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Shape and instability of free-falling liquid globules

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Abstract – The velocity of a falling raindrop depends on its size, and thus so does its shape. Here we describe the different simple shapes which model drops falling in air. While millimetric drops remain spherical, owing to the action of surface tension, drops larger than the capillary length get flattened, as sessile drops on solids. Air penetrates still larger globules, which are observed to be unstable. They inflate till they burst, generating myriads of fragments.

We all know that drops or bubbles tend to be spherical, owing to surface tension: a sphere minimizes the surface area of a given volume. Here we discuss if this property still holds for drops falling in air, as a function of the drop size. In the limit of large velocities, inertial friction can exceed surface forces, which leads to spectacular deformations of the fluid globules [1], in both the drop and bubble cases [2]. If we restrict to raindrops, which mainly form from the coalescence of smaller globules, it must first be emphasized that they are polydisperse, with a typical size of a few millimeters [3,4]. Most studies on their shapes focused on sizes below 6 mm in diameter, which seems to be the largest drops hitting the ground [5,6]. The terminal velocity of raindrops was measured, and shown to result from a balance between weight and inertia [1,7]. These objects are nearly spherical, except the largest ones (about 4 mm in diameter), which are slightly flattened [1,8,9]. We first recall the well-known shapes and velocities in these regimes [1], using simple scaling laws. We then focus on centimeter-size drops, which can be flat objects if not too large, and instable globules if larger, which eventually break them in smaller fragments [1,10,11]. This, literally, is an avatar: this Sanskrit word originally meant "descent", before getting its modern meaning of "transformation". We describe and model the kinetics of these avatars, on which very few data are available.

The experiment is straightforward. A funnel filled with water is placed at the top of a building (height of 20 m). A jet of 10 mm in diameter comes out, and falls down, transforming in drops of different sizes (between a fraction of a millimeter and one centimeter), owing to the Plateau-Rayleigh instability and to air friction. Along the fall, high-speed cameras are placed to take images of the falling drops. The rate of imaging is 1000 frames per second. The drops fall at a velocity up to 10 m/s, requiring very short exposure times. The shutter time is typically set at 1/8000 s (*i.e.* less than 1 mm of fall), and the size of the image is about 30 cm allowing us to follow not only the shape, but also its evolution over about 30 ms.

The globule is characterized by its height h and equatorial diameter D, after 8 to 12 m of fall, allowing drops to reach their final velocity. The results are displayed in fig. 1a, where the full symbols correspond to stationary shapes. For small sizes ($D < 6$ mm), h and D are equal: droplets are spherical. For larger globules, h gets smaller than D, indicating a flattening of the profile; the average value of h saturates around 6 mm, whatever the drop size. These different shapes are displayed in fig. 1b. Figure 1a also shows the existence of a few "very" large objects (empty symbols), even bigger than the (at most) centimeter-size drops released from the funnel. These globules are unstable, and inflate during the experiment, which will be described hereafter.

If a drop has reached its terminal velocity V, air friction balances the weight. The fall takes place at a high Reynolds number (10^2 to 10^4), so that air viscosity can be ignored; the force balance can be dimensionally written $\rho_a V^2 R^2 \sim \rho g R^3$, for a spherical drop of radius R and density ρ, and denoting ρ_a as the air density [1]. Hence the terminal velocity can be classically written:

$$V \sim (\rho g R/\rho_a)^{1/2}, \quad (1)$$

V is found to be a few meters per second for millimeter-size drops. For a drop starting from the rest, V is reached after a distance $\rho R/\rho_a$ (about 1 m) much smaller than the observation distance (about 10 m). If falling drops are spherical, air friction must be weaker than surface forces, which tend to preserve the cohesion of liquid. This can be

