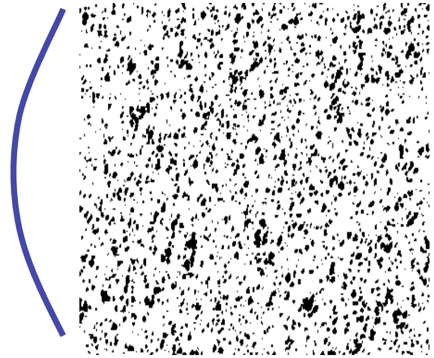
Content

Acoustic of porous media

Physics of porous medium

$$\mathrm{div}(\mathbf{v}) = -i\omega \frac{\phi}{\gamma P^e} p \quad ; \quad \mathbf{v} = -\frac{\boldsymbol{\kappa}(\omega)}{\mu} . \boldsymbol{\nabla} p$$

Gaz stiffness — Dynamic permeability

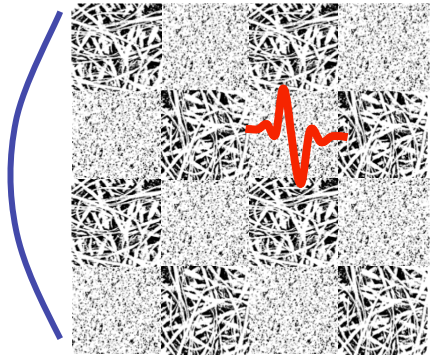


$$\frac{\Lambda_C(\omega)}{2\pi} = \frac{1}{\omega}\sqrt{\frac{i\omega\boldsymbol{\kappa}(\omega)\gamma P^e}{\mu\phi}} = \frac{1}{\omega}\sqrt{\frac{\gamma P^e}{\tau(\omega)\rho^e}} >> l$$

Features of acoustic P_2 waves

Diffusive at low frequency
(viscous dominated)

Propagative at high frequency
(inertia dominated)



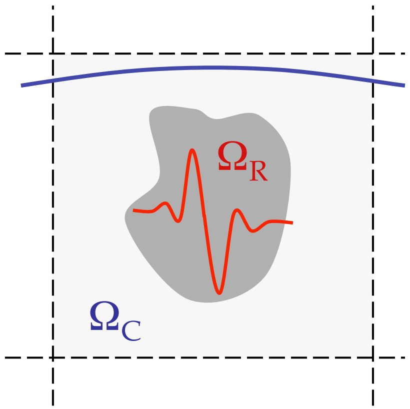
Contrasted porous media ?

Double porosity media

Porous media - C

$$L = O(\frac{\Lambda_c}{2\pi}) = \frac{1}{\omega}\sqrt{\frac{|i\omega\boldsymbol{\kappa}_c|\gamma P_e}{\mu\phi_c}} \gg \ell \quad \text{i.e.} \quad \frac{\omega^2}{\gamma P_e} = \frac{|i\omega\boldsymbol{\kappa}_c|}{\mu\phi_c L^2}$$

Micro-porous media - R



$$\frac{\Lambda_r}{2\pi} = \frac{1}{\omega}\sqrt{\frac{|i\omega\boldsymbol{\kappa}_r|\gamma P_e}{\mu\phi_r)}} = O(\ell) \quad \text{i.e.} \quad \frac{\omega^2}{\gamma P_e} = \frac{|i\omega\boldsymbol{\kappa}_r|}{\mu\phi_r\ell^2}$$

Permeability Contrast / Pore size contrast

$$\frac{|\boldsymbol{\kappa}_r|\phi_c}{|\boldsymbol{\kappa}_c|\phi_r} = O(\frac{\ell^2}{L^2}) = \varepsilon^2 \ll 1$$

Homogenization

Auriault et Boutin, TIPM, 1994 ; Boutin et al ,IJSS, 1998

Homogenization

Scaled governing equations

$$
\begin{cases}
(i\omega/\mu)\mathrm{div}(\boldsymbol{\kappa}_c.\boldsymbol{\nabla} p_c) = -\omega^2 \dfrac{p_c}{\beta} & \text{in } \Omega_c \\
(i\omega/\mu)\mathrm{div}(\varepsilon^2 \boldsymbol{\kappa}_r.\boldsymbol{\nabla} p_r) = -\omega^2 \dfrac{p_r}{\beta} & \text{in } \Omega_r \\
(i\omega/\mu)(\boldsymbol{\kappa}_c.\boldsymbol{\nabla} p_c - \varepsilon^2 \boldsymbol{\kappa}_r.\boldsymbol{\nabla} p_r).\mathbf{n} = 0 & \text{over } \Gamma \\
p_r - p_c = 0 & \text{over } \Gamma
\end{cases}
$$

C constituent

$p_c^{(0)} = P^{(0)}(x)$ & ≈ Impervious micropores

R constituent

$$
\begin{cases}
\dfrac{1}{\mu}\mathrm{div}_y(\boldsymbol{\kappa}_r.\boldsymbol{\nabla}_y(p_r^{(0)})) = i\omega p_r^{(0)} \dfrac{\phi_r}{\beta} & \text{in } \Omega_r \\
p_r^{(0)} = \mathbf{P}^{(0)}(\mathbf{x}) & \text{over } \Gamma
\end{cases}
$$

$$
p_r^{(0)} = P^{(0)} + \boldsymbol{\zeta}(\mathbf{y}, \omega) P^{(0)}
$$

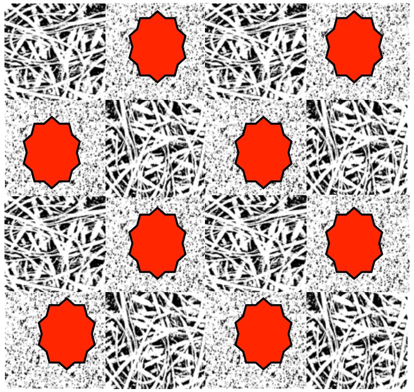
Visco-inertial : diffusive regime

Double porosity model

Macro – description

$$\mathrm{div}_x(\mathbf{V}^{(0)}) \quad + \quad i\omega P^{(0)} \left(\frac{\phi_m(1-c)}{\gamma P^e} + \frac{\phi_r c}{\gamma P^e} \int_{\Omega_r} \zeta d\Omega \right) = 0$$

$$\mathbf{V}^{(0)} \quad = \quad -\frac{\kappa_m(\omega)}{\mu} \mathbf{A}.\nabla_x P^{(0)}$$



Non-conventional effect

Source on **mass balance**

Usual permeability ; **Unconventional stiffness, complex valued**

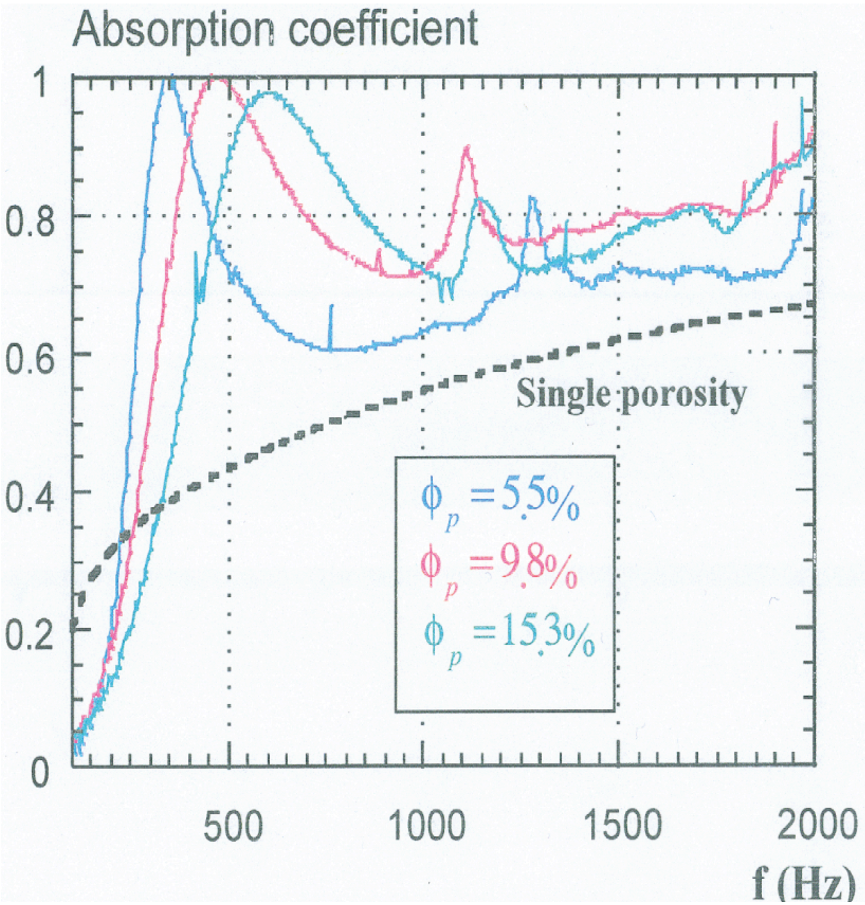
Visco-inertial "resonator" : No band gap

