# Content

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# How to reach high contrasts ?

## Basic/implicit assumptions

Contrast of properties : O(1)    Inner Geometry : O(1)

## Possible candidates

Contrast $\neq$ O(1)

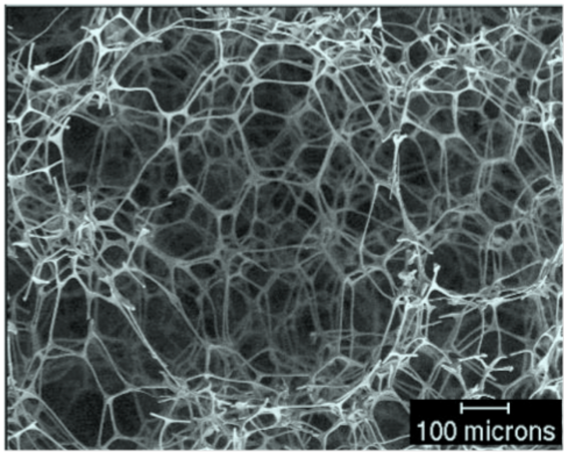
Geometry : O(1)

$\Rightarrow$ Highly contrasted continuous media

Contrast : O(1)

**Geometry $\neq$ O(1)**

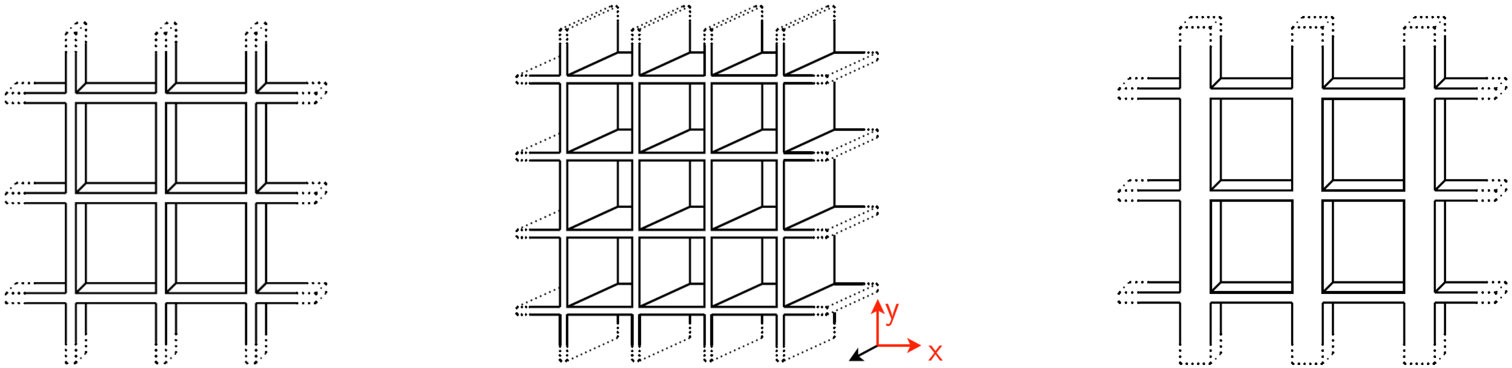
$\Rightarrow$ **Discrete Reticulated media**



# Specificity of reticulated systems

## Geometric contrast

Truss of beams or plates : Structural dynamics of buildings




Mechanical contrast : Bending versus Extension

## Consequences

Inner resonance

& Enriched local kinematics

***Chesnais, Boutin, Hans, JASA, 2012***

# Co-dynamic regime ?

## Local state

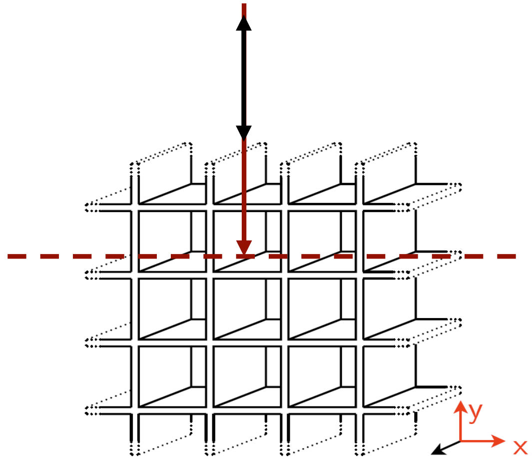

Compression $\quad N(u\,;\,l/\lambda_c) \quad \sqrt{\dfrac{\rho\,\omega^2}{E}} = \dfrac{2\pi}{\lambda_c}$

Bending $\quad T, M(v\,;\,\theta\,;\,l/\lambda_f) \quad \sqrt[4]{\dfrac{\rho A\,\omega^2}{EI}} = \dfrac{2\pi}{\lambda_f} \qquad \dfrac{\lambda_f}{\lambda_c} = O\left(\sqrt{\dfrac{a}{\lambda_c}}\right) << 1$

| Local **Quasi-statism** | Local **Dynamics** |
| --- | --- |
| $l << \lambda_f << \lambda_c$ | $\lambda_f \cong l << \lambda_c$<br>Inner resonance in bending |
| **Classical** Mechanics | **Non-conventional** Mechanics |

**Homogenization of discrete media ....**

# Non-conventional longitudinal vibrations




## Local Dynamics

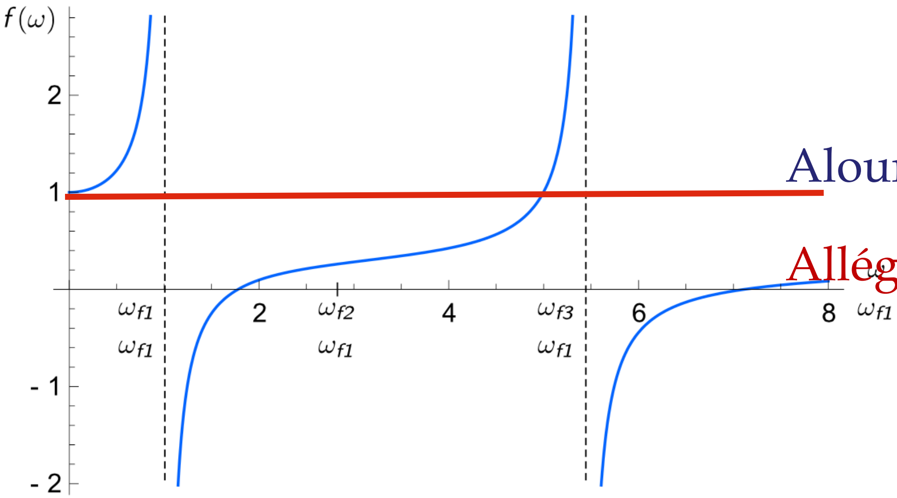
$\lambda_f \cong l << \lambda_c$

Bending inner Resonance

$$E_x \frac{d^2 U_x}{dx^2} + \rho\left[\frac{A_m}{\ell_p} + \frac{A_p}{\ell_m} f\left(\frac{\omega}{\omega_{f1}}\right)\right]\omega^2 U_x = 0$$

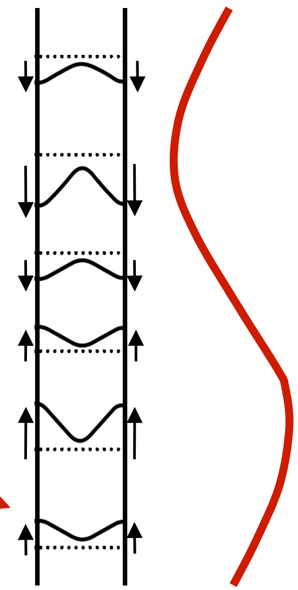
Apparent mass $\Lambda(\omega)$!

