Acoustic Waves with flow as a Gravity Analogue

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Experimental Black-Hole Evaporation?

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It is shown that the same arguments which lead to black-hole evaporation also predict that a thermal spectrum of sound waves should be given out from the sonic horizon in transsonic fluid flow.

$$\frac{1}{\sqrt{-g}}\sum_{ij}\frac{\partial}{\partial x^i}\sqrt{-g}g^{ij}\frac{\partial}{\partial x^j}\phi = 0.$$

$$g_{tt} = \rho^2(c^2 - |\boldsymbol{v}|^2),$$
$$g_{ti} = \rho^2 v^i,$$
$$g_{ij} = \rho^2(\delta_{ij} - v^i v^j),$$

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2D
$$\begin{cases} D_t^2\phi - \nabla^2\phi = 0 \\ \partial_y\phi = 0 \quad \text{for} \quad y = 0 \\ \partial_y\phi = -D_t(b(x)D_t\phi) \quad \text{for} \quad y = 1 \end{cases}$$

Integrate the 2D equation along y leads to the exact expression:

$$D_t^2\left(\int_0^1 \phi\, \mathrm{d}y\right) - \partial_x^2\left(\int_0^1 \phi\, \mathrm{d}y\right) - \partial_y\phi(x,1) = 0$$

+ BC $\quad \partial_y\phi = -D_t(b(x)D_t\phi) \quad \text{for} \quad y = 1$

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assuming $\partial_y \phi(x,1) = a_1 V(x) + a_2 F(x)$ where

$a_1,\, a_2 \in \mathbb{R}$ $\quad V(x) = \phi(x,1)$ $\quad F(x) = \int_0^1 \phi \, \mathrm{d}y$

$$\begin{cases} D_t^2 F - \partial_x^2 F = a_1 V + a_2 F \\ D_t(\, b\, D_t V) = -(a_1 V + a_2 F) \end{cases}$$

for a parabolic approximation

$$\phi = C_1 + C_2 y^2$$
$$a_1 = -a_2 = 3$$

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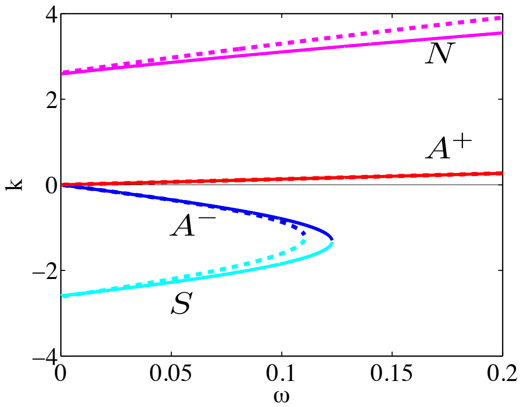
dispersion relation:

$$\left(\Omega^2 - k^2 + a_2\right)\left(b\,\Omega^2 - a_1\right) + a_1 a_2 = 0$$

Comparison
1D-2D model



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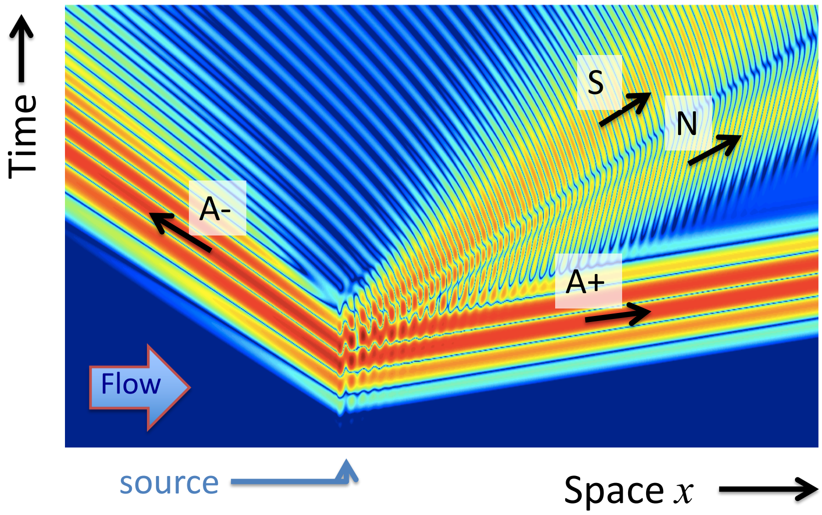