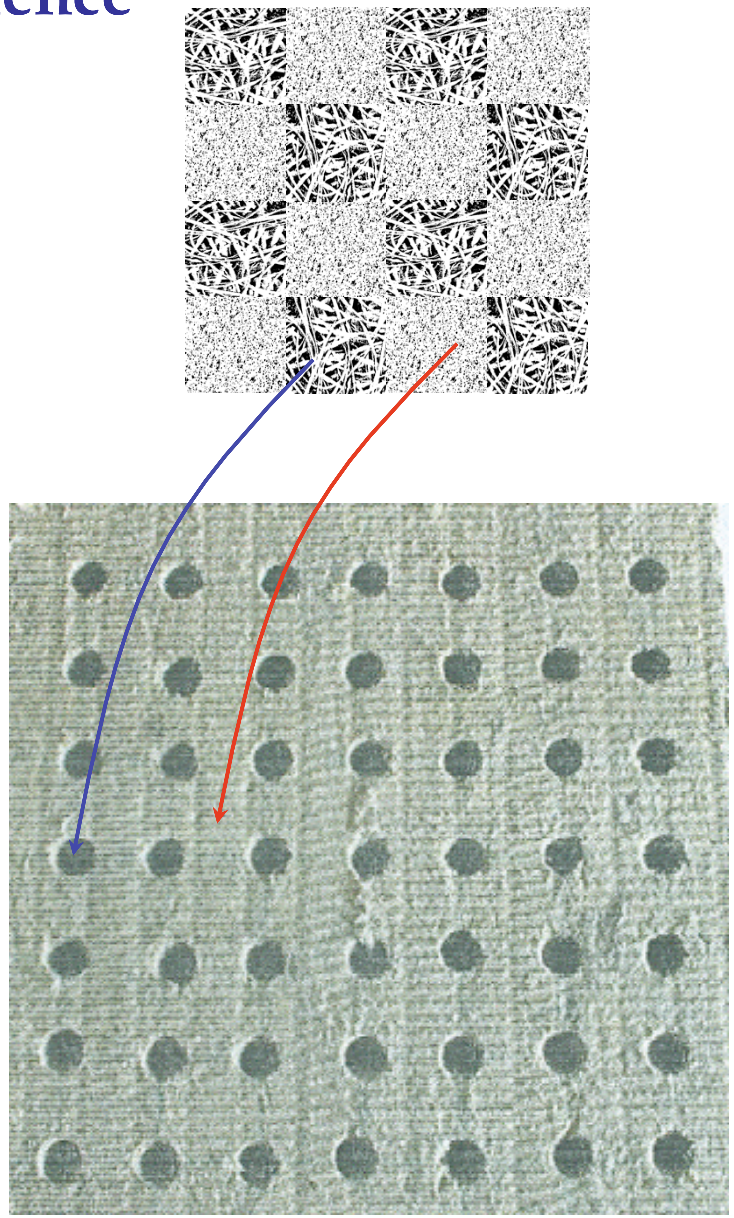
Limited dispersion

Enhanced damping

Experimental Evidence

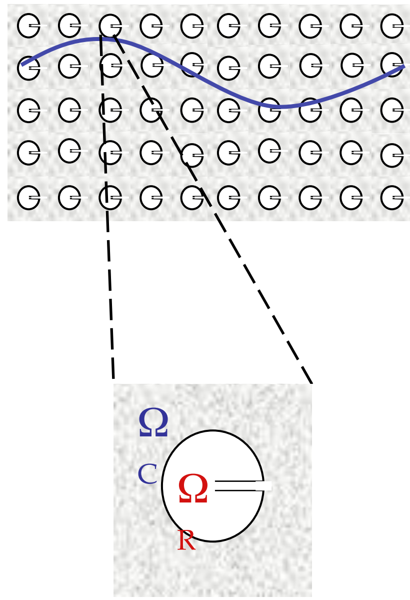
[Olny, Boutin, Jasa 03]





Porous media with resonators

C-constituent   Porous medium

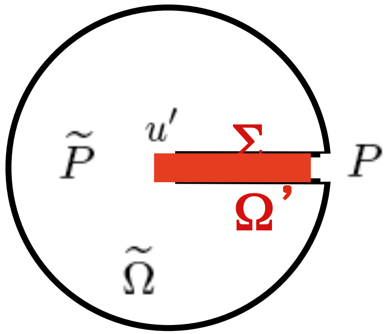


$$\mathrm{div}(\mathbf{v}) = -i\omega \frac{\phi}{\gamma P^e} p \quad ; \quad \mathbf{v} = -\frac{\boldsymbol{\kappa}(\omega)}{\mu} . \boldsymbol{\nabla} p$$

Gaz stiffness ;   Dynamic permeability

$$\frac{\Lambda_C(\omega)}{2\pi} = \frac{1}{\omega} \sqrt{\frac{i\omega \boldsymbol{\kappa}(\omega) \gamma P^e}{\mu \phi}} = \frac{1}{\omega} \sqrt{\frac{\gamma P^e}{\tau(\omega) \rho^e}}$$ >> l

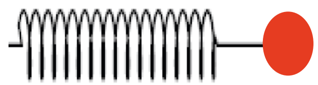
R-constituent   Helmholtz resonator $\omega_0 \approx \omega$



Inner resonance by morphological contrast

$$\omega_0 = 2\pi f_0 = \sqrt{\frac{\mathrm{k}}{\rho^e \Omega'}} = \mathcal{C}^e \frac{|\Sigma|}{\sqrt{\widetilde{\Omega} \Omega'}}$$

$$\omega'_c = \frac{8\mu}{\rho^e (\alpha R)^2} \ll \omega_0 = \mathcal{C}^e \sqrt{3} \frac{\alpha}{R} \ll \omega'_d = \frac{\mathcal{C}^e}{\ell}$$



Inner resonance poro-acoustics

Homogenization Boutin, Jasa, 2013

$$q^{(1)} = i\omega \frac{P^{(0)}}{\gamma P^e} \frac{|\widetilde{\Omega}|}{1-(\omega/\omega_0)^2 \tau'(\omega)}$$

$$\frac{1}{|\widehat{\Omega}|} q^{(1)} + \mathrm{div}_x(\mathbf{V}^{(0)}) + i\omega \frac{P^{(0)}}{\gamma P^e} \phi_m \frac{|\Omega|}{|\widehat{\Omega}|} = 0$$

$$\begin{cases} \mathrm{div}_x(\mathbf{V}^{(0)}) & + \quad i\omega P^{(0)} \left(\frac{\phi_m(1-c)}{\gamma P^e} + \frac{1}{1-(\omega/\omega_0)^2\tau'(\omega)} \frac{c}{\gamma P^e} \right) = 0 \\ \mathbf{V}^{(0)} & = \quad -\frac{\kappa_m(\omega)}{\mu} \mathbf{A}.\nabla_x P^{(0)} \end{cases}$$

Helmoltz resonator : Source on **mass balance**

Usual permeability ; **Apparent stiffness**

$$E(\omega) = \left(\frac{1-c}{E_m} + \frac{c}{E_r} \right)^{-1}$$

E(ω) < 0 $\omega_0 < \omega < \omega^*_0$ E(ω_0) << $\gamma P^e/c$ E(ω^*_0) >> $\gamma P^e/c$