$$f\left(\frac{\omega}{\omega_{f1}}\right) = \frac{8}{3\pi\sqrt{\frac{\omega}{\omega_{f1}}}\left[\coth\left(\frac{3\pi}{4}\sqrt{\frac{\omega}{\omega_{f1}}}\right) + \cot\left(\frac{3\pi}{4}\sqrt{\frac{\omega}{\omega_{f1}}}\right)\right]}$$




Alourdissement

Allégement



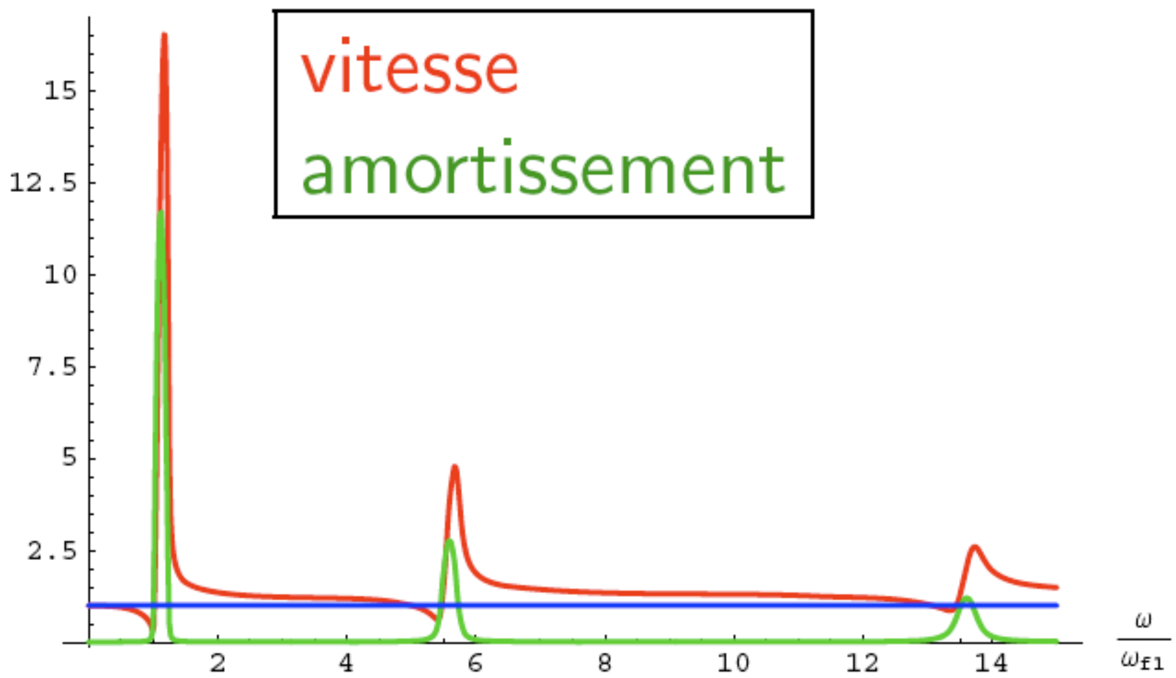
# Non-conventional wave features

## Dispersion and band-gap

$V^2(\omega) = E_x/\Lambda(\omega)$

Band gaps $\Lambda(\omega) = \infty$

Odd bending modes of Horizontal beams



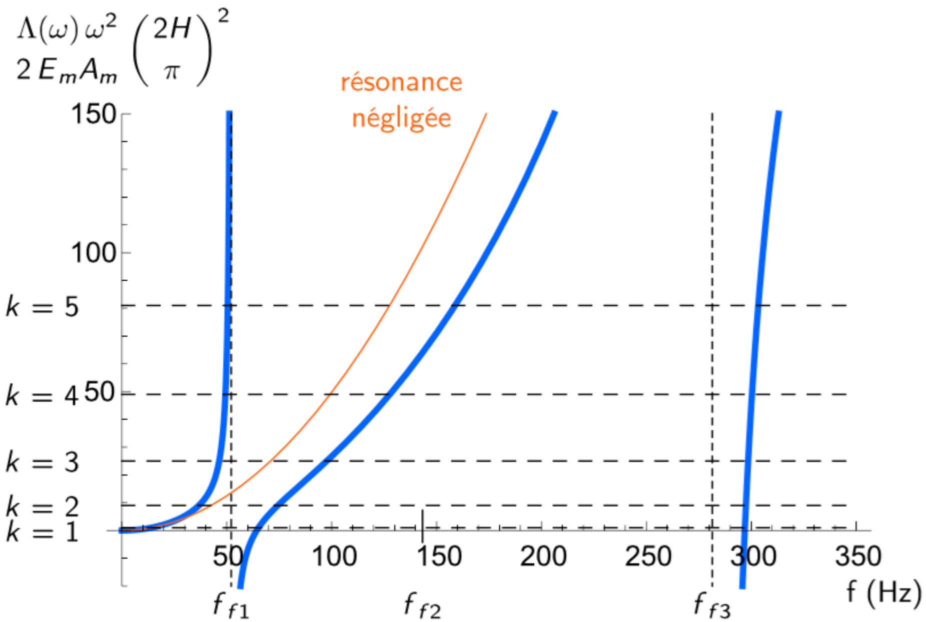
# Non-conventional Modal response

## Modal analysis

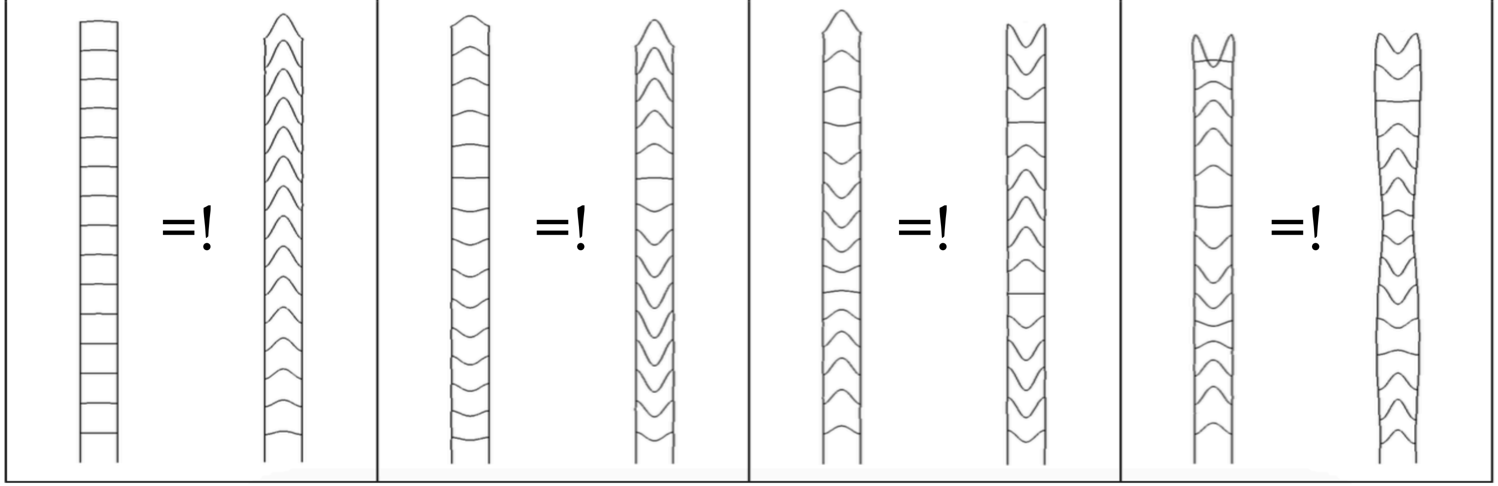
Clamped-free structure =

V(x) = Vsin($\alpha$ x)