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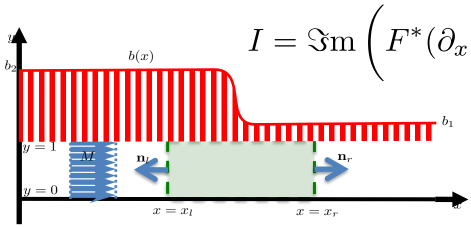
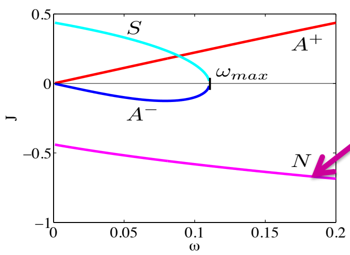


$$I = \Im\mathrm{m}\left(F^*(\partial_x F - M D_\omega F) + \frac{a_1}{a_2} M\, bV^* D_\omega V\right)$$

is conserved along x

$$I_m = (k_m(1-M^2) + M\omega)|F_m|^2 + M\, b(\omega - k_m M)|V_m|^2$$

(with $a_1 = -a_2$)



Negative Energy Wave

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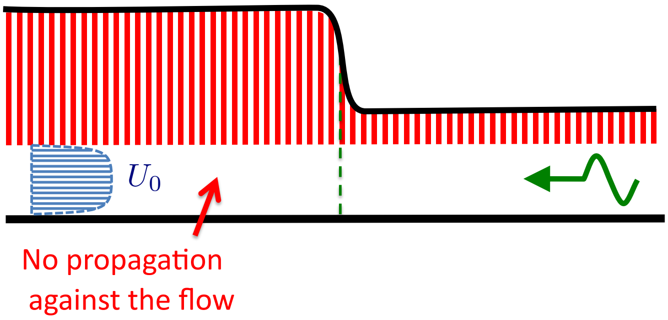
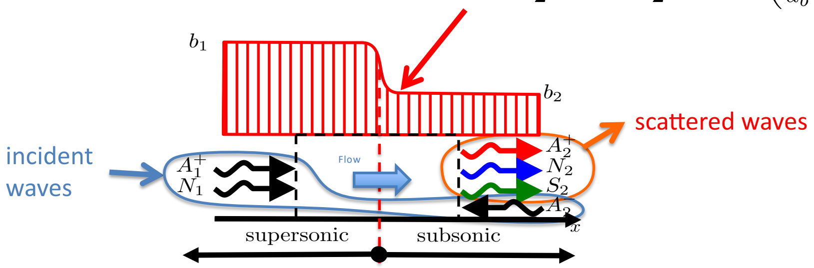
Scattering problem



White hole
(Deaf hole)

$$c_S(b_1) = \frac{1}{\sqrt{1+b_1}} < M < \frac{1}{\sqrt{1+b_2}} = c_S(b_2)$$

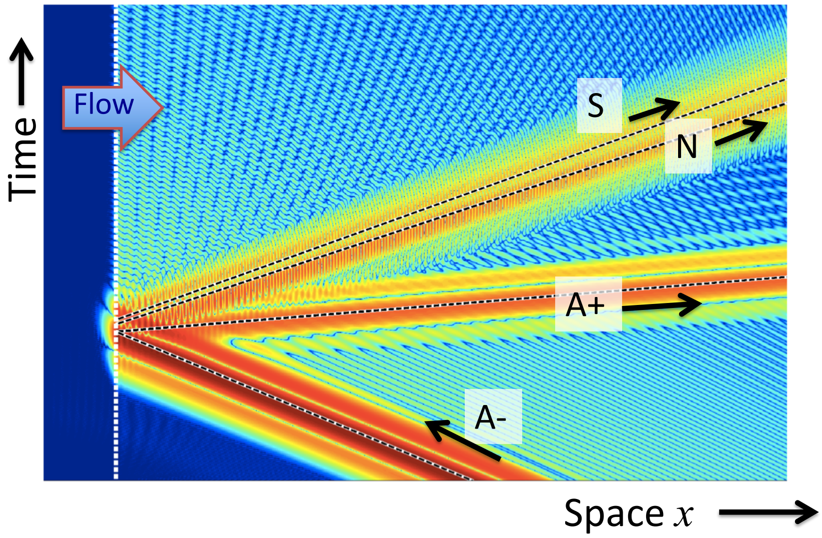
$$b(x) = \frac{b_1+b_2}{2} + \frac{b_2-b_1}{2}\tanh\left(\frac{x}{d_b}\right)$$