Long wavelenght

Dispersion
Band gaps

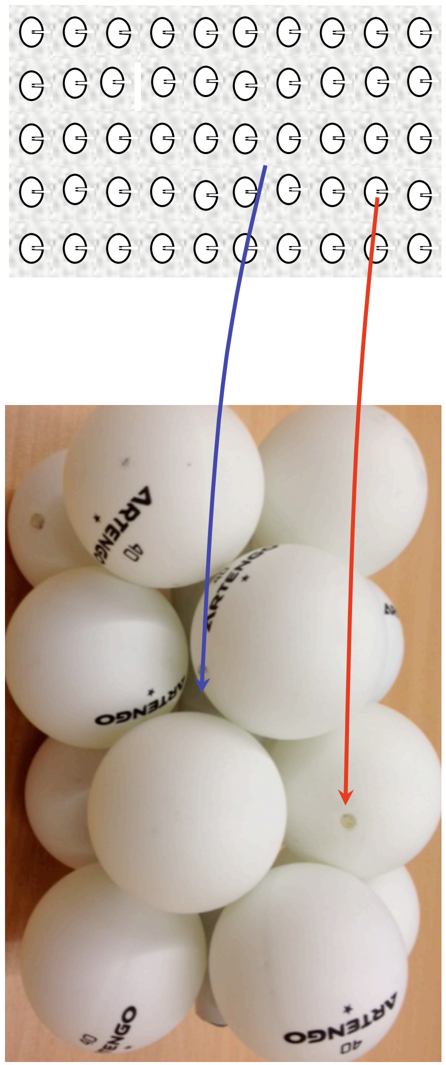
$$\omega_0^* = \omega_0 \sqrt{1 + \frac{c}{(1-c)\phi_m}}$$

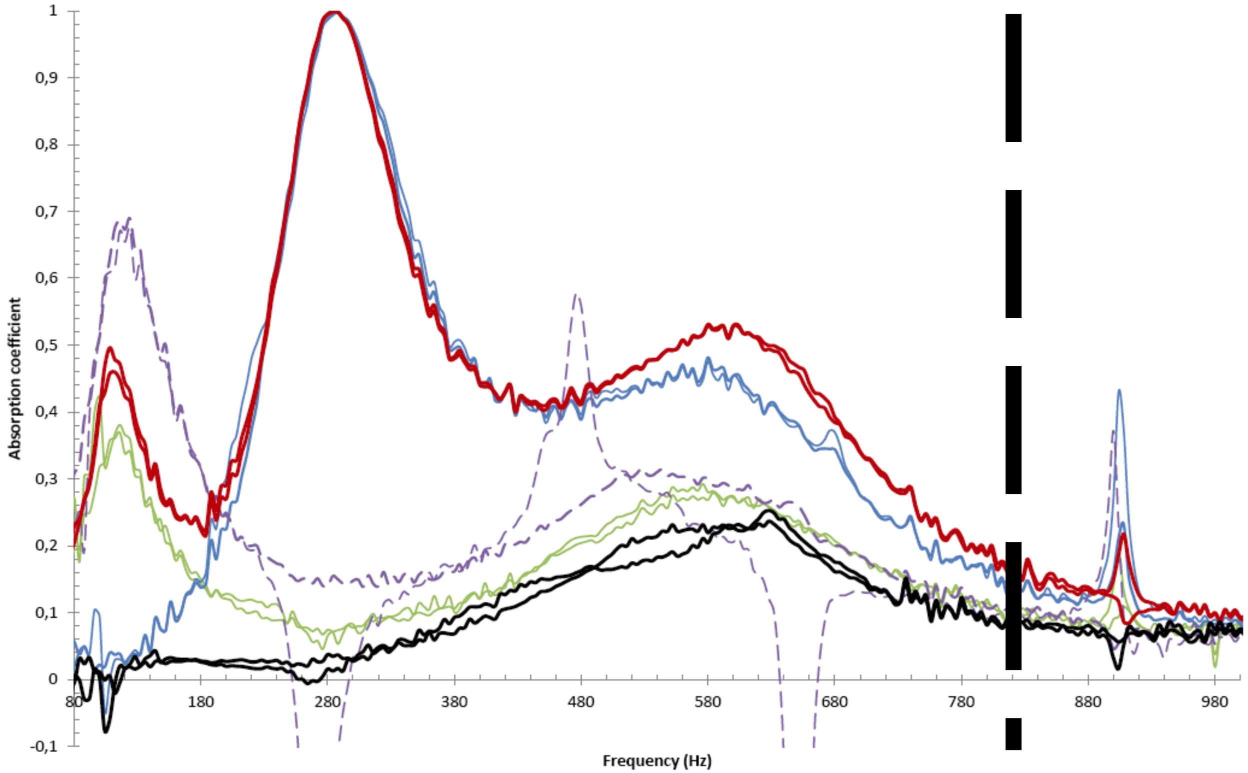
Experiments on Prototype(s)

Spheres of diameter 4cm - Resonator or Impervious

Resonnator frequency : 125 Hz ; 314 Hz

Same Periodic arrangement ; H = 13.8cm





Boutin & Becot, Wave Motion, 2015

Learnings

Same general principles $\varepsilon = 2\pi\, l/\Lambda \approx \lambda_R / \Lambda_C$