$$\begin{cases} \alpha^2 = \dfrac{\Lambda(\omega)\,\omega^2}{2E_m A_m} \\ \cos(\alpha H) = 0 \end{cases} \quad \Rightarrow \quad \alpha_k H = \frac{(2k-1)\pi}{2}$$




| | $k = 1$ | | $k = 2$ | | $k = 3$ | | $k = 4$ | |
|---|---|---|---|---|---|---|---|---|
| H | 14,00 | 65,14 | 36,73 | 74,35 | 46,42 | 97,68 | 49,36 | 127,8 |
| C | 13,76 | 64,83 | 35,99 | 74,22 | 45,46 | 97,37 | 48,28 | 127,0 |

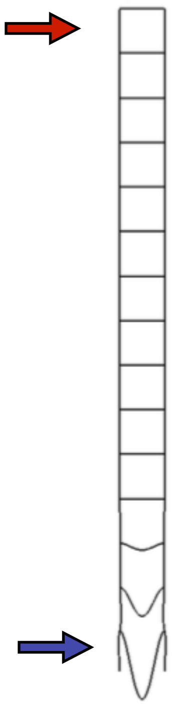


# Band–gap versus modes

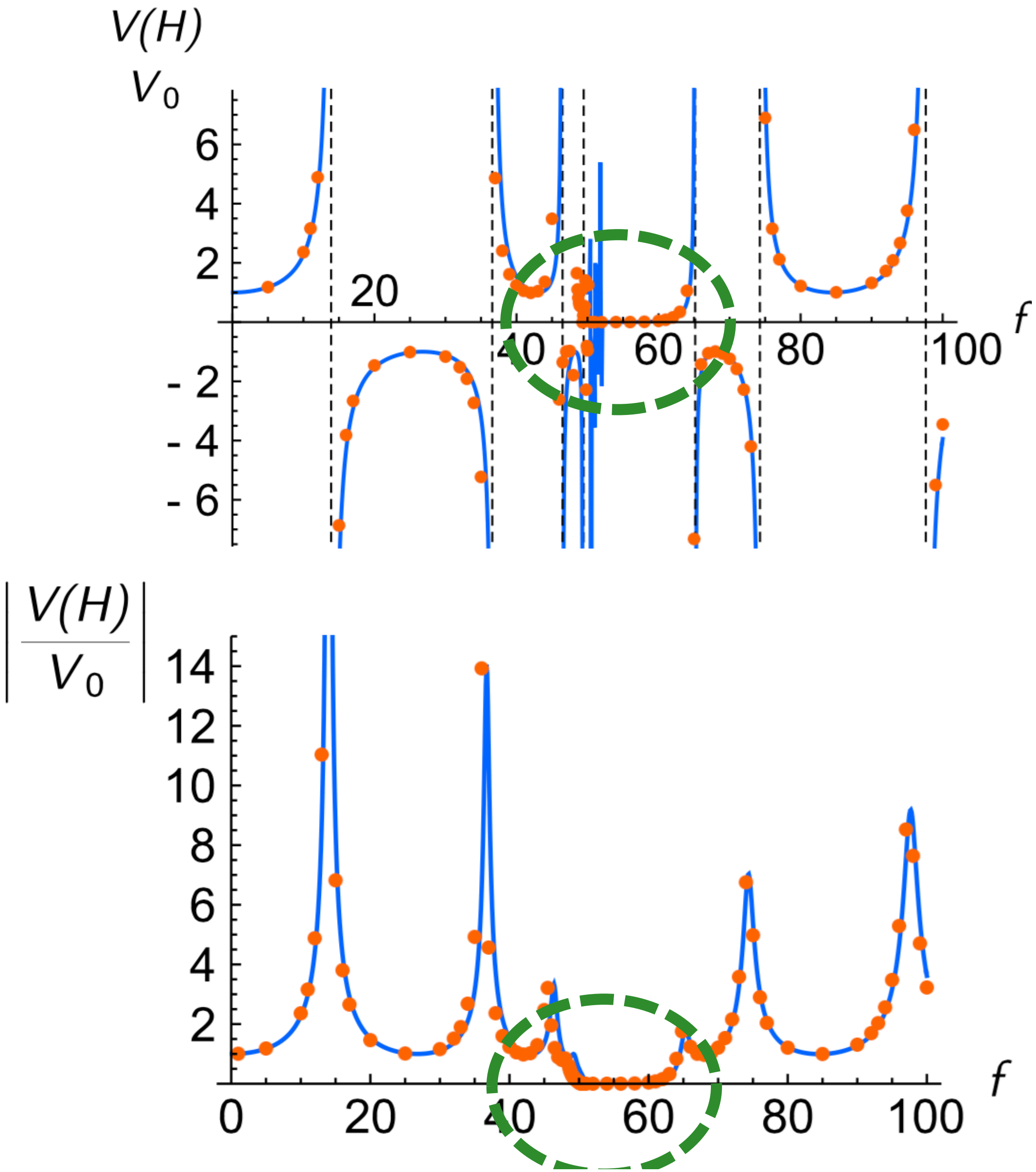
## Transfer function

Bandgaps : no transmisson

With 0 and 2% damping



Déformée
à 52 Hz



# Learnings

**Similar to composites** $\varepsilon = 2\pi\, l/\Lambda \approx \lambda_R / \Lambda_C$

Carrying constituent (connected) & Resonant constituent

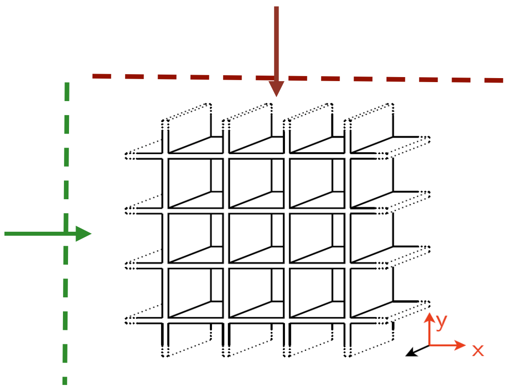
Forcing motion — Forced regime

High contrast

Atypical mass — High dispersion and band gaps

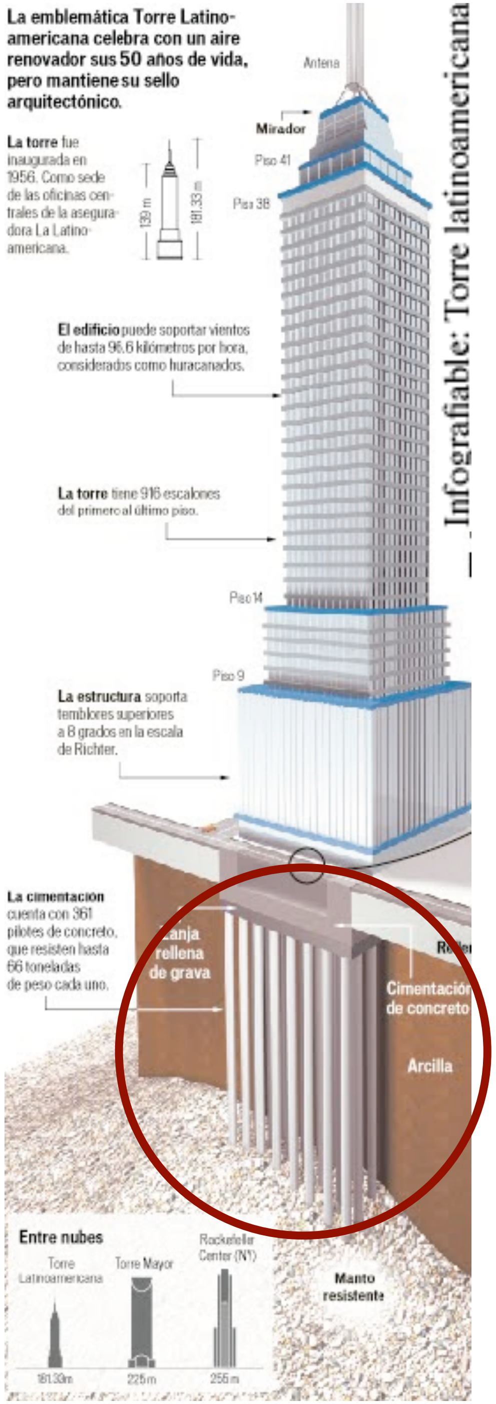
**But**

Versatile Resonant constituent — Depends on the direction of the wave

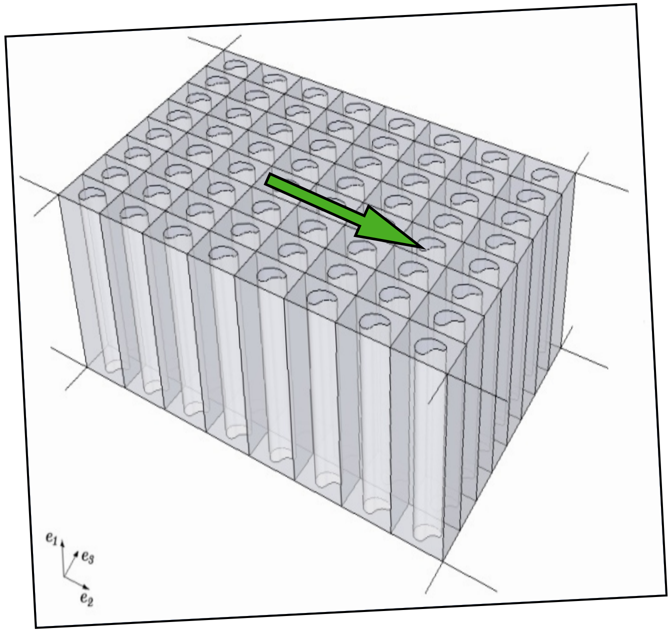


# Content

# Dynamics of soft soils reinforced by piles




# Transverse dynamics of reinforced composites

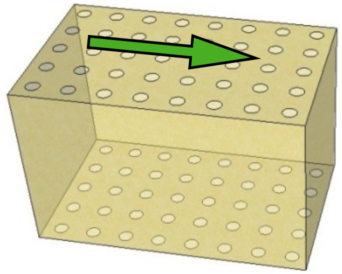



J. Soubestre, C. Boutin, J. Mech of Mat, 2012

# Physical analysis

## Scale separation

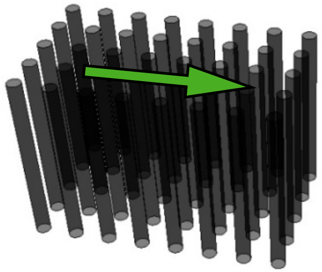
*l/H* = $\varepsilon$ << *1*    2D-In plane Periodicity



## Shear  : Soil

$$\tau = G\frac{\partial U}{\partial x} \qquad \frac{\partial \tau}{\partial x} = -\rho_s \frac{\partial^2 U}{\partial t^2}$$

## Bending : Pile network

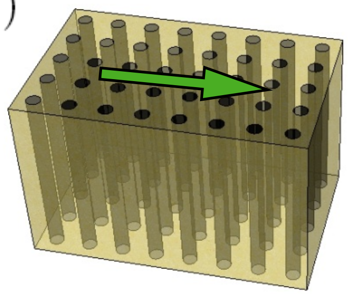