$$\begin{pmatrix} a_2^+ \\ n_2 \\ s_2 \end{pmatrix} = \begin{bmatrix} \tilde{A}_\omega & b_\omega & A_\omega \\ \tilde{\beta}_\omega & a_\omega & \beta_\omega \\ \tilde{\alpha}_\omega & \tilde{a}_\omega & \alpha_\omega \end{bmatrix} \begin{pmatrix} a_1^+ \\ n_1 \\ a_2^- \end{pmatrix}$$

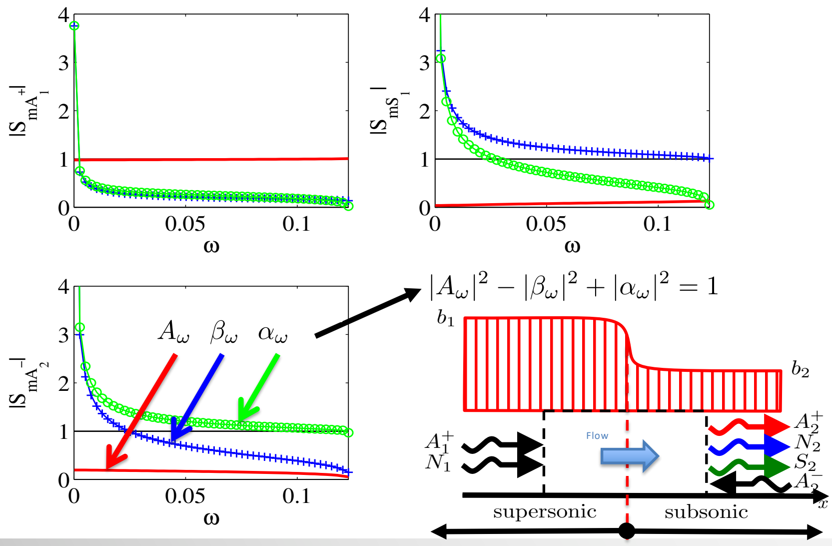
White hole

$$|A_\omega|^2 - |\beta_\omega|^2 + |\alpha_\omega|^2 = 1$$



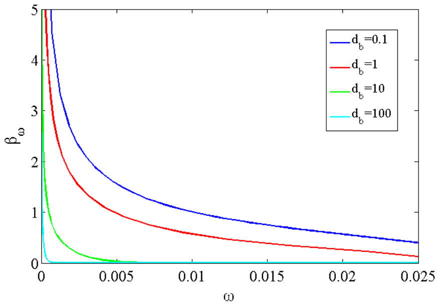
White hole

—: A_2^+, +++: N_2 and ooo: S_2

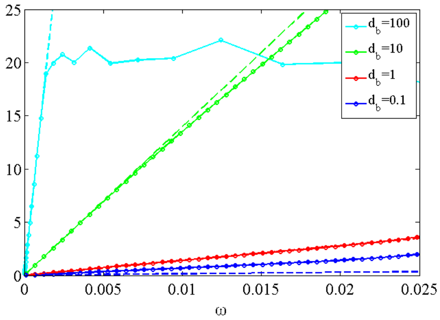


White hole

$$b(x) = \frac{b_1 + b_2}{2} + \frac{b_2 - b_1}{2} \tanh\left(\frac{x}{d_b}\right)$$



$$\log\left(\frac{1}{|\beta_\omega|^2} + 1\right)$$



Planck law:

$$|\beta_\omega|^2 = \frac{1}{e^{\omega/T} - 1}$$

White hole

$$T_H = \frac{1}{2\pi} \partial_x c_S|_{c_S = M} = -\frac{M^3}{4\pi} \partial_x b|_{c_S = M} = \frac{M^3(b1 - b2)}{8\pi d_b}$$