Carrying constituent (connected) & Resonant constituent

Forcing motion — Forced regime

High contrast

Effect of the resonator

Source term on the **macroscopic balance**

Momentum balance — **Atypical mass**

Mass balance — **Atypical stiffness**

Physics of the resonator

Visco-inertial : No poles — Limited dispersion NO band gaps

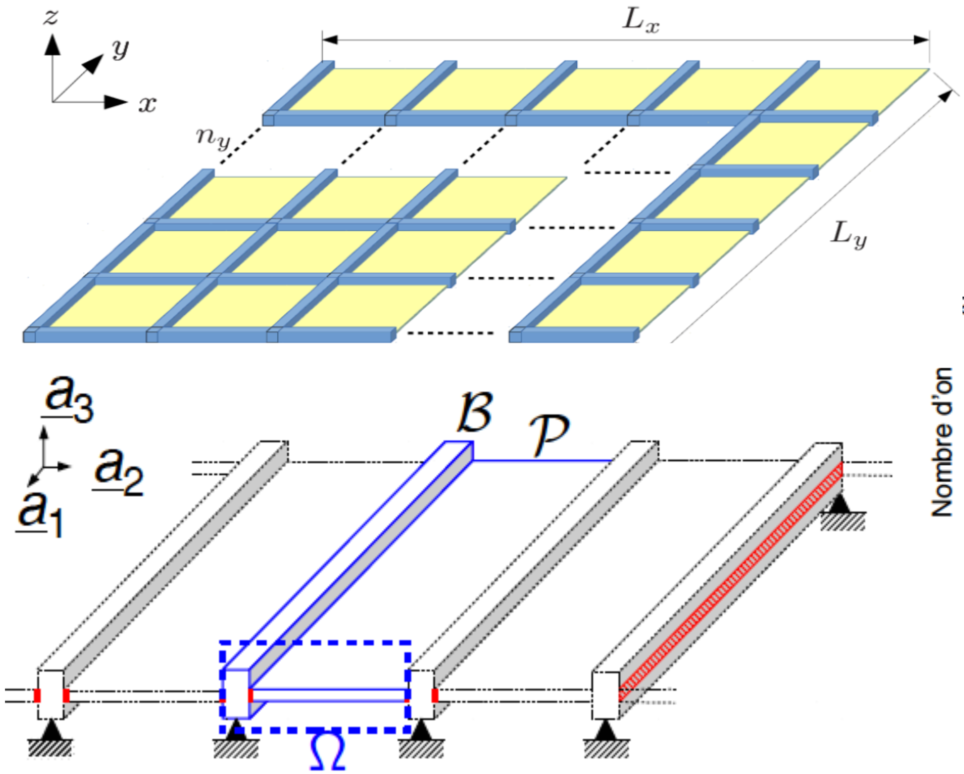
Enhanced damping

Elasto-inertial system : poles — High dispersion and band gaps

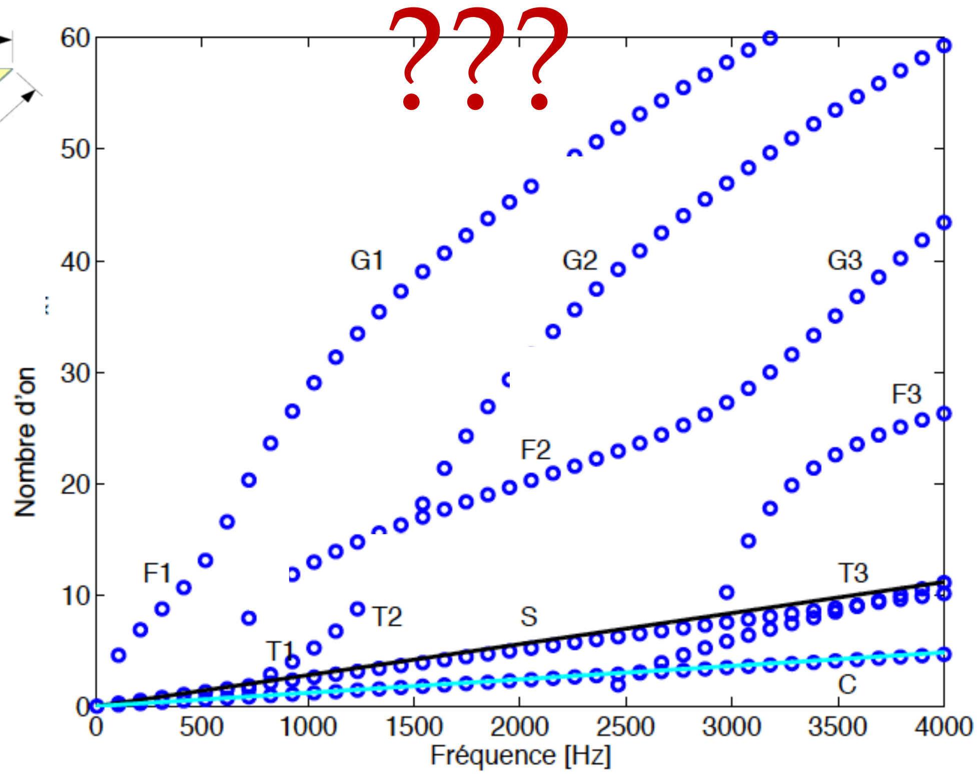
Content

Ribbed Plates

1D – 2D Ribbed plates

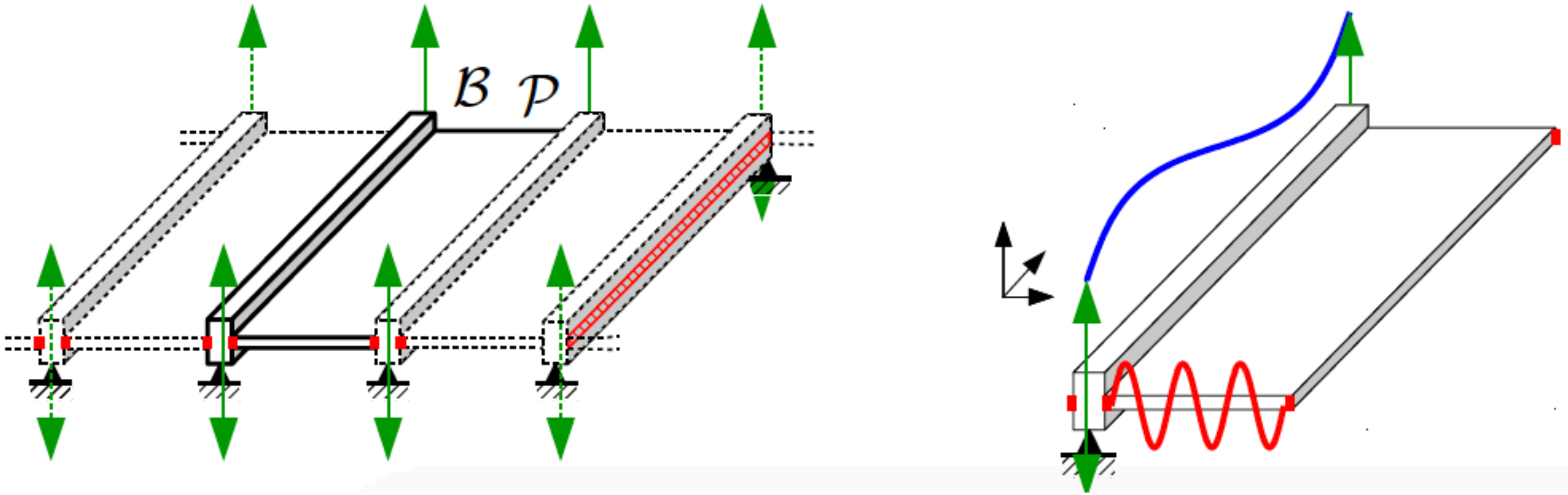


Complex wave dispersion



Fossat, Boutin, Ichchou ; IJSS, 2018

Inner resonance ?



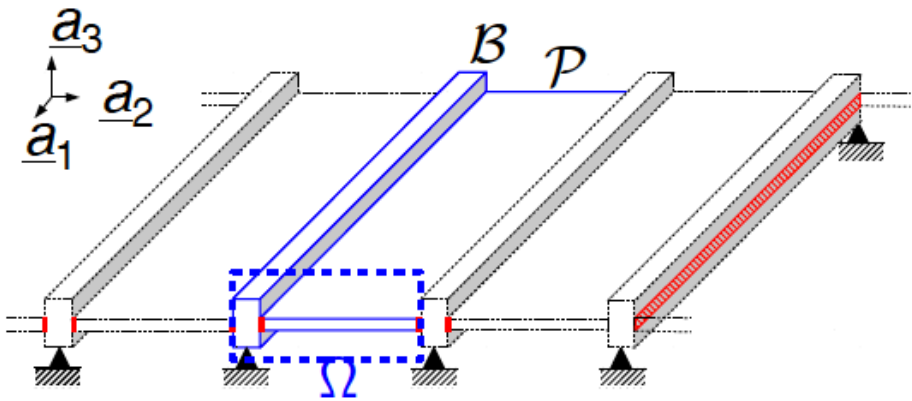
At the same frequency

Vibration at large scale in the beam B
Vibration at small scale in the plate P

Conditions of contrasts ?

What is the corresponding plate model ?

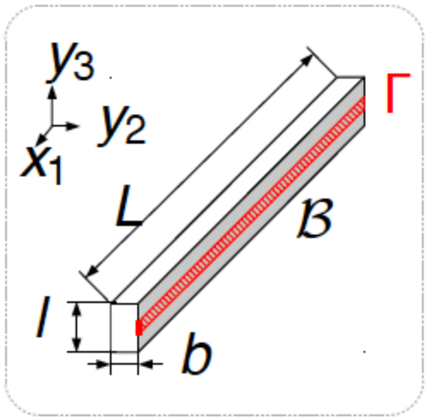
The mechanical system

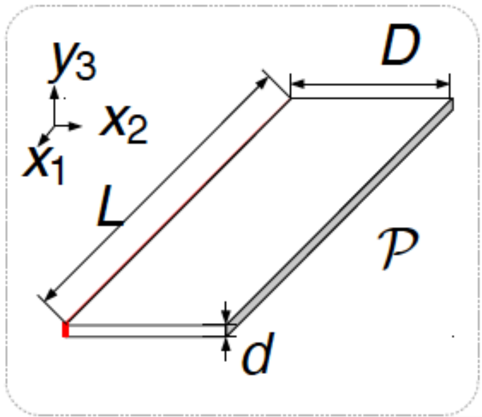


Two small parameters

Beam $\varepsilon_b = l/L << 1$

Plate $\varepsilon_p = d/D << 1$





Possible contrasts ?