$$M = -EI\frac{\partial^2 U}{\partial x^2} \qquad T - \frac{\partial M}{\partial x} = 0$$

$$\frac{\partial T}{\partial x} = -\rho_p \frac{\partial^2 U}{\partial t^2}$$

## Reinforced media

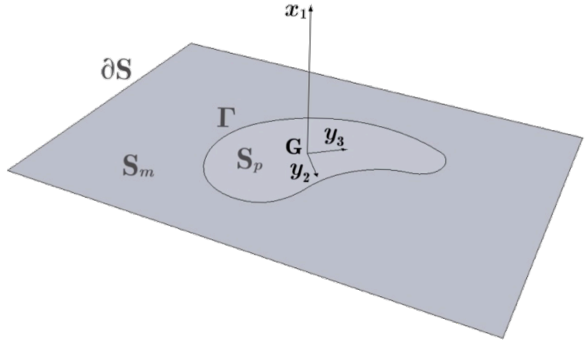
Inertia balanced by Soil & Piles   $O(EI\frac{\partial^3 U}{\partial x^3}) = O(GS\frac{\partial U}{\partial x})$



Shear / Bending Coupling   $EI/H^3 \cong GS/H$

$\Rightarrow$ $G \cong EI/SH^2 \cong E(l/H)^2 \cong E\varepsilon^2$

# Homogenization Process




## Formalism

Space variables $x_1$ ; $y_2 = \varepsilon x_2$; $y_3 = \varepsilon x_3$

Parameter $\mu\ \mu_m = \varepsilon^2 \mu_p$

Motions

Strain, Stress **Normal** **Out of plane** **In plane**

$$\underline{\underline{A}} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

## Expansions

In $\varepsilon^{2}$ $^{2}$powers $$^{q}\underline{u} = \sum_{i=0}^{\infty} \varepsilon^{2i} \left( {}^{q}u_{\alpha}^{2i} \underline{e}_{\alpha} + \varepsilon\, {}^{q}u_{1}^{2i+1} \underline{e}_{1} \right) \qquad q = m, p$$

## Balance equations

Jump of $\varepsilon$ between **In plane** and **Axial** balance

## Resolutions order by order

# Macroscopic transverse dynamics

## Harmonic behaviour of the Soil - Piles system

$$\underbrace{-E_p I_{p\alpha} \frac{d^4 U_\alpha}{dx_1^4}}_{\text{flexion}} + \underbrace{G\left(S_m + S_p\right)\frac{d^2 U_\alpha}{dx_1^2}}_{\text{cisaillement}} + \left(\rho_p S_p + \rho_m S_m\right)\omega^2 U_\alpha = 0$$

Analytical ! Involves multiple interactions

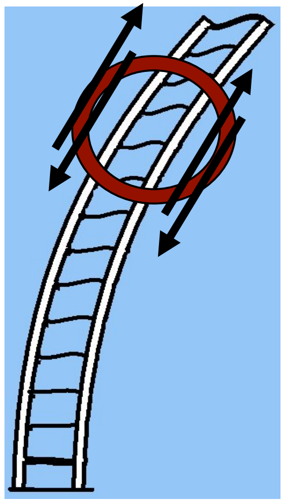
## Second gradient behaviour at leading order

