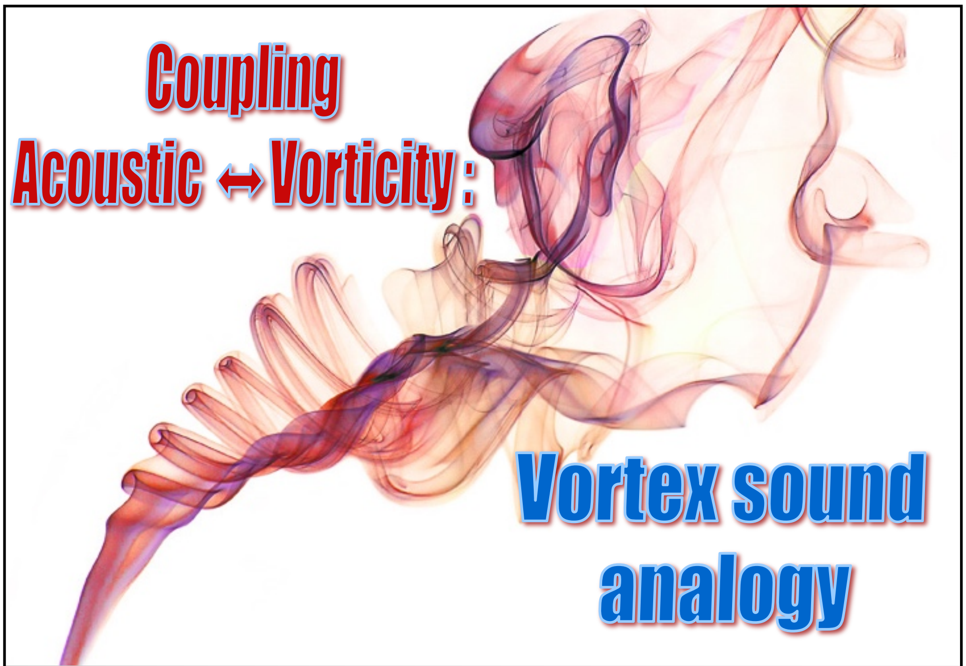


Vortex sound analogy

(Powell 1964, Howe 1975, Möhring 1979, Doak 1995)

For a perfect fluid

Continuity equation $\frac{D\rho}{Dt} = -\rho \vec{\nabla} \vec{u}$

Euler equation in Crocco's form $\frac{\partial \vec{u}}{\partial t} + \vec{\nabla} B = -\vec{\omega} \times \vec{u}$ where $B = \int \frac{dp}{\rho} + \frac{u^2}{2}$ is the stagnation enthalpy

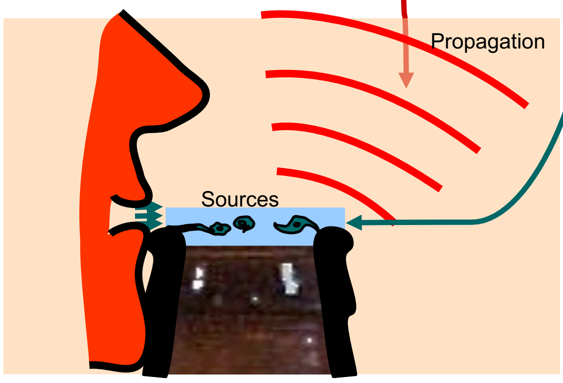
Acoustical Analogy for stagnation enthalpy (Powell-Howe analogy)

$$\left(\frac{D}{Dt} \left(\frac{1}{c_0^2} \frac{D}{Dt} \right) - \frac{1}{\rho} \vec{\nabla} \left(\rho \vec{\nabla} \right) \right) B = \frac{1}{\rho} \vec{\nabla} \left(\rho \, \vec{\omega} \times \vec{u} \right)$$

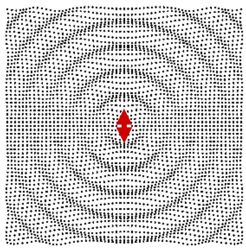
Vortex sound analogy

For low Mach number, at the order $\mathcal{O}(M)$
Powell's simplification

$$\underbrace{\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p}_{} = \underbrace{\vec{\nabla}\left(\rho_0\,\vec{\omega}\times\vec{u}\right)}_{}$$



Vortex sound analogy



For Powell's simplification

$$\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \vec{\nabla}\underbrace{\left(\rho_0\,\vec{\omega}\times\vec{u}\right)}_{-\vec{f}_c}$$