Thickness d/l

Modulus E_p/E_b

Density ρ_p / ρ_b

Inner resonance conditions

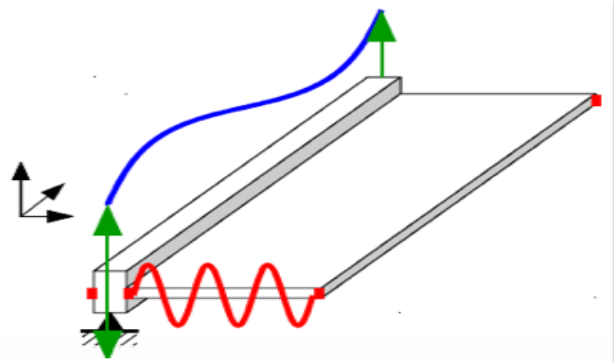
Resonances

Beam $\quad E_b I_b \partial^4_{x1} U = \rho_b l b \, \omega_b^2 U \quad \omega_b^2 = O(E_b l^2/\rho_b L^4)$

Plate $\quad E_p J_p \partial^4_{x1} U = \rho_p d \, \omega_p^2 U \quad \omega_p^2 = O(E_p d^2/\rho_p D^4)$

$\omega_b = O(\omega_p)$ → $\frac{E_p}{E_b} \frac{\rho_b}{\rho_p} \left(\frac{d}{l}\right)^2 = O\left(\frac{D}{L}\right)^4$

"Non-homogeneous" kinematics



→ Non-symmetric beam/plate coupling

Beam : forcing ; Plate : forced

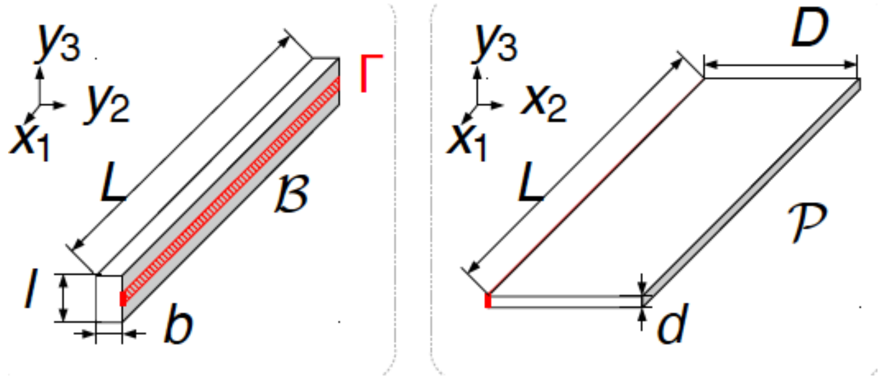
→ $\partial_{x1} T_b = T_p \quad E_b I_b U/L^4 = O(E_p J_p \partial^3_{x1} U) = O(E_p d^3/D^3) U$

→ $E_b l^4/L^4 = O(E_p d^3/D^3)$

Scaling

Requirements

$E_b\, l^4/L^4 = O(E_p d^3/D^3)$



$$\frac{E_p}{E_b}\frac{\rho_b}{\rho_p}\left(\frac{d}{l}\right)^2 = O\left(\frac{D}{L}\right)^4 \ll 1$$

Geometrical contrast

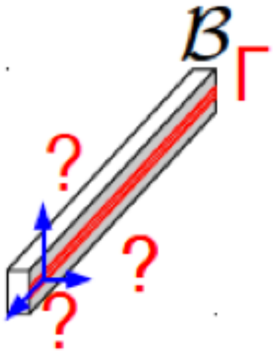
Slenderness	$\varepsilon = \varepsilon_b = l/L = \varepsilon_p = d/D$	Similar
Thickness	$d/l = D/L = O(\sqrt{\varepsilon})$	Thin & narrow plate

Mechanical contrast

Modulus	$E_p/E_b = O(\varepsilon)$	Soft plate
Density	$\rho_p/\rho_b = O(1)$	Similar

Asymptotic process

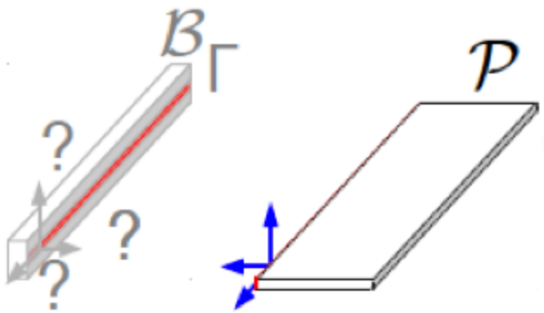
Beam



$$\underline{\underline{\sigma}} = \left[\begin{array}{c|c} \sigma_N & \underline{\sigma}_T \\ \hline \underline{\sigma}_T & \underline{\underline{\sigma}}_S \end{array}\right]$$

$$\begin{cases} \partial_{x_1} T_3 + \boxed{\int_\Gamma \sigma^{(3)}_{S32}} = -\Lambda_b \omega^2 \int_S U(x_1) \\ \partial_{x_1} M_3 + \boxed{\int_\Gamma y_3 \sigma^{(2)}_{T12}} - T_3 = 0 \\ M_3 = -E_b I_b \dfrac{d^2 U(x_1)}{dx_1^2} \end{cases}$$

Plate



$$\underline{\underline{\sigma}}^{\mathcal{P}} = \left[\begin{array}{c|c} \underline{\underline{\sigma}}_p & \underline{\sigma}_t \\ \hline \underline{\sigma}_t & \sigma_n \end{array}\right]$$

$$\begin{cases} div_\alpha(\underline{T}) = -\lambda_p \omega^2 w \\ \underline{div_\alpha}(\underline{\underline{M}}) + \underline{T} = 0 \\ \underline{\underline{M}} = E'_p I_p ((1-\nu_p) \underline{\underline{e}}_\alpha (\underline{\nabla}_\alpha w) + \nu_p \Delta_\alpha w \underline{\underline{I}}_P) \end{cases}$$

AND $\quad \mathbf{w = U(x_1)}$ **on** $\mathbf{\Gamma}$

$$w(x_1 \rightarrow x_2) = \phi_0(x_2)\, U(x_1)$$

Coupling

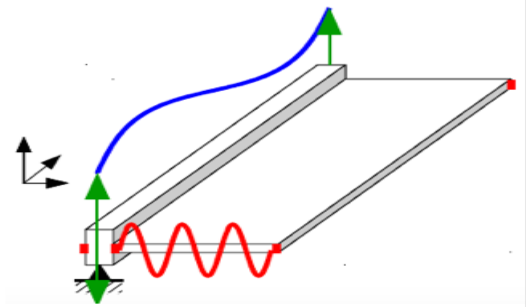


Plate : Moment **M =** $\int_\Gamma y_3 \sigma^{(2)}_{T12} = 0$ (clamping on Γ : $\sigma_{12} = 0$)

Force **F =** $\int_\Gamma \sigma^{(3)}_{S32}$

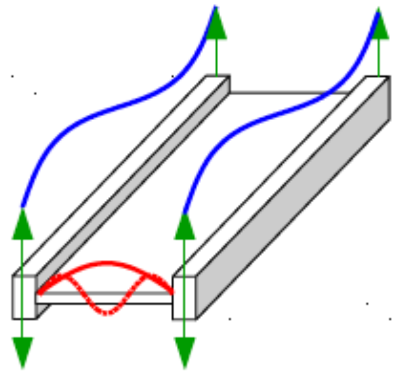
Integration of plate dynamic balance

$w(x_1, x_2) = \phi_0(x_2)\, U(x_1)$

$F = \int_\Gamma \sigma^{(3)}_{S23} = -\rho_p \omega^2\, U(x_1) \int_{x_2} \phi_0(x_2)$

➡ **Effective apparent inertia (ω)**

Effective hybrid beam-plate model

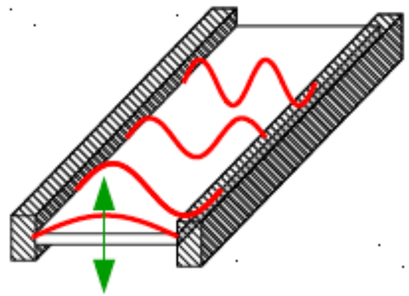


Active beams $U(x_1) \neq 0$

$$\begin{cases} E_b I_b \partial^4 x_1 U = \omega^2 (\mathbf{\Lambda_b} + \lambda_p D \langle \phi_\omega \rangle) U(x_1) \\ \langle \phi_\omega \rangle = \frac{2}{\delta^*} \frac{1}{\coth(\delta^*) - \cot(\delta^*)}; \quad \delta^* = \frac{\delta D}{2} \end{cases}$$