Non local in space ≈ **Sandwich beam**

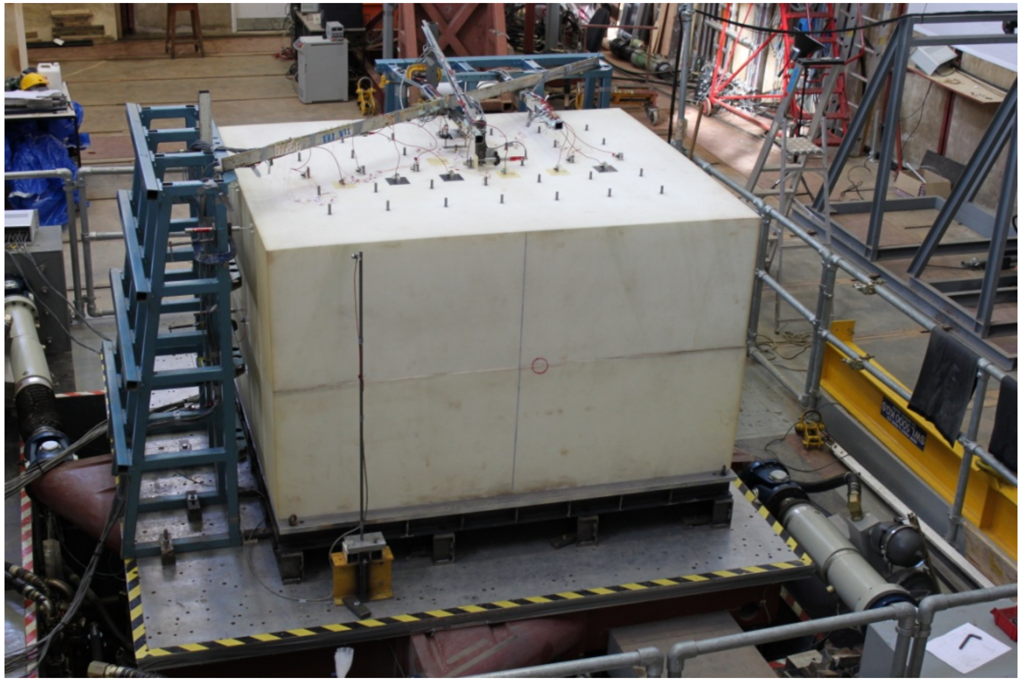
Rotation of beams but not Cosserat !

Pile motion = Soil motion

Pile rotation ⇨ Shear in soil



# Experiments



Shaking Table (Blade) - Bristol University

Series project (EU)

# Design of the specimen

## Conditions for full shear-bending coupling

$E_p I_p \, U^{(4)} + CS \, U^{(2)} = <\rho> \omega^2 \, U$ $\quad$ $C \approx \mu_m$ $\quad$ Specimen high = H

