Mesoscopic Length Scale Controls the Rheology of Dense Suspensions

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From the flow properties of dense granular suspensions on an inclined plane, we identify a mesoscopic length scale strongly increasing with volume fraction. When the flowing layer height is larger than this length scale, a diverging Newtonian viscosity is determined. However, when the flowing layer height drops below this scale, we evidence a nonlocal effective viscosity, decreasing as a power law of the flow height. We establish a scaling relation between this mesoscopic length scale and the suspension viscosity. These results support recent theoretical and numerical results implying collective and clustered granular motion when the jamming point is approached from below.

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Particulate fluids such as dense granular suspensions show a diverging viscosity at increasing particle volume fraction [1,2]. This critical slowing down of the dynamics in response to shear corresponds to a well-known phenomenology shared by supercooled liquids entering the glassy regime [3]. This was recently associated with the so-called “jamming transition” [4] as it has been shown that, besides temperature, stress and density also play a determinant role in the passage from flow to arrest.

Within the class of soft-glassy materials [4] this is exemplified by dry granular flows [5], dense emulsions [6], foams [7], or dense colloidal suspensions [8]. Recent theories of jamming relate the vicinity of the transition to the emergence of large scale structures (see, for example, [9] and references therein). These are usually called dynamical heterogeneities and are thought to control the dynamical processes of flow arrest or flow onset. These structures were postulated or observed in many different statistical models and experiments [9–11]; however, the relation with the macroscopic rheological properties remains unclear and there is, so far, no fully accepted conceptual picture of flow properties around arrest.

In the domain of soft-glassy systems, dry granular materials have a special status since, contrarily to others, there is no relevant thermal or cohesion energy and the resistance to motion depends directly on the confining stress applied onto the solid matrix. Recently, the analogy between granular suspensions and dry granular matter has attracted a lot of interest [12,13] as it could provide a common vision of the dynamical processes at work around jamming. Recent numerical simulations of granular packing in the overdamped limit [14–16] have served as toy models to study the dynamics of nearly jammed granular suspensions interaction via lubrication forces. This connection, originally proposed by Ouaguenouni and Roux [17], was made explicit, and the emerging picture is that of a continuous transition with a diverging length scale appearing on both sizes of the jamming transition viewed as a rigidity transition. In this framework, the structural length scale is explicitly related to the existence of an isostatic point, i.e., a marginally stable mechanical structure that controls the mechanical rigidity of the granular packing.

Note however, that for suspensions, the presence of an incompressible surrounding fluid is likely to strongly constrain the restructuration possibilities of the granular packing. Thus, it could influence the reorganization processes close to jamming in a way that is somehow different from the simple picture revealed by granular packing dynamics.

Along those lines, we carry out a systematic exploration of this question by studying avalanche flows of isodense suspensions on an inclined plane. As it was already shown for granular materials [5], pastes [18], and more recently for foams [19] and dense suspensions [20], this setup is a valuable rheometer for complex fluids. The reason being that under gravity the inclined plane geometry fixes quite naturally an important rheological quantity: the ratio between shear stress and confining stress, namely, the “effective dynamical friction.”

The avalanche setup consists of a plane inclined at an angle $\theta$ with respect to the horizontal direction [see Fig. 1(a)]. The avalanche plane dimensions are $L = 1$ m in length and $W = 38$ cm in width. The suspension is initially poured in a reservoir connected to the avalanche plane by a gate that can be lifted as high as $e = 1$ cm such as to let the suspension flow on the avalanche track. A CCD camera observes the suspension surface at a distance $X = 10$ cm from the inlet. The observation field is 12 cm by 16 cm. The flowing layer height $h(t)$ is monitored using the trace of a laser sheet inclined at low angle and reflecting on the surface. From time to time, black powder tracers are sprinkled on the surface and, as they pass in the observation field, the surface velocity field is obtained by particle image velocimetry analysis. We then compute the mean velocity: $V(t)$ for a time resolution $\delta t = 5$ s corresponding to a height $h(t)$ [see Fig. 1(b)]. The suspensions are made of quasimonodisperse spherical polystyrene beads from Dynoseeds with diameter $d = 40 \pm 5 \mu$m. The particle density is $\rho = 1.05–1.06$ g · cm$^{-3}$. The inter-
stational fluid is a modified silicone oil (Shin Etsu SE KF-6011) \((\gamma_k = 116\, \text{mPa} \cdot \text{s} \text{ at } T = 20^\circ \text{C})\). This oil guarantees a close density matching \((\rho = 1.07 \, \text{g} \cdot \text{cm}^{-3})\) with the suspended particles. Volume fraction is varied from \(\phi = 35\%\) up to \(\phi = 57.8\%\). Special care has been taken when preparing the suspensions to control the volume fraction and its homogeneity. The volume fraction accuracy is then \(\Delta \phi = 1\%\). Before each experiment, the material is well mixed. The mixing plays the role of a preshear and reduces the effect of the slight density mismatch that can lead to creaming of the suspension stored at rest for a long time. In this way, we obtain reproducible initial states and experimental results up to large concentrations. For a given experiment, at a fixed inclination and volume fraction, the flow heights \(h(t)\) and the surface velocities \(V_x(t)\) are monitored locally. Because of the reservoir discharge, the input suspension flux decreases with time. However, in spite of the local evolution of height and velocity, we verified that time and angular variations, as well as inertia, could be neglected in the momentum equation balance, and thus we are in the standard condition of homogeneous and quasistationary flow (see discussion in Ref. [20]). For a stationary and homogeneous flow of height \(h\) along the flow direction \(x\) (\(y\) being the direction perpendicular), momentum balance gives that the shearing stress is \(\sigma_{xy} = \rho g \sin(\theta)(h - y)\) and the perpendicular confining stress is: \(\sigma_{yy} = \rho g \cos(\theta)(h - y)\). Here, \(\rho\) is the material density and \(g\) the gravity. Therefore, the working conditions correspond, everywhere in the bulk, to an effective friction coefficient \(\mu = |\sigma_{xy}/\sigma_{yy}|\) simply determined by the inclination angle \((\mu = \tan(\theta))\).