$$\delta^4 = \omega^2 \lambda_p / (E'_p I_p)$$

Passive beams $U(x_1) = 0$



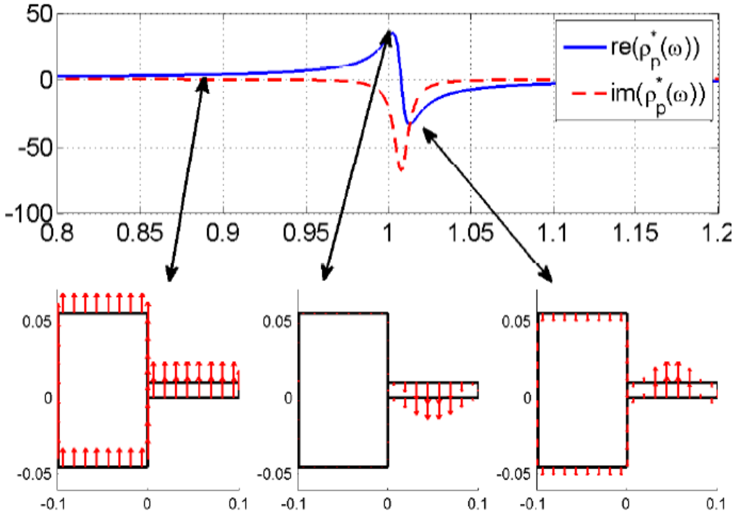
$$\begin{cases} E'_p I_p \partial^4 x_2 w = \lambda_p \omega^2 w(x_2) \\ w(x_1, x_\Gamma) = 0 \text{ on } \Gamma \end{cases}$$

$$w(x_1, x_2) = (\phi_0(x_2) - 1)\exp(ikx_1)$$

Symetric eigenmode : Guided wave

Inner resonance effect

Heterogenous Kinematics



Effective mass

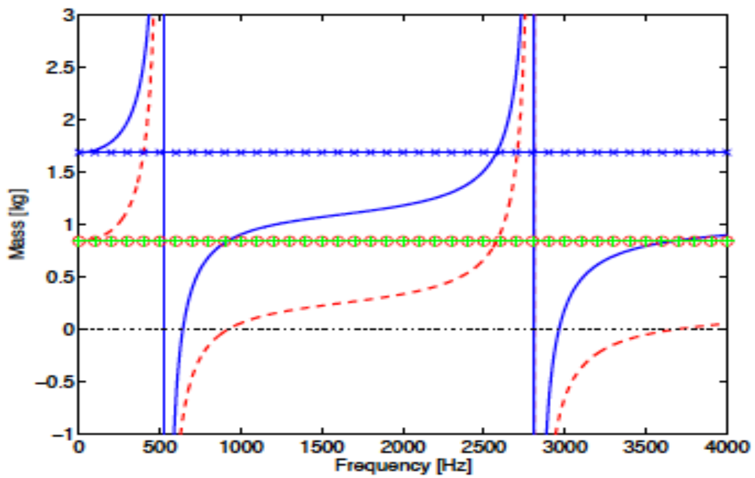
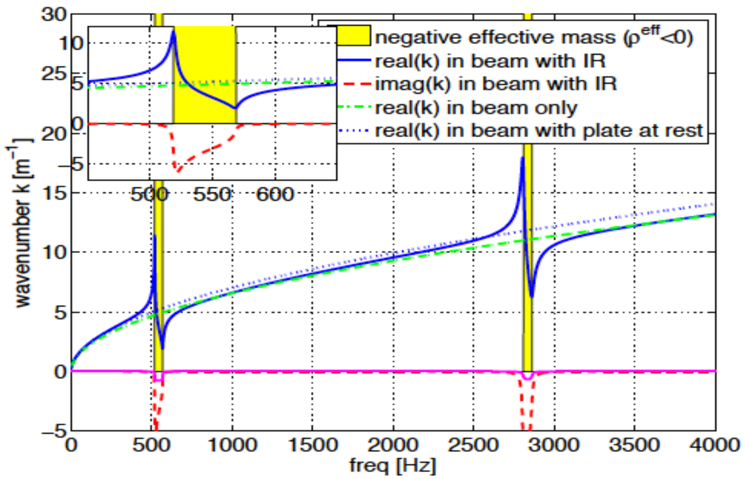
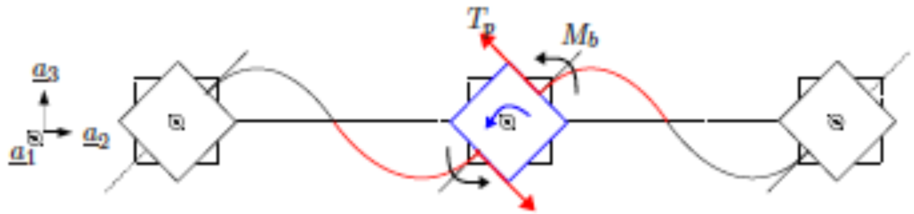


Figure 4: Real part of the effective mass of the plate (−−) and effective mass of the whole structure (−). Static mass of the plate (−○−), static mass of the beam and (−+−), Static mass of the whole structure (−×−) with structural damping $\eta = 0.5\%$

Band gap



Hybrid beam-plate model - Torsion



Active beams $\theta(x_1) \neq 0$

$$G_b \mathcal{I}_b \partial^2_{x_1}\theta = -\omega^2 \left(\rho_b J_b + \Lambda_p D^3 J^*_\omega\right)\theta + \frac{E'_p I_p (D+b)}{D^2} C^*_\omega \, \theta$$

$$J^*_\omega = \frac{1}{(2\delta^*)^2}\frac{\coth(\delta^*) + \cot(\delta^*) - 2/\delta^*}{\coth(\delta^*) - \cot(\delta^*)} \qquad C^*_\omega = (2\delta^*)^2 \frac{\coth(\delta^*) + \cot(\delta^*)}{\coth(\delta^*) - \cot(\delta^*)}$$

Passive beams $\theta(x_1) = 0$

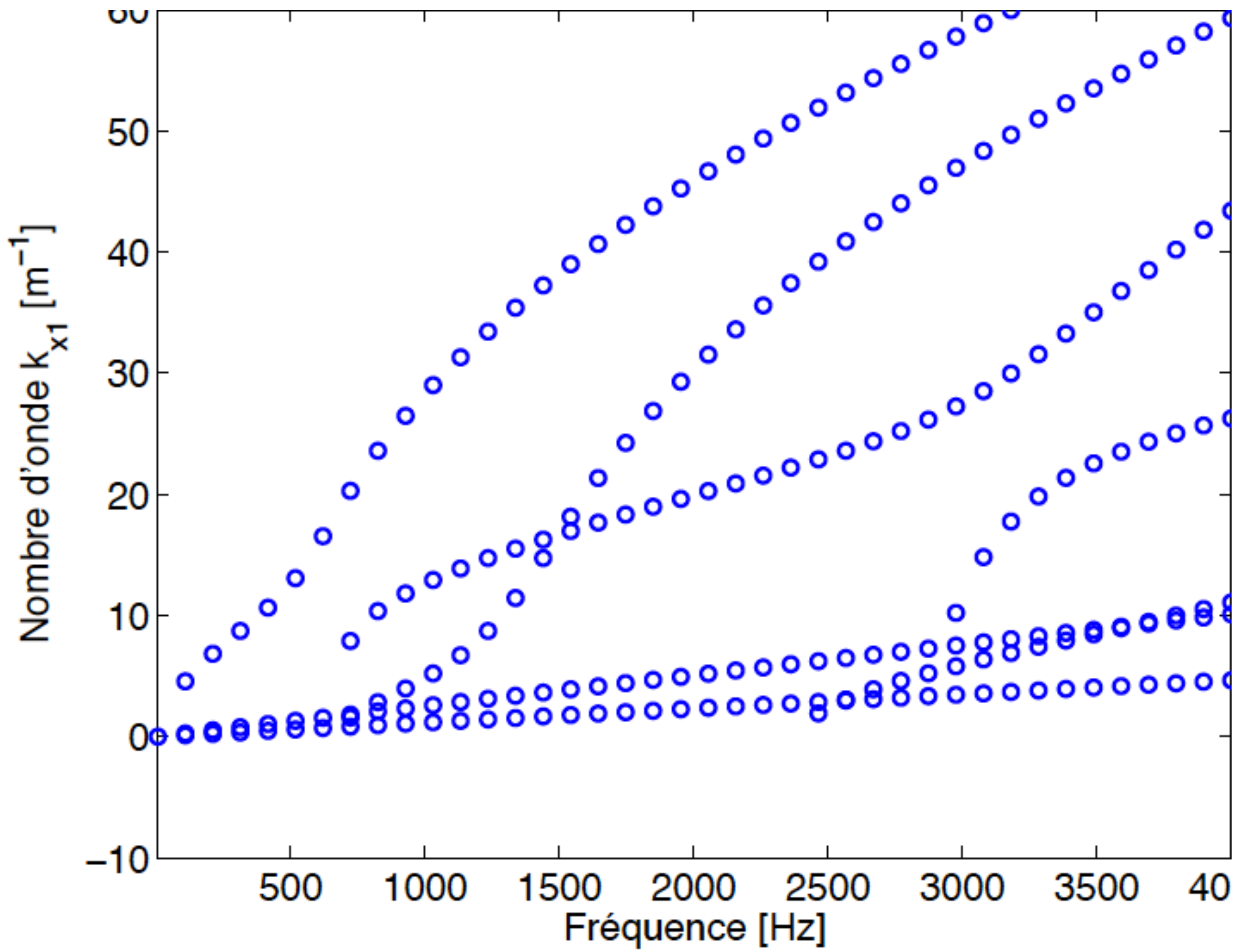
$$E'_p I_p \, \partial^4_{x2} w = \lambda_p \omega^2 w(x_2)$$