$E_p I_p / H^4 = \mu_m S / H^2 = <\rho> \omega^2$

## Sample

**Foam** matrix $\mu_m$ = 30 kPa

**Steel** hollow piles $\quad E_p$ = 21 Gpa

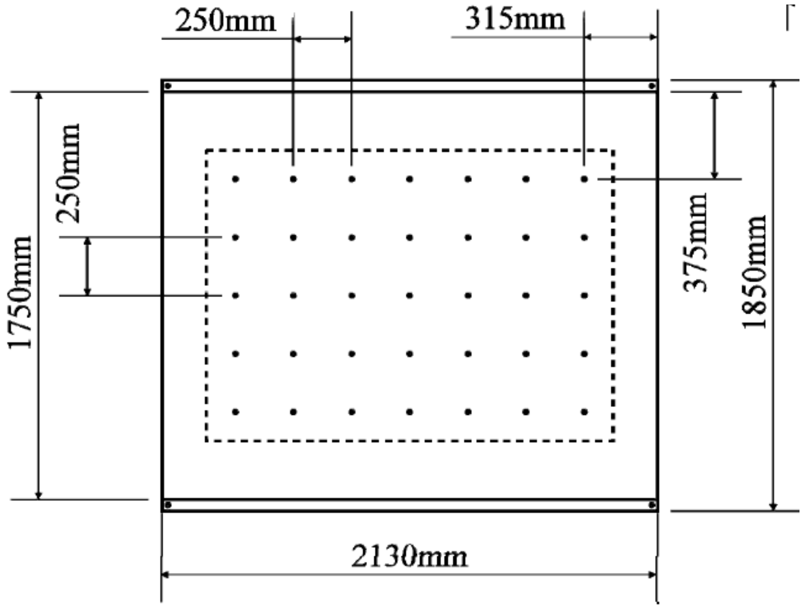
Dimensions $\quad$ H = 1.25m

*l* = 25 cm

Configurations $\quad$ 35, 17, 9, 0 piles

**Clamped - Free**

Hinge ; Sliding

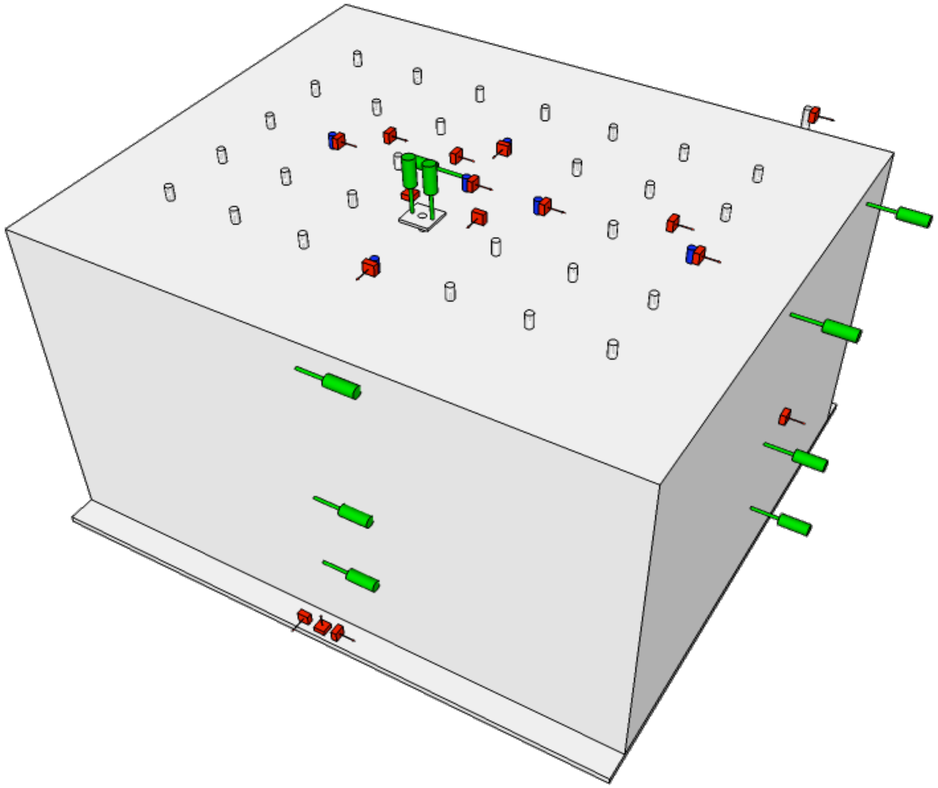


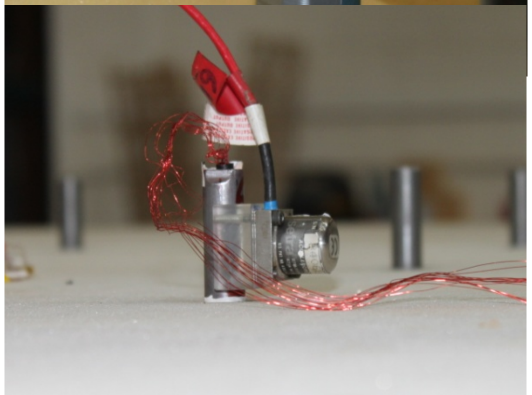
# Phenomena - Instrumentation

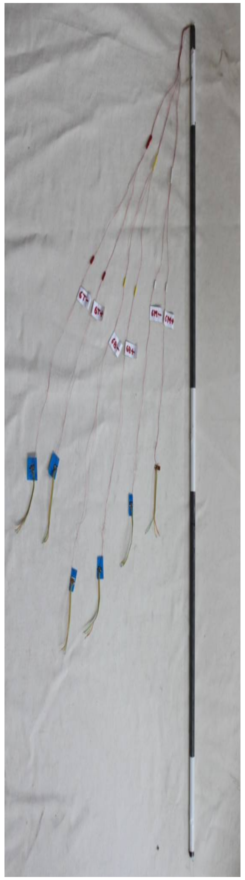
**Eigen frequency - Homogeneity** — 16 Accelerometers (pile & foam)

**Bending of piles** — 36 Gauges on 6 piles

**Mode shape** — 10 LVDT



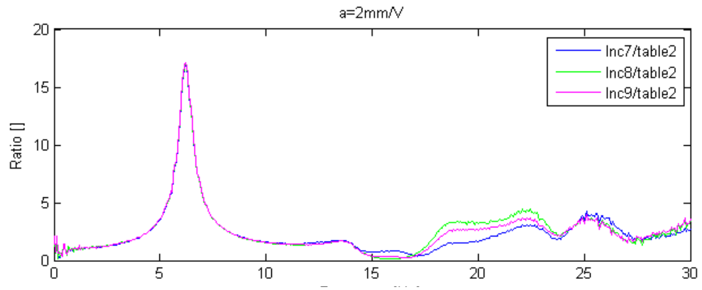




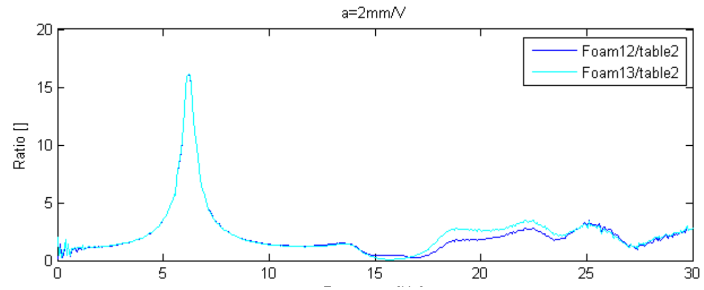
# White noise(s) Response

## Piles (3)

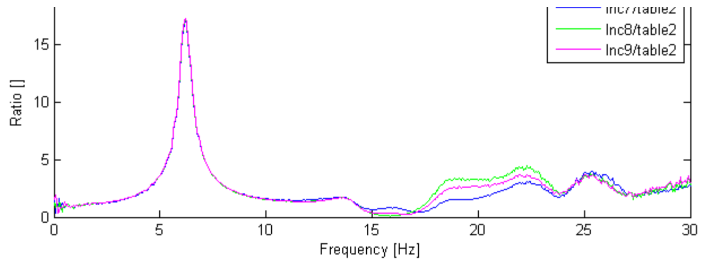
## Foam (2)