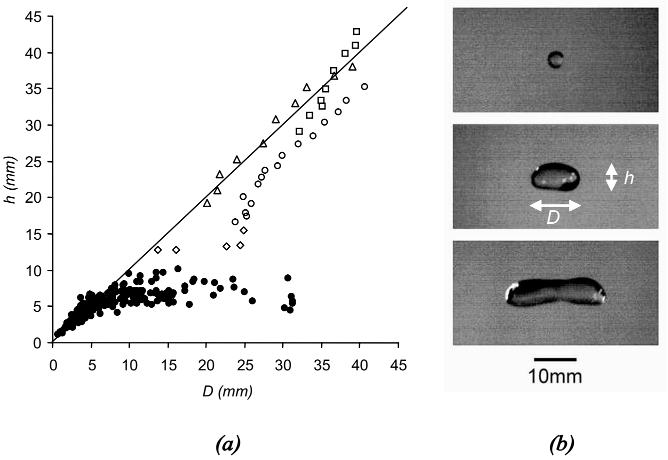

Fig. 1: Shape of a falling raindrop: the height h of the globule is plotted as a function of its equatorial diameter D, after 8 to 12 meters of free fall in air (a). Full symbols are for stationary states. Small drops are found to be spherical ($h = D$), while large ones tend to be flattened by the action of air (h becomes constant). The shapes are shown for three falling water drops of radius smaller, comparable, and larger than the capillary length (b). Empty symbols hold for larger globules; several data are displayed for each symbol, corresponding to different times. The data remain close to the line $h = D$, indicating an isotropic inflation of these very large objects (shown in fig. 4).

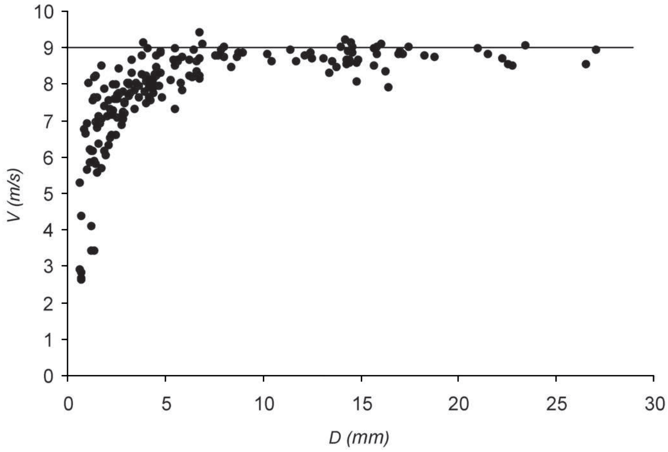

Fig. 2: Terminal velocity V of a water drop falling in air, as a function of its equatorial diameter D. For drops smaller than the capillary length, the velocity quickly increases with the size (eq. (1)). For larger drops, which are flattened (fig. 1a), the velocity saturates (eq. (4)).

written dimensionally: $\rho_a V^2 R^2 < \gamma R$, where γ is the liquid surface tension. Introducing the terminal velocity (eq. 1), the condition for sphericity simply reads [1]:

$$R < \kappa^{-1}, \qquad (2)$$

where κ^{-1} the capillary length $= (\gamma/\rho g)^{1/2}$, that is, 2.7 mm for water at 20 °C. As observed in fig. 1a, a falling raindrop of characteristic size smaller than the capillary length (3 mm for water) indeed remains quasi-spherical.

What happens if inequality (2) is not satisfied? Figure 1 indicates that the drops are stretched along the horizontal plane, forming liquid "coins". The falling velocity of such an object is once again given by balancing the drag force with the weight. This writes $\rho_a V^2 R^2 \sim \rho g R^2 h$, but does not lead directly to the fall velocity, h and R being unknown (and related to each other by the volume conservation $\Omega \sim R^2 h$). A second equation expresses the equilibrium of the distorted shape. In the reference frame of the drop, air stream imposes a Bernoulli depression, which tends to stretch the globule in the horizontal plane. This pressure applies on a surface area which scales as Rh, so that the "stretching" force can be written $\rho_a V^2 Rh$. On the other hand, a force γR due to surface tension tends to restore a globular shape. Balancing these two forces ($\rho_a V^2 Rh \sim \gamma R$) provides (together with the fall equation) the drop height:

$$h \sim \kappa^{-1} \qquad (3)$$

This thickness is independent of the drop size, as observed in fig. 1. The velocity of these stretched drops can be deduced from the same force balances. It reads

$$V \sim (\rho g \kappa^{-1}/\rho_a)^{1/2}, \qquad (4)$$

V is the limit of eq. (1) as the drop reaches its maximum size ($R = \kappa^{-1}$) in the spheroidal state (eq. (2)). For larger objects ($R > \kappa^{-1}$), V becomes independent of the drop size. This final velocity is reached after a distance of about $\rho\kappa^{-1}/\rho_a$, *i.e.* a few meters (the drops are observed after 10 m of fall). Figure 2 shows the results for the terminal velocity as a function of the drop equatorial diameter. The velocity of a small drop (eq. (2)) is observed to strongly depend on its size (as expected from eq. (1)), while V indeed saturates for globules larger than the capillary length. The maximum velocity is around 9 m/s, of the order of the 5 m/s predicted by eq. (4) without any numerical coefficients.

We now wonder what happens for still larger globules. Such objects are not easy to generate: as they form, drops have a size often limited by the capillary length. However, owing to the centimetric aperture of our funnel, we could produce a few large drops of about 1 cm. These drops are unstable. In fig. 1a, we report the height h and diameter D of three of them (empty symbols); several data are displayed, corresponding to different times. All the data remain close to the line $h = D$, indicating an isotropic inflation. The largest observed diameter (not reported in fig. 1a) was 65 mm. The shape of such a globule is shown in fig. 3: the drop, initially full of water, becomes hollow, consisting of a thin aqueous shell surrounding air, and bounded at the bottom by a rim. These bag-shaped drops have been reported in wind tunnels [12] and in shock tubes [13,14]. They were also found to show up when accelerating a liquid drop either by a constant body force, or impulsively, and yielding backward-facing bags [15,16]. However, to the best of our knowledge, there

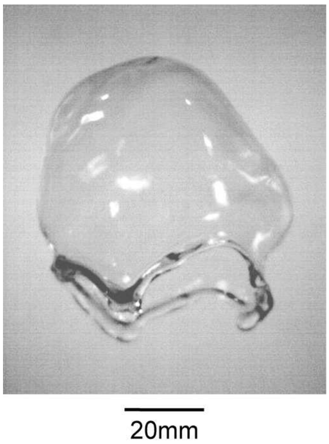

Fig. 3: Transient shape of a very large falling water drop (of initial diameter $D_o = 1.8 \pm 0.2\,\mathrm{cm}$, and falling velocity $V = 6.2 \pm 0.6\,\mathrm{m/s}$). The drop is a bag, consisting of a water shell surrounding air and bounded at the bottom by a rim. The bag quickly inflates as a function of time (fig. 4), allowing it to expand by a factor of order 3 (that is, 30 for the volume), before bursting.

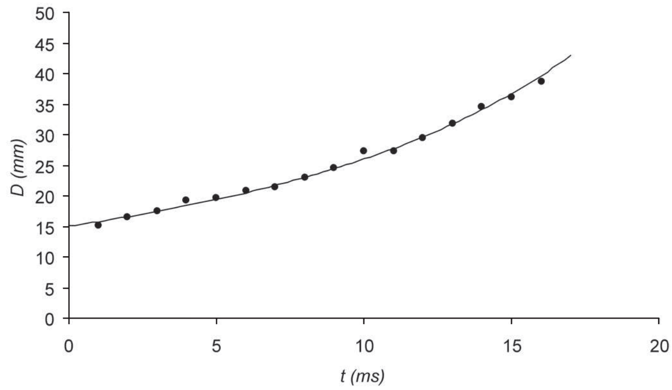

Fig. 4: Inflation of a bag-shaped drop: the bag diameter is plotted as a function of time. The non-linear growth is observed to be well fitted by eq. (5), drawn in full line with $\alpha = (6/\pi)^{1/2}$ (the value expected for a isotropic growth) and $m = 0.94\,\mathrm{g}$, in fair agreement with the mass unprecisely deduced from the bag-shaped drop ($m = 1.0 \pm 0.7\,\mathrm{g}$).