$$\partial_{x2} w(x_1, x_\Gamma) = 0 \text{ on } \Gamma$$

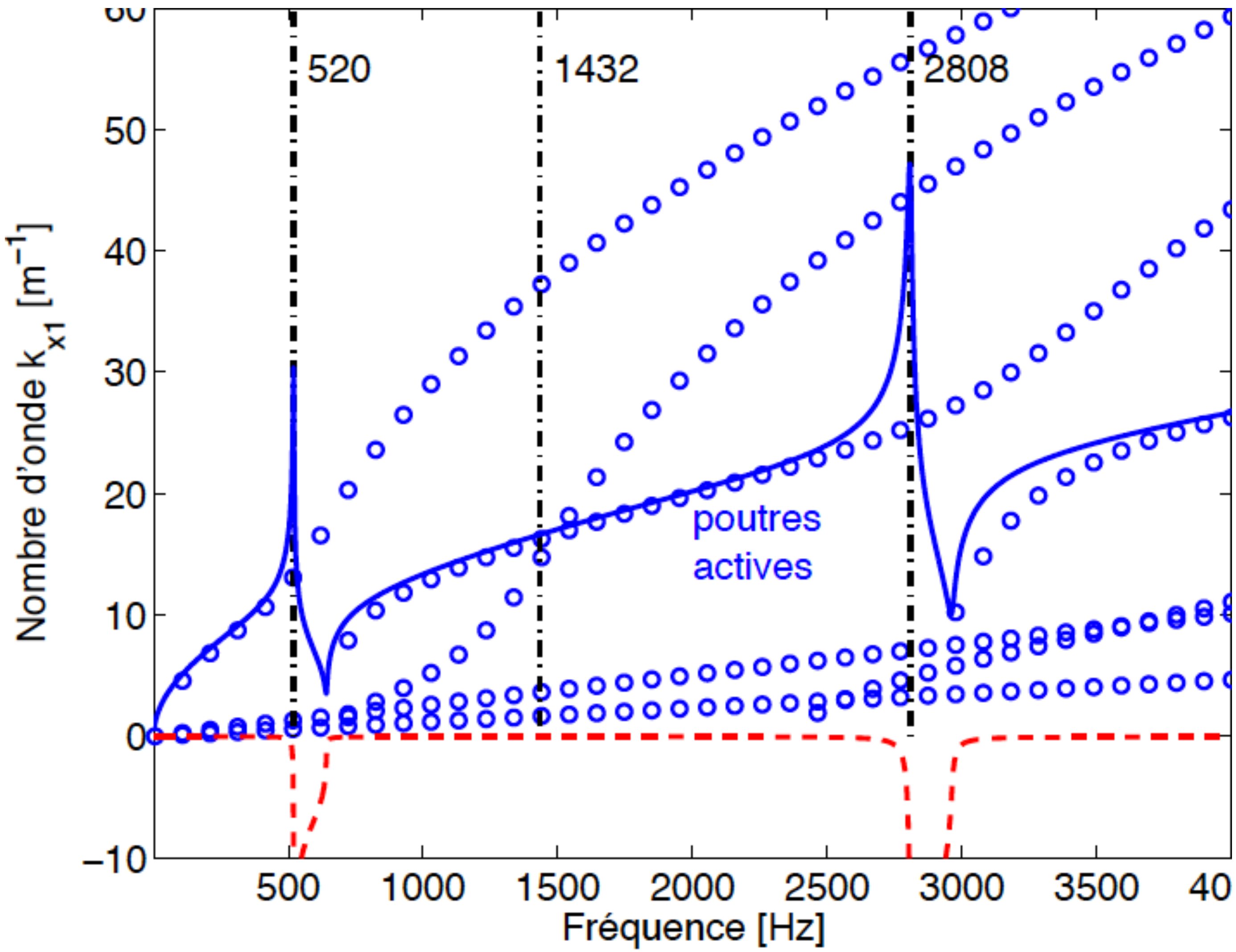
$$w(x_1, x_2) = \phi_1(x_2)\exp(ikx_1)$$