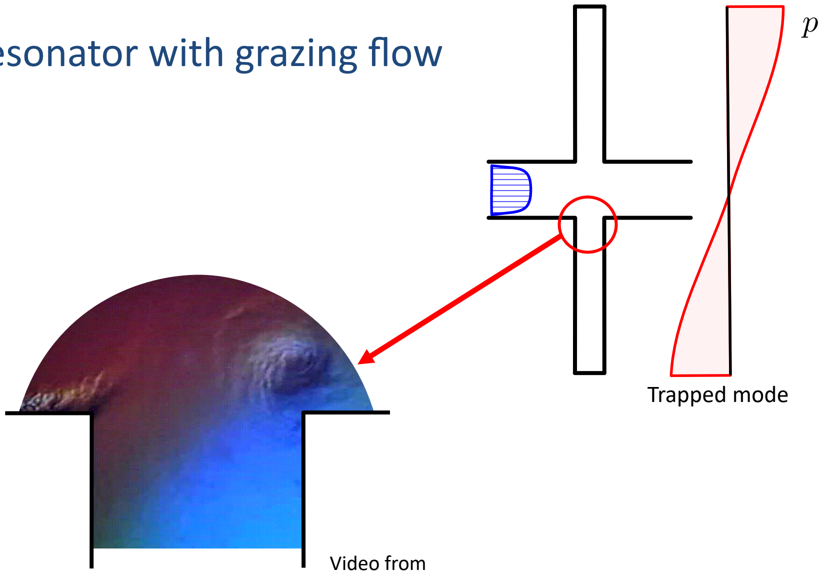
Coriolis force

It acts as a dipole

For a periodic acoustical field, the acoustic power is given by

$$\mathcal{P} = \int_\Omega \left\langle \vec{f}_c\,.\vec{u}_a \right\rangle \mathrm{d}V = -\int_\Omega \rho_0\left\langle (\vec{\omega}\times\vec{u}).\vec{u}_a\right\rangle \mathrm{d}V$$

Resonator with grazing flow



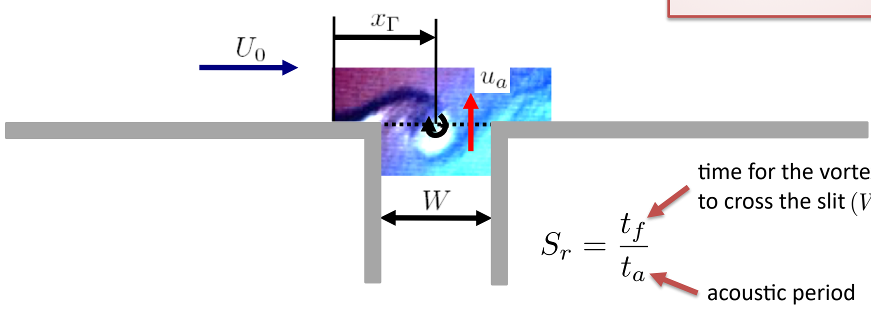
Resonator with grazing flow

Concentrated vortex model hypothesis :

- A vortex is created as soon as u_a becomes negative
- The vortex move with a constant velocity $U_\Gamma = 0.4\,U_0$
- The vorticity increases linearly

Strouhal number

$$S_r = \frac{fW}{U}$$

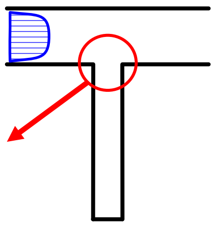


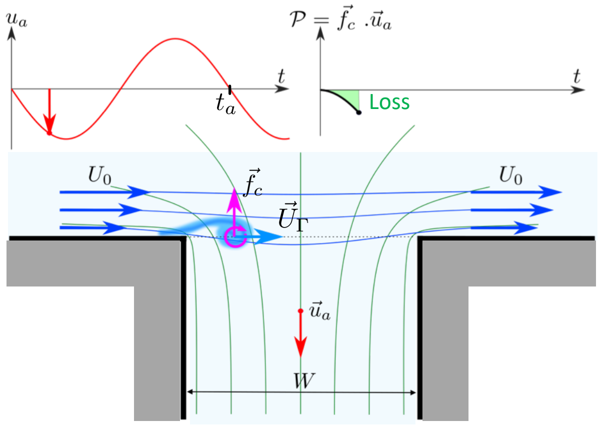
$$S_r = \frac{t_f}{t_a}$$

t_f: time for the vortex to cross the slit (W)