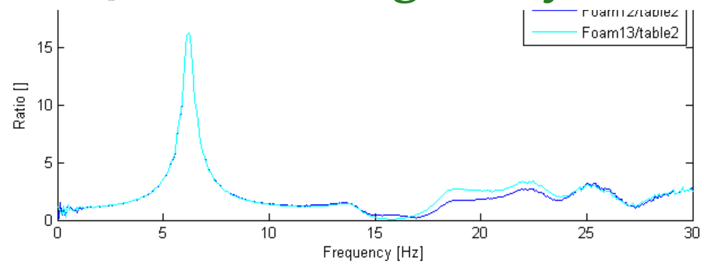

1

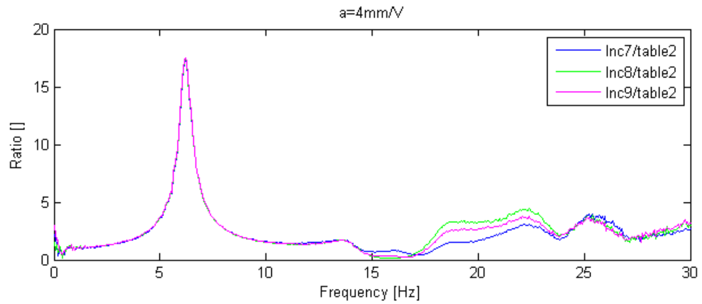



## Fundamental frequency - Foam / Pile homogeneity

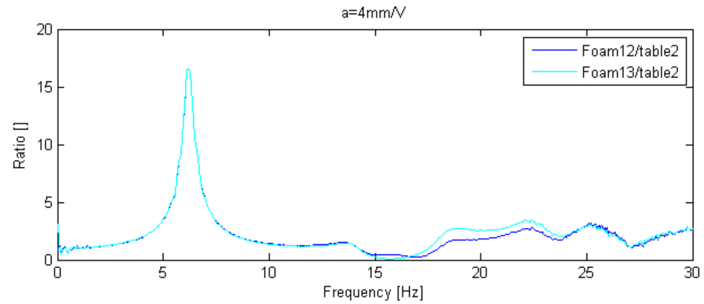



1.5




2




## Linearity - Repetability

# Eigen frequency vs Pile concentration

**Eigen frequency**

**Lower pile concentration**

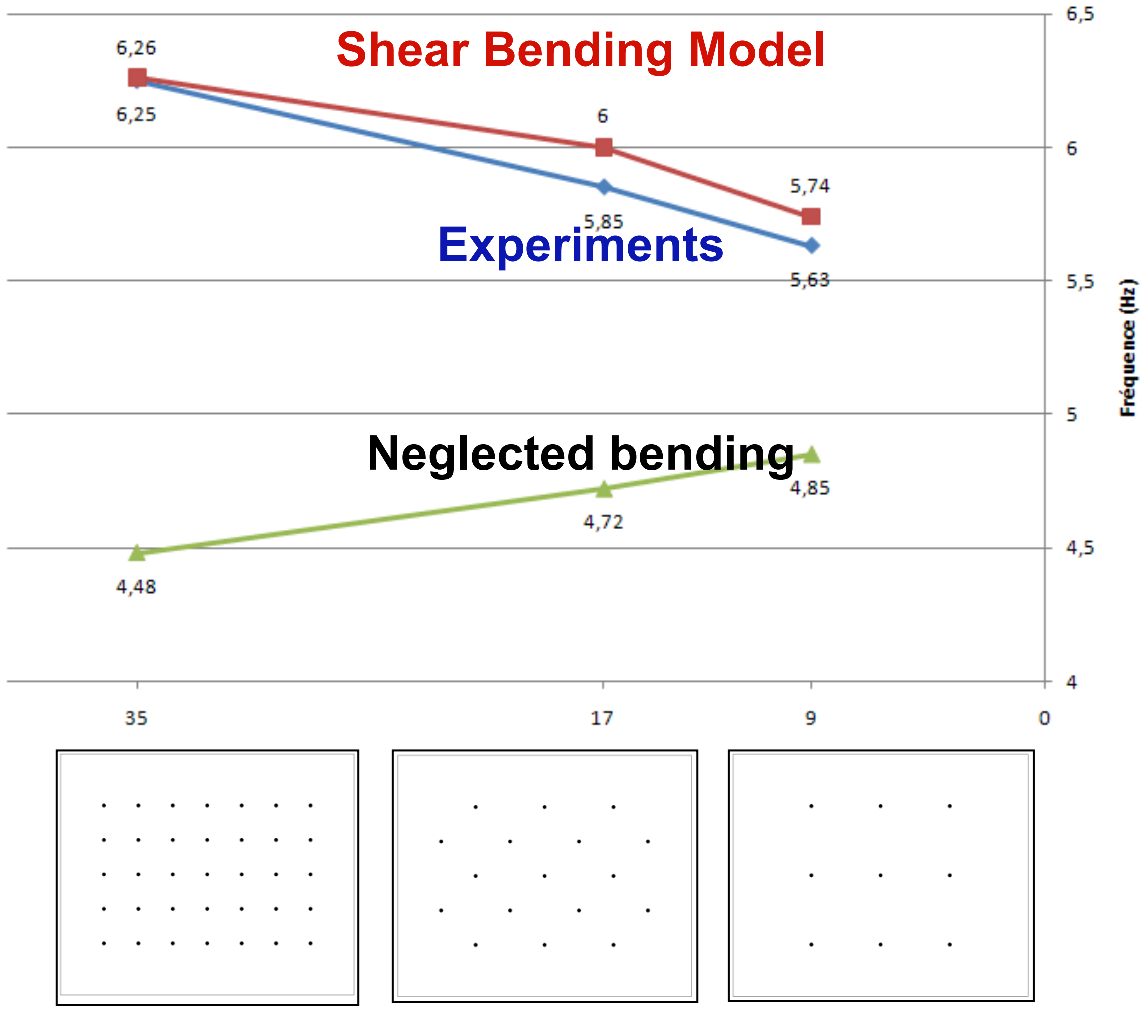
E I ➔ C ≈ G ➔
E I / S ↘ < ρ > ↘

**Neglected bending**

C ≈ G ➔
< ρ > ↘ ω ↗

**Shear Bending Model**

C ≈ G ➔
$E_p I_p$ / S ↘ ω ↘
< ρ > ↘




**Clamped - Free**

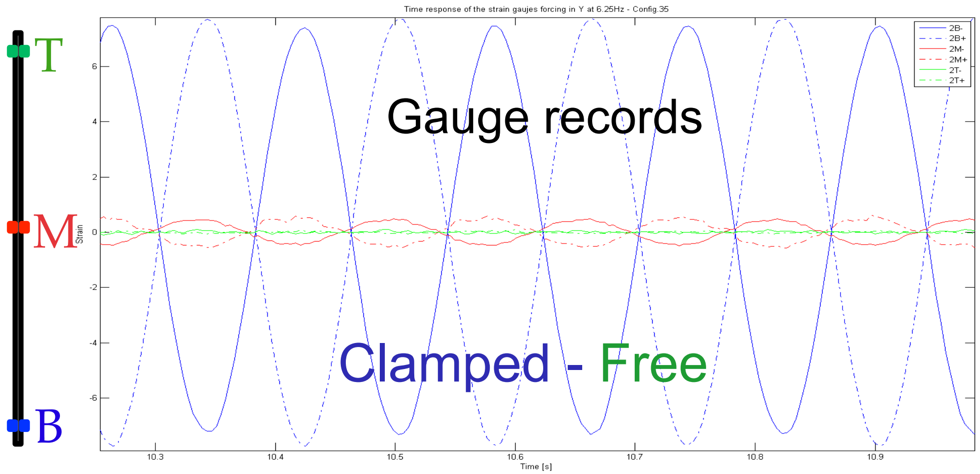
# Fundamental mode - Moment distribution

## Pile response (6.25Hz)

+ - : Bending

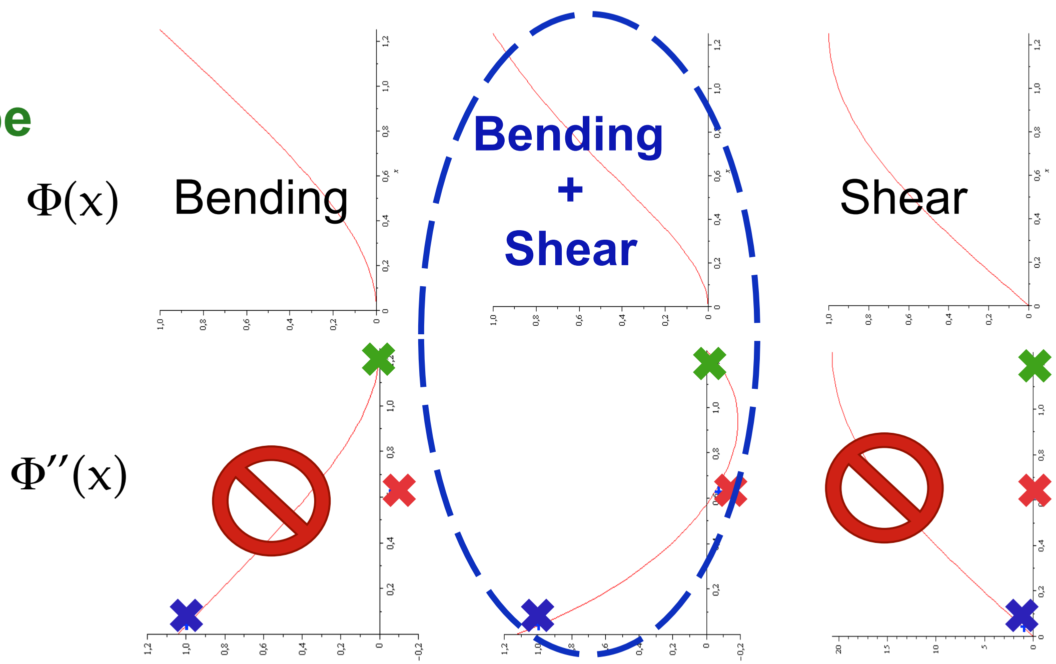
Moment inversion B/M

Zero Top moment

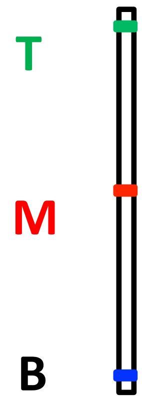



## Strain / Moment / Mode shape

e(x) ~ M(x) ~ Φ''(x)



