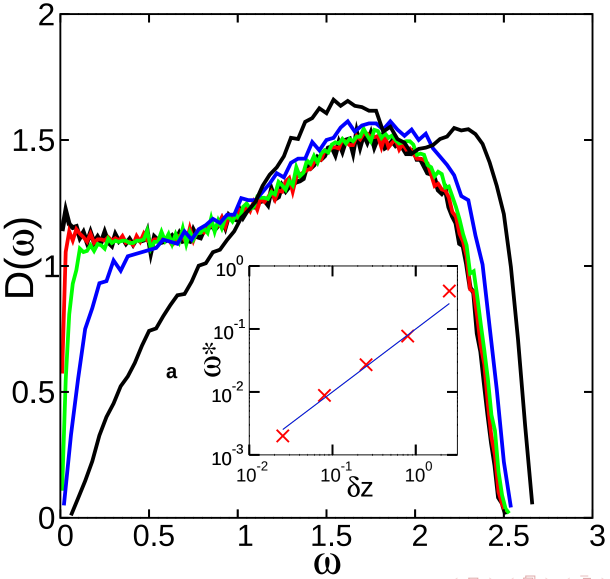
ISOSTATIC POINT

$g(\omega)$ model.



ISOSTATIC POINT

$g(\omega)$ Wyart et al.



ISOSTATIC POINT

M

Number of contacts