t_a: acoustic period

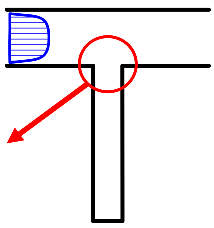
Resonator with grazing flow

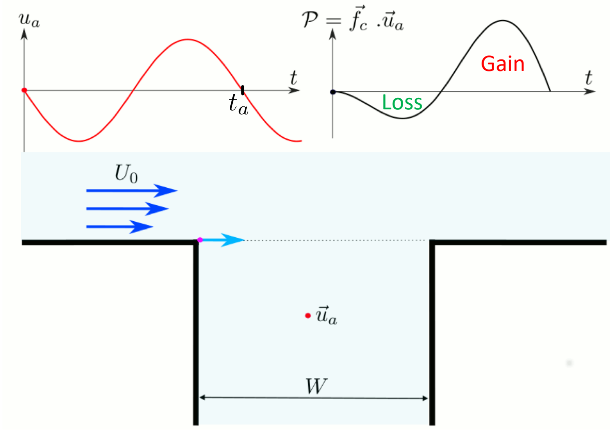




$$\vec{f}_c = -\rho_0 \, \vec{\omega} \times \vec{u}$$

Resonator with grazing flow

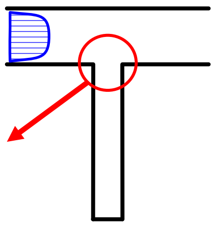


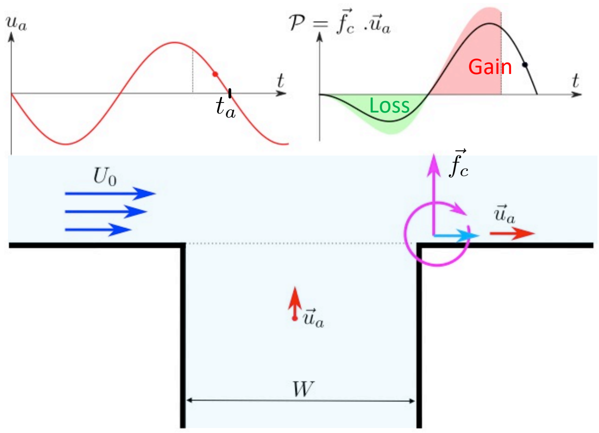


$$t_f = t_a$$

$$S_\Gamma = \frac{fW}{U_\Gamma} = 1$$

Resonator with grazing flow



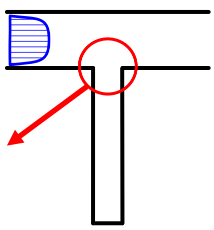


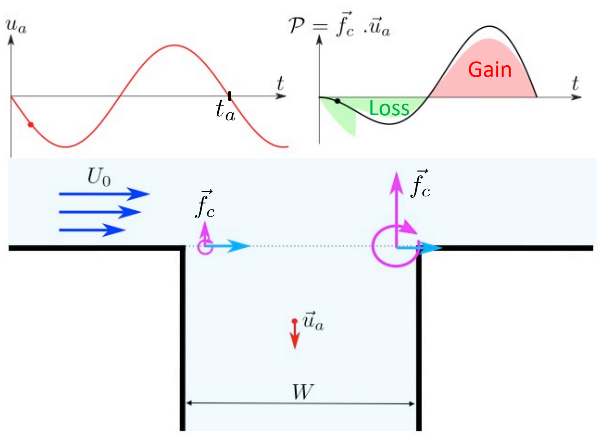
speedy vortex

$$t_f < t_a$$

$$S_\Gamma = \frac{fW}{U_\Gamma} < 1$$

Resonator with grazing flow



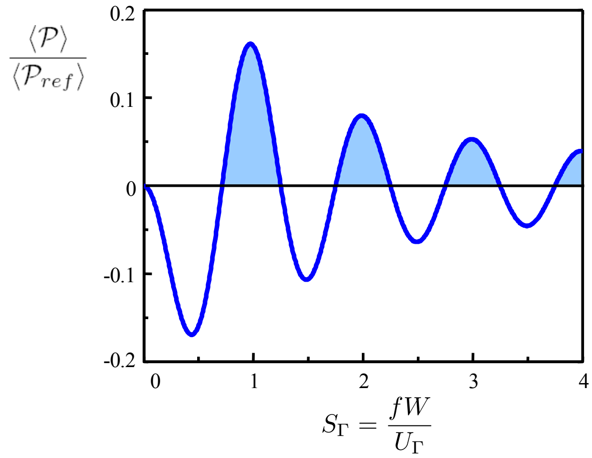


slow vortex

$$t_f > t_a$$

$$S_\Gamma = \frac{fW}{U_\Gamma} > 1$$

Resonator with grazing flow



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Y. Aurégan: *Guided Waves & Flow Interactions*