For the present study, experiments were performed for packing fractions \(47.4\% \leq \phi \leq 57.8\%\) and angles \(5^\circ \leq \theta \leq 39^\circ\). We already reported [20] for large \(h\) values a regime where \(V_x \propto h^2\) [see Figure 1(b), regime I]. This scaling is characteristic of a viscous flow, and we tested that the inclined plane can be used for a reliable and practical measurement of the viscosity. The viscosity extracted diverges in the vicinity of the jamming packing fraction [see Fig. 1(c)]. However, we observe in Fig. 1(b) that this regime breaks down at a certain height below which one observes a new regime characterized by an effective power-law scaling: \(V_x \propto h^\beta\) with \(\beta < 2\). For thinner flowing layers, one observes an arrest of the flow and the development of erosive structures as already noticed by Daerr et al. [21] and Timberlake et al. [22]. This height is about 10 grain sizes and is marginally accessible at our experimental precision; in the following, we do not discuss this regime.

We display in Fig. 2(a) the surface velocities for the second regime versus the flow height. We now demonstrate that all data can be rescaled within a common flow rule. For a given value (arbitrarily chosen) of \(V_0 = V_x/\sin(\theta) = 4.0 \times 10^{-3} \, \text{m/s}\), we associate to each flow curve, i.e., for each experiment, a length \(l\) such that a data collapse can be made onto a single curve: \(V_x/\sin(\theta) = V_0(h/l)^\beta\) with \(\beta = 1.32 \pm 0.02\) [see Fig. 2(b)] obtained from a power-law best fit. Note, however, that the precise value of \(\beta\) might depend slightly on the exact range of heights used to perform the fit. To give a visual impression of this uncertainty, we also display the best fit curve on our data forcing a power law with \(\beta = 1.5\). We also tested that changing the value of \(V_0\) does not change the subsequent analysis. The identification of this new regime allows us to establish a general flow rule for the inclined plane rheology. We now present our data in the form of an effective viscosity \(\eta_{\text{eff}} = \frac{\rho g \sin(\theta)}{2V_0} h^2\).

Accounting for the scaling relation previously determined, we obtain for each experiment a scaling length \(\xi\) through the relation \(\eta_{\text{eff}} = \rho g h^{\beta-2}\beta\xi^\beta/2V_0 = \eta(\phi)(h/\xi)^{2-\beta}\).

We recall that \(\eta(\phi)\) is obtained from the viscous regime identified at large flowing layer height [20] [the corresponding viscosity values are displayed in Fig. 1(c)].
gravitational force. Hence, in Fig. 3(b), we display the length of macroscopic quantities left such as viscosity, density, and power-law fit there must be a relation between them, if no other scale such as particle stiffness is present, this scale is called mesoscopic. From a dimensional point of view, scaling explanations of granular flows are much larger than the grain size (as large as the volume fraction ρ). On the other hand, when the flowing layer height is below this mesoscopic scale, the apparent suspension kinematic viscosity η(φ)/ρ increases strongly when packing fraction approaches the jamming limit and in apparent direct relation with the suspension kinematic viscosity η(φ)/ρ. When the flowing layer height is below this mesoscopic scale, the apparent suspension rheology becomes nonlocal, i.e., the flow height appears in the effective viscosity expression (ηeff ∝ hα, with α = 0.68), and the effective viscosity is then reduced at smaller thicknesses. It is interesting at this point to discuss this fact in relation with similar behavior found for dense granular flows. For assemblies of dry grains flowing on a plane, experimental measurements and numerical simulations show that the friction coefficient μ depends on the local confining pressure P (here P(y) ∝ ρg(h − y)). Note that, according to the general relation existing between viscosity and confining

FIG. 2 (color online). (a) Flow curves Vs(h) in the pseudogranular regime (II), for different packing fraction φ and slope angles θ. (b) Collapse of the flow curves on the graph Vs(h)/sinθ as a function of the scaled height h/l(θ, φ) [see text for explanations of l(θ, φ)]. The solid gray (red) curve is the best power-law fit y = V0(h/l(θ, φ))β for all data points (β = 1.32 ± 0.01). The black curve corresponds to a best fit forcing the exponent β = 1.5.