# ≠ Boundary conditions

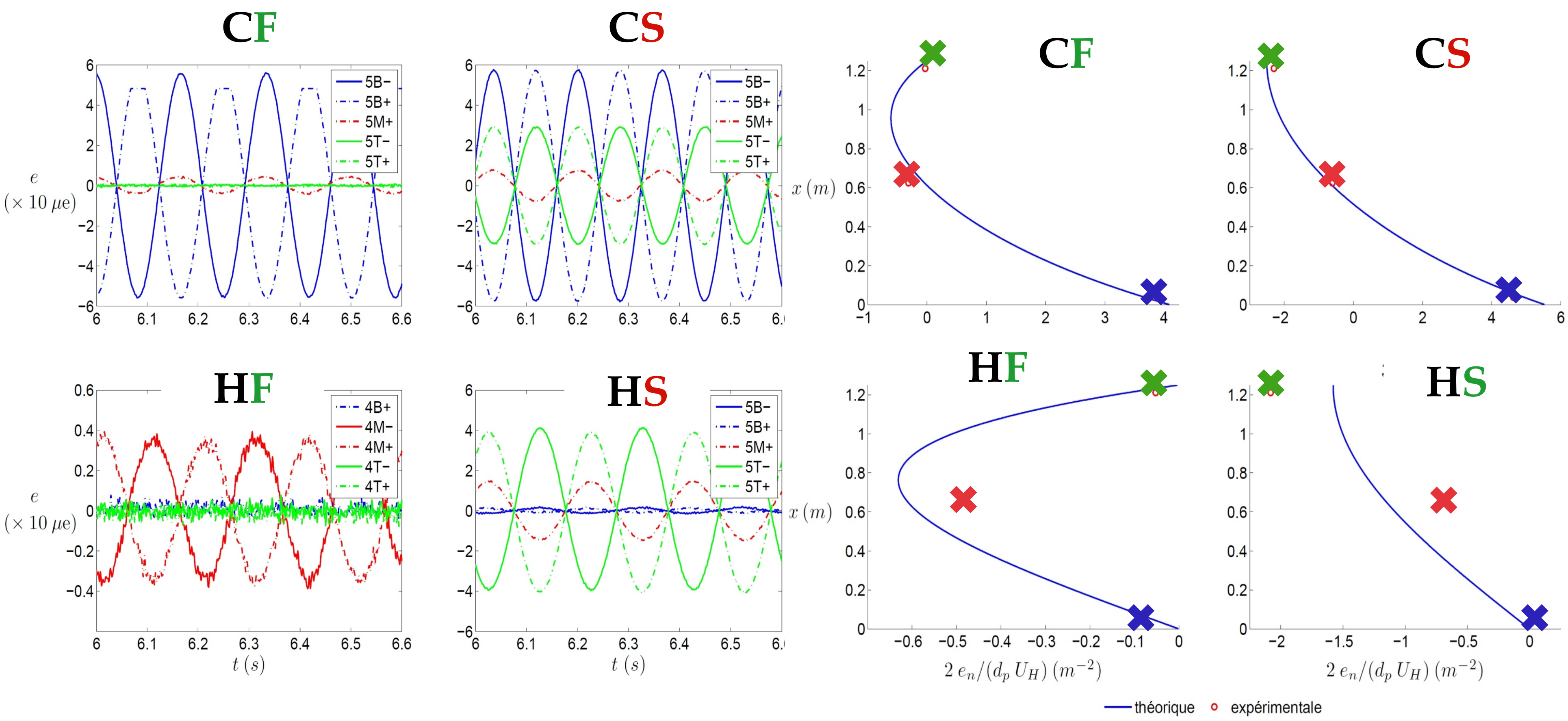



Clamped - Free

Clamped - Sliding

Hinge - Free

Hinge - Sliding



# Macro-dynamics in compression

## Axial balance

$$\begin{cases} \partial\, {}^{P}\sigma^{1}_{11}/\partial x_1 + \mathrm{div}_y([{}^{P}\sigma^{2}]) = -{}^{P}\rho\omega^2 U_1 \\ \mathrm{div}_y\,([{}^{m}\sigma^{2}]) = -\,{}^{m}\rho\omega^2\,\zeta(y,\omega)U_1 \end{cases}$$

**Beam**

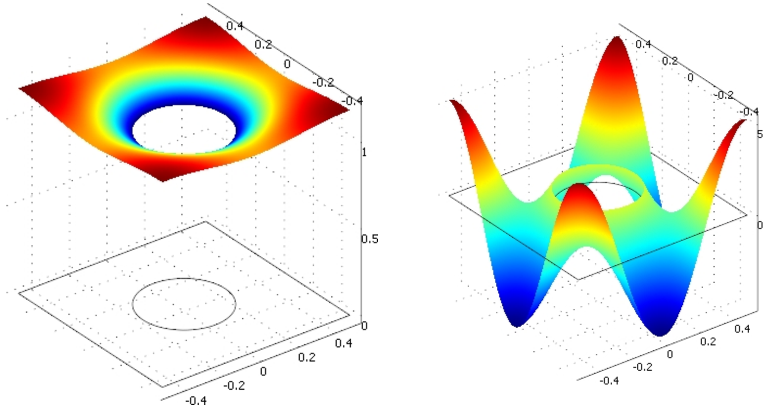
y - Periodicity

**Matrix**

Integration

$$\partial < \sigma^1{}_{11} > /\partial x_1 = <{}^{P}\rho + {}^{m}\rho\zeta(y,\omega)>\omega^2 U_1$$

$$< \sigma^1{}_{ii} > = -E_p\partial U_1/\partial x_1$$

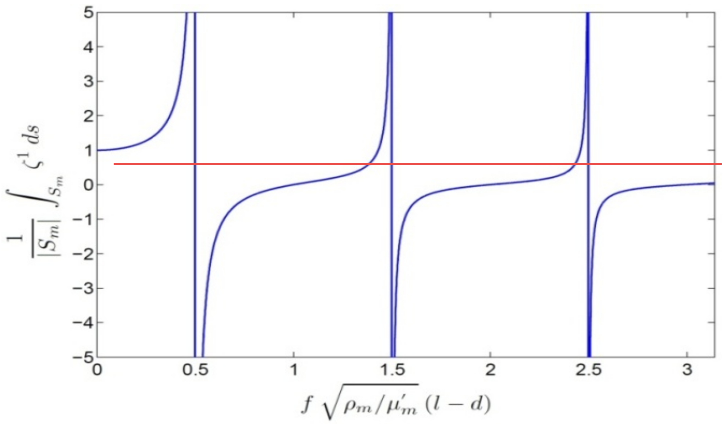


## Meta material

**Non local in time** **Local in space**

Apparent density $<{}^{P}\rho + {}^{m}\rho\zeta(y,\omega)>$

Band gaps : Eigen frequencies



# Learnings

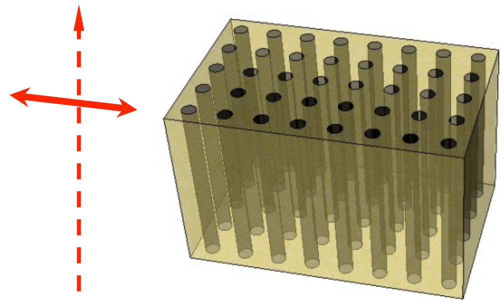
**Combined geometric and mechanical contrasts**

Multi faceted behaviour

**Second gradient behaviour**

SH wave

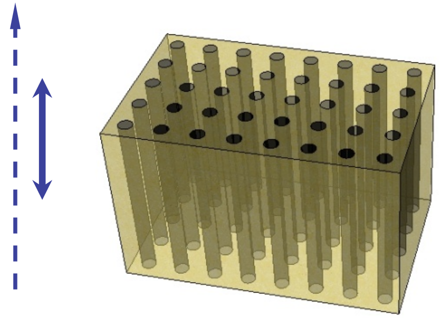
Wave dispersion ; NO band gaps



**Inner resonance behaviour**

P axial wave

Wave dispersion and band gaps



**Standard elastic behaviour**

SV Wave

NO Wave dispersion and NO band gaps