is no description of the kinetics of the instability for a drop moving at a constant velocity in air, and we intend here to describe the formation and death of these "bubbly" globules.

The formation of a bubbly state might be understood as follows: in the reference frame of the drop, air is coming at a velocity V, which tends to flatten the drop base with a force $\rho_a V^2 D^2$. For large drops, air should even induce a deformation of this bottom surface by a quantity δ (visible in the last photo in fig. 1). This implies a Laplace pressure of the order of $\gamma\delta/D^2$ (for $\delta \ll D$), and thus an elastic restoring force of the order of $\gamma\delta$. A balance between aerodynamic and surface forces yields: $\delta \sim \rho_a V^2 D^2/\gamma$. Our criterion for the invagination of the bottom surface, leading to the formation of a bubble, is $\delta > \kappa^{-1}$ (since we are in the regime where the thickness of the globule is κ^{-1}, as shown in eq. (3)). Introducing the law for the velocity of fall (eq. (4)), we immediately deduce that the bubbly state will form if $D > \kappa^{-1}$, where we expect a coefficient of the order of π, by analogy with the criterion of development of the Rayleigh-Taylor instability [1]: drops larger than 1 cm should transform in bags, as indeed observed experimentally.

At this large scale ($D \gg \kappa^{-1}$), surface tension cannot resist the deformation, and air penetrates. These "bags" should then inflate, as does the jacket of a biker on a highway. In the drop reference frame, the air inside the bag can be considered as still, at the atmospheric pressure. Outside the bag, air is moving at the falling velocity V, which yields a depression scaling as $\rho_a V^2$. The pressure difference between the outside and the inside of the bag makes it inflate like a balloon [17]. For a bag of diameter D and mass m, the balance between inertia and Bernoulli pressure can be written $m\partial^2 D/\partial t^2 \sim \rho_a V^2 D^2$. This equation is integrated, assuming a constant fall velocity (despite the change of size). Denoting D_o as the bag diameter at the origin of time, this provides an equation for the evolution of the bag diameter:

$$D \sim D_o/(1 - t/\tau^*)^2, \tag{5}$$

where $\tau^* = \alpha(m/\rho_a V^2 D_o)^{1/2} \sim (\rho/\rho_a)^{1/2} D_o/V$, of the order of 30 ms for centimetric globules. Equation (5) implies that the bag explodes in a finite time τ^*, corresponding to the divergence of D. The numerical coefficient α in eq. (5) can be calculated, assuming that the bag remains close to spherical (as suggested by the isotropic inflation reported in fig. 1a): we find $\alpha = (6/\pi)^{1/2}$.

It is implicitly assumed for deriving eq. (5) that the velocity remains constant during the inflation. This might contradict the force balance during the fall ($\rho_a V^2 D^2 \sim \rho g D_o^3$), which suggests that the drop should slow down as inflating. However, the characteristic time τ associated with changes in the velocity scales as $\rho D_o^3/\rho_a V D^2$, so that τ is larger than τ^* if $(\rho/\rho_a)^{1/2} D_o^2/D^2$ is larger than unity. This should generally be the case: typical inflation factors D/D_o are of the order of 3, making τ^* at least three times smaller than τ. On the other hand, for $\tau < \tau^*$ (which might occur for small density contrast between the two fluids), the inflation equation together with the force balance can be easily integrated, yielding a remarkable result: $D \sim gt^2$! The globule also inflates, but its size does not diverge anymore in a finite time.