Figure 3(a) shows that this effective viscosity can be written in the form ηeff = η(φ)F(X) with X = (h/ξ(φ))α, with α = 2 − β = 0.68. The function F(X) has two limits: when X ≫ 1, F(X) = 1, and when X ≪ 1, F(X) = X. Thus, the length scale ξ defines a crossover length below which the scaling regime [F(X) = X] starts. Several complementary tests were performed, changing the grain size (d = 80 μm) and the observation position on the incline (25 cm from the reservoir). It appears that the previous picture remains robust to all these changes. We could not detect any systematic variation of ξ with the inclination angle θ, and the length ξ seems to depend on the volume fraction φ only.

Note that the characteristic lengths ξ obtained from the flow curves are much larger than the grain size (as large as 250d for a suspension at φ = 0.58). This is why we call this scale “mesoscopic.” From a dimensional point of view, if no other scale such as particle stiffness is present, there must be a relation between ξ and the three other macroscopic quantities left such as viscosity, density, and gravity. Hence, in Fig. 3(b), we display the length ξ as a function of the only length scale that can be constructed:

\[ ξ(φ) = C \left( \frac{η(φ)}{ρ} \right)^{2/3} g^{-1/3}, \]  

and we obtain a linear relation with C = 0.217 ± 0.001. Error bars are essentially due to uncertainties in determining the viscosities. Therefore, for the range of packing fractions spanned (0.47 < φ < 0.58), we identify a mesoscopic length scale (20d < ξ < 250d) which increases strongly when packing fraction approaches the jamming limit and in apparent direct relation with the suspension kinematic viscosity η(φ)/ρ. When the flowing layer height is below this mesoscopic scale, the apparent suspension rheology becomes nonlocal, i.e., the flow height appears in the effective viscosity expression (ηeff ∝ hα, with α = 0.68), and the effective viscosity is then reduced at smaller thicknesses.
pressure $\eta = \mu P / \dot{\gamma}$, this would yield for a dry granular flow with no yield stress (i.e., a packing in isostatic conditions) an effective viscosity depending on the flowing layer height $\eta \propto h^{1/2}$. In granular materials, collective motions of surface grains with a mesoscopic correlation length for the velocity field were also reported [23,24]. Ertas and Halsey [25] had proposed for granular materials that correlations could be viewed as a “Prandtl mixing length” entering into the momentum transfer picture of the effective viscosity. A similar picture of “granular clusters” has been developed by Mills and Snabre [12] to account for the viscosity divergence at jamming. We propose that our measurements could reflect the presence of correlated granular motion in the flow which form dynamical clusters. When the system size is smaller than the correlation length, clusters are smaller, and thus the suspension flow is easier. Another important point concerns the relation between the mesoscopic length scale and the viscosity ($\xi \propto \eta^{2/3}$). For dimensional consistency, we introduced density and gravity acceleration in the relation. In recent numerical models on dense suspensions [14–16], a critical behavior was identified below the transition. In this framework, gravity naturally comes into play if the shearing stress becomes comparable to the effective grain elastic modulus. Actually, no such 3D simulations with Hertzian contact interactions are available so far and no direct comparison is possible. However, we may estimate for our polystyrene particles (stiffness $E_0 = 10^{10}$ Pa) the effective stiffness of the Hertzian contacts under a typical experimental stress to be $\approx 3 \times 10^7$ Pa. We found that the dimensionless parameter proposed by Olsson and Teitel [15] to account for the role of particle stiffness near jamming is in our case small, and this corresponds to a regime where elastic modulus dependence of the viscosity essentially scales out.

In conclusion, we studied the flow of dense granular suspensions, approaching the jamming transition, by the use of an original setup: an inclined plane. In this way we have shown for the first time that a mesoscopic length scale controls the rheology of these suspensions. This length scale is a function of the grain fraction only and increases strongly when approaching the maximum volume fraction. It is thus directly linked to the divergence of the viscosity. In the range of the concentrations explored, this experimental finding gives important insights into the critical character of the jamming transition from below the jamming point [14–16]. Our findings agree qualitatively with theoretical predictions that the collective character of par-

ticle motion near jamming is at the origin of the divergence of the viscosity [12,25]. Our clear experimental results characterizing the flow of dense suspension when approaching the jamming transition might help to establish a consensus in this field, where a large amount of theoretical work exists, but which is currently lacking experimental results.

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