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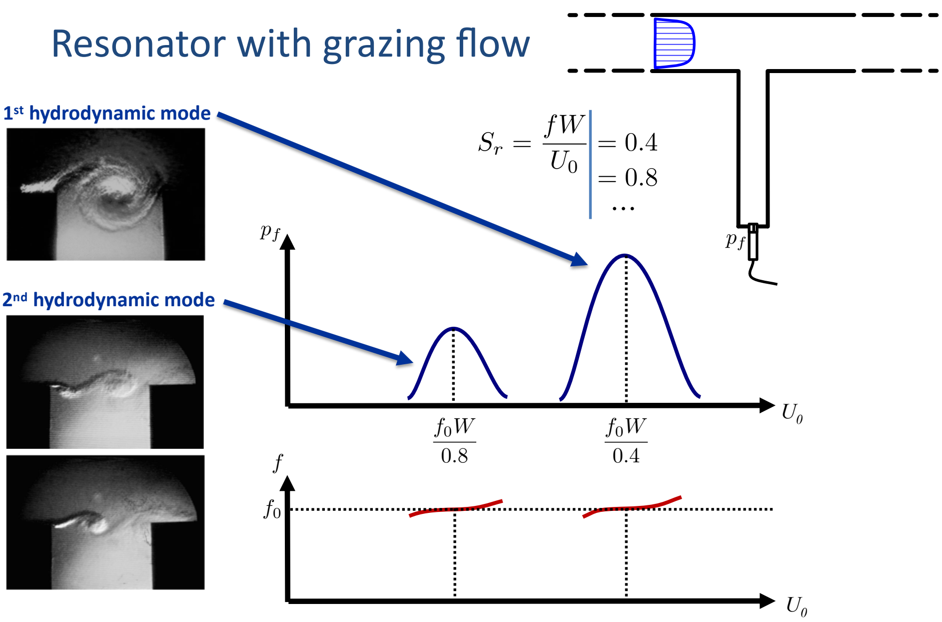
Resonator with grazing flow



Wave Propagation in Complex and Microstructured Media
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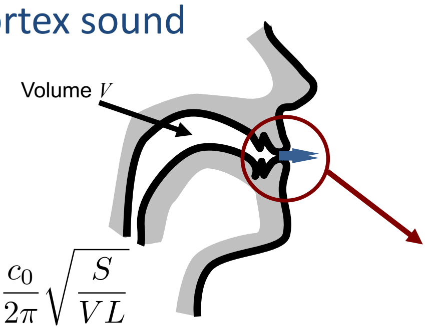
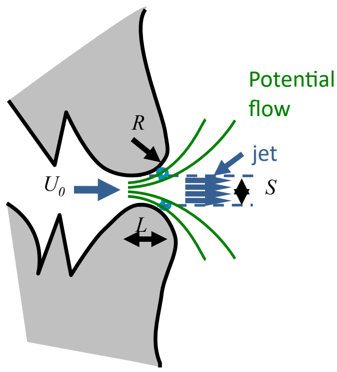
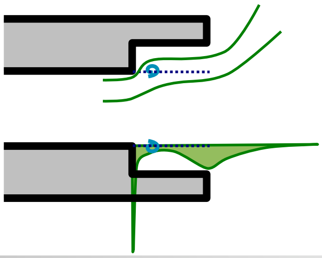
Vortex sound



$$f_0 = \frac{c_0}{2\pi}\sqrt{\frac{S}{VL}}$$

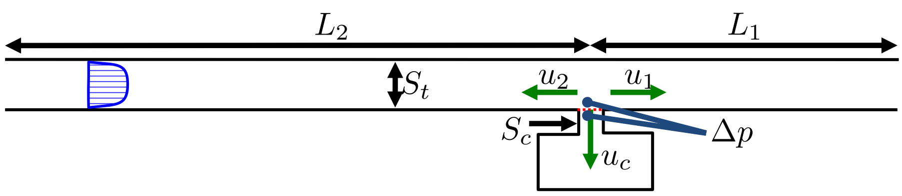




Whistling condition :

$$S_r = \frac{f_0 R}{U_0} = 0.3;\ 0.6;\ \dots$$

Vortex sound



$$Z_R = jkl_c + \frac{S_c}{jkV} \qquad k_R = \sqrt{\frac{S_c}{L_c V}}$$

$$\frac{1}{Z_1} = j\tan(kL_1)$$

$$\frac{1}{Z_2} = j\tan(kL_2)$$

$$\frac{u_c}{\Delta p} = \left(Z_R + \left(\frac{S_t}{S_c}\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right)\right)^{-1}\right)^{-1}$$