We checked experimentally that the fall velocity of an inflating globule remains nearly constant. If the force balance $\rho_a V^2 D^2 \sim \rho g D_o^3$ were satisfied, V should drop by a factor of about 3, while the velocity was observed to decrease by only 20%. Figure 4 shows the time evolution of the size of a bag-shaped globule. Equation (5) is

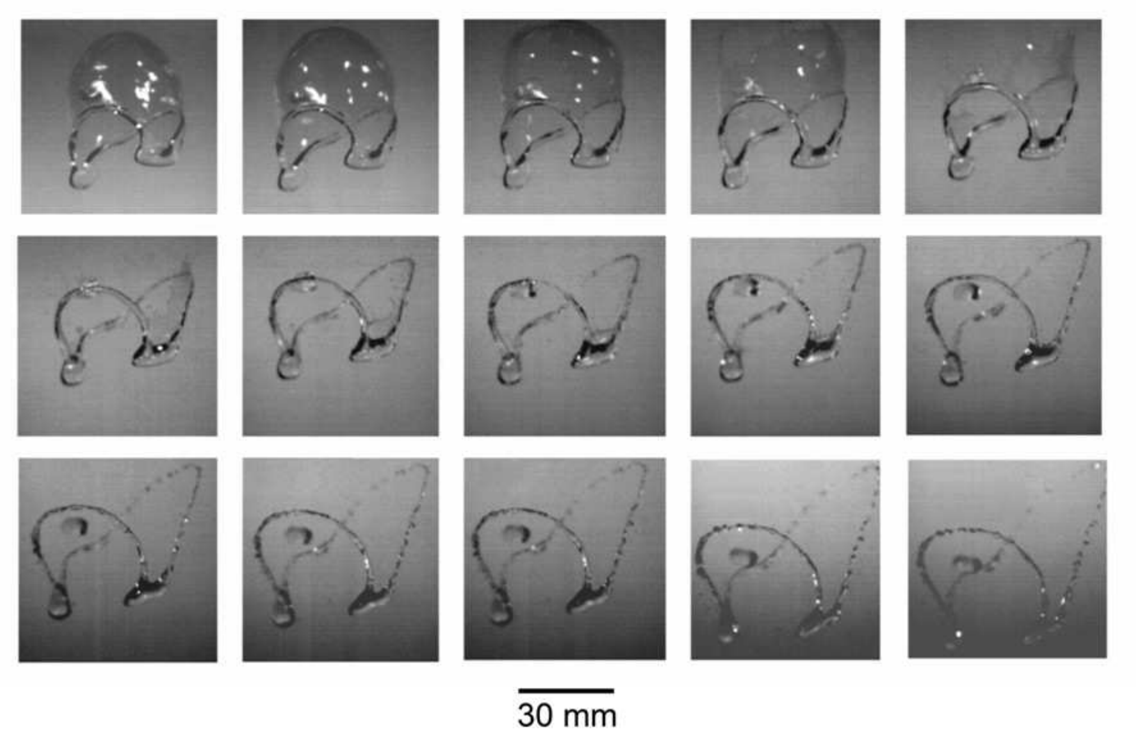

Fig. 5: Death of a bag-shaped drop. As the instability takes place, the shell becomes very thin so that, as a bubble, it bursts once a hole appears. The remaining liquid rim, at the bottom of the shape, later destabilizes because of Plateau-Rayleigh instability. This drop, of initial size 12 mm and falling velocity 7.5 m/s, eventually decays in multiple fragments of individual size smaller than the capillary length. The interval between two images is 1 ms.