Non symmetric eigen mode : Guided wave

Theory versus direct WFEM computation



Theory versus direct WFEM computation

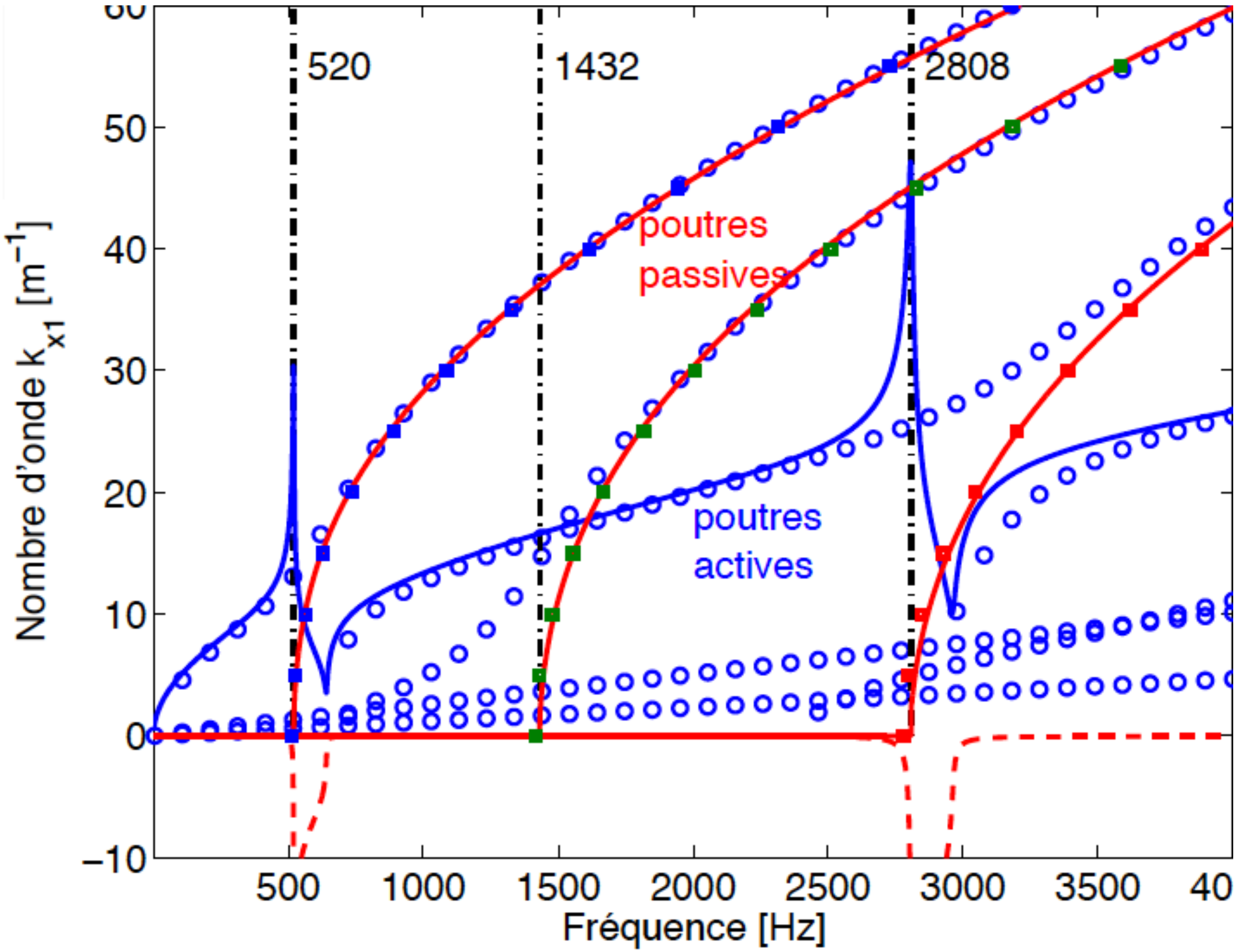


Theory versus direct WFEM computation

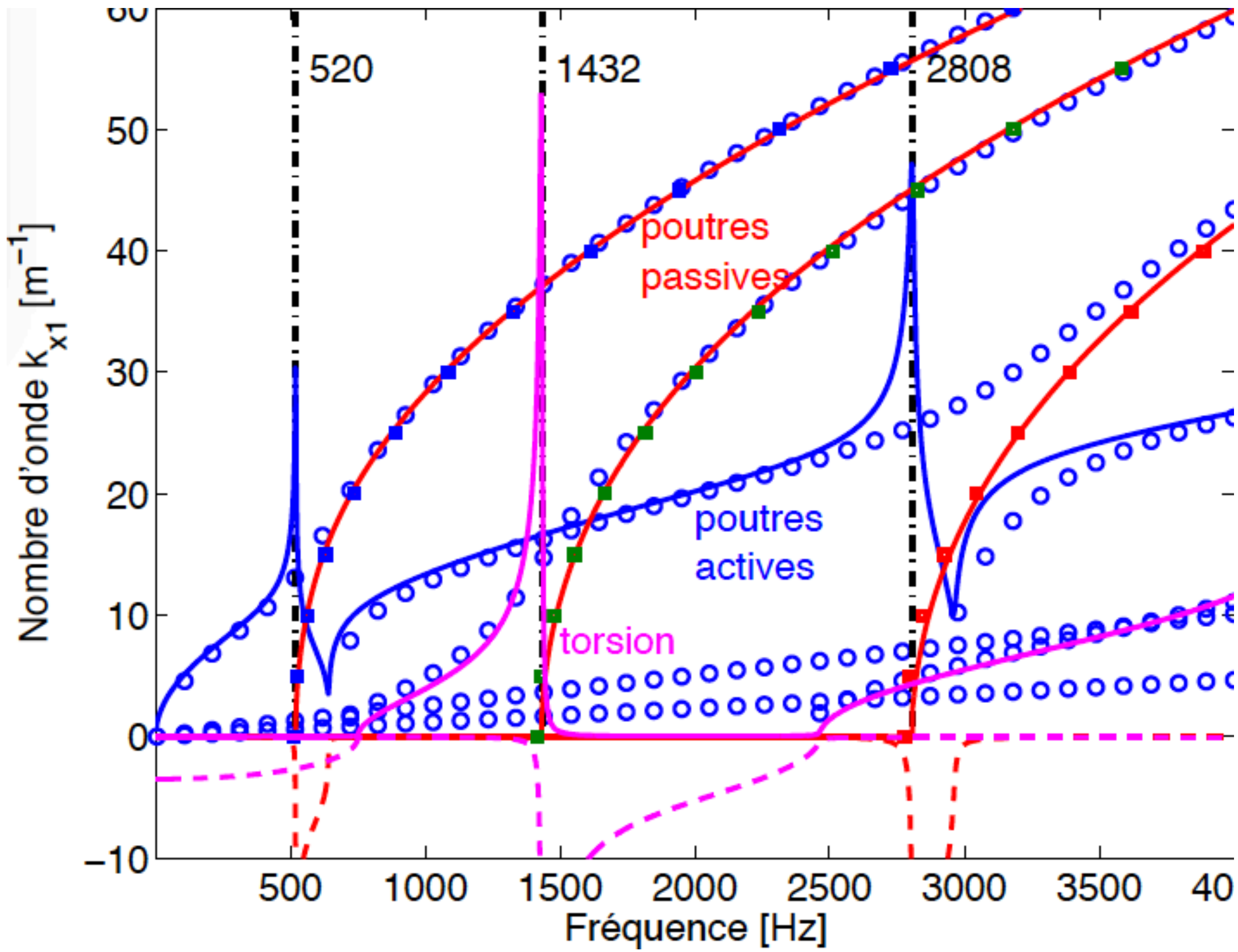
≠ Modes

Slow
Wave
(guided)

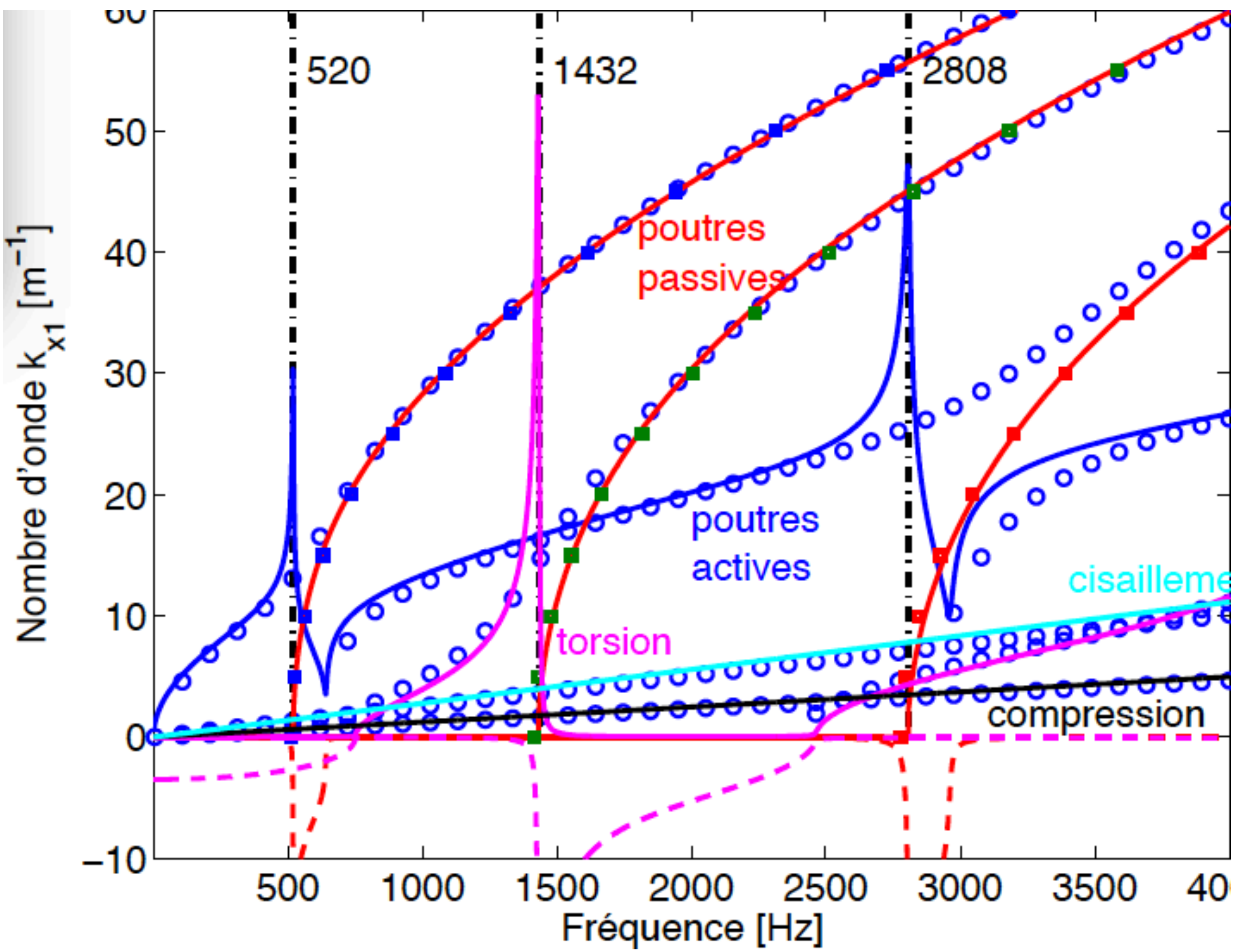
Fast
Wave