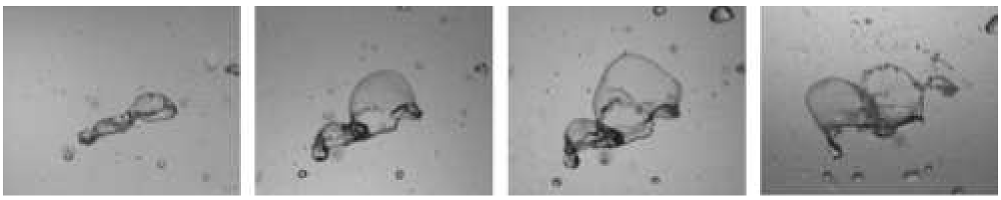

Fig. 6: Evolution of a very large globule of water (equatorial diameter larger than 3 cm). As expected from a Rayleigh-Taylor–like instability, several bags form. As single ones, they inflate and burst, leading to the destruction of the globule. Time is $t = 0$, 10, 13 and 20 ms.

superimposed to the data, and found to describe the non-linear growth of the globule, for $\alpha = (6/\pi)^{1/2}$ and $m = 0.9$ g (in agreement with the mass deduced from the globule shape, and corresponding to an initial diameter $D_o = 1.2 \pm 0.2$ cm). It is observed in particular that the speed of inflation ($\mathrm{d}D/\mathrm{d}t$) increases as a function of time, as predicted by eq. (5). This physically comes from the increase of surface area of the globule, which makes the action of the Bernoulli depression more and more efficient.

However, the globule diameter is not observed to diverge. As the bag grows, the shell gets thinner so that it eventually bursts: as shown in fig. 5, holes appear in the shell and expand. As proposed by Culick for the speed of retraction of a soap film, a balance between surface tension and inertia leads to a constant opening velocity $V = (2\gamma/\rho\varepsilon)^{1/2}$, denoting ε as the film thickness [18]. The holes are indeed observed to open at a constant velocity, of the order of 5 m/s, from which we deduce a shell thickness of about 10 μm. Later, the rim at the bottom also destabilizes, because of the Plateau-Rayleigh instability. Hence, if a large raindrop ever forms, it eventually decays in a spray of droplets, explaining why big liquid globules cannot reach the Earth, unlike hailstones.

Similar observations were made with liquid nitrogen. Because of a much smaller capillary length (about 1mm, instead of 3 mm for water), bag-shaped drops are much smaller than the aqueous ones. Starting from a drop of diameter around 5 mm, the diameter of the bag is about 14 mm before bursting. Multiplying these values by 3, one falls in the typical range of water bags. Our scenario also suggests that if still larger, a globule might generate several bags (as a large suspended drop generates several drips). We succeeded in making a globule of a few centimeters in diameter. As seen in fig. 6, it is indeed observed to have two bags, which grow and explode nearly at the same time. It would be worth making even larger globules (of 10 cm in diameter) to see if many bubbles develop at the same time. As a natural complement, it would be useful to conduct simulations of these falling globules, in order to confirm the different scalings (some parameters are nearly impossible to vary in a large manner, while arbitrarily small surface tensions can be used in simulations); this would also allow to specify the numerical coefficients in the different laws. Questions remain about the stretched drops. We are not aware of the existence of such puddles in rain. These drops might be sensitive to turbulence and collisions, which both tend to break them [4,13], but the stability of these puddles remains to be discussed.

It is worth noticing that the succession of shapes reported here is very similar to what can be observed by depositing liquids on a very hot plate, so that they float on their own vapour. There again, drops smaller than the capillary length are quasi-spherical; if larger, they make puddles whose thickness scales as κ^{-1}, independently of their size. This similarity may be understood very simply: for a falling drop in the terminal velocity regime, air friction balances the weight, so that the shape results from a balance between surface tension and weight, as for a static non-wetting drop. Surprisingly, the analogy persists for very large puddles: on a hot plate, a centimetric puddle gets bubbly, because the underlying vapour can raise at this scale [19]; as for a falling globule, the number of bubbly chimneys is fixed by the size of the drop: in both cases, the instability can thus be seen as a variety of the Rayleigh-Taylor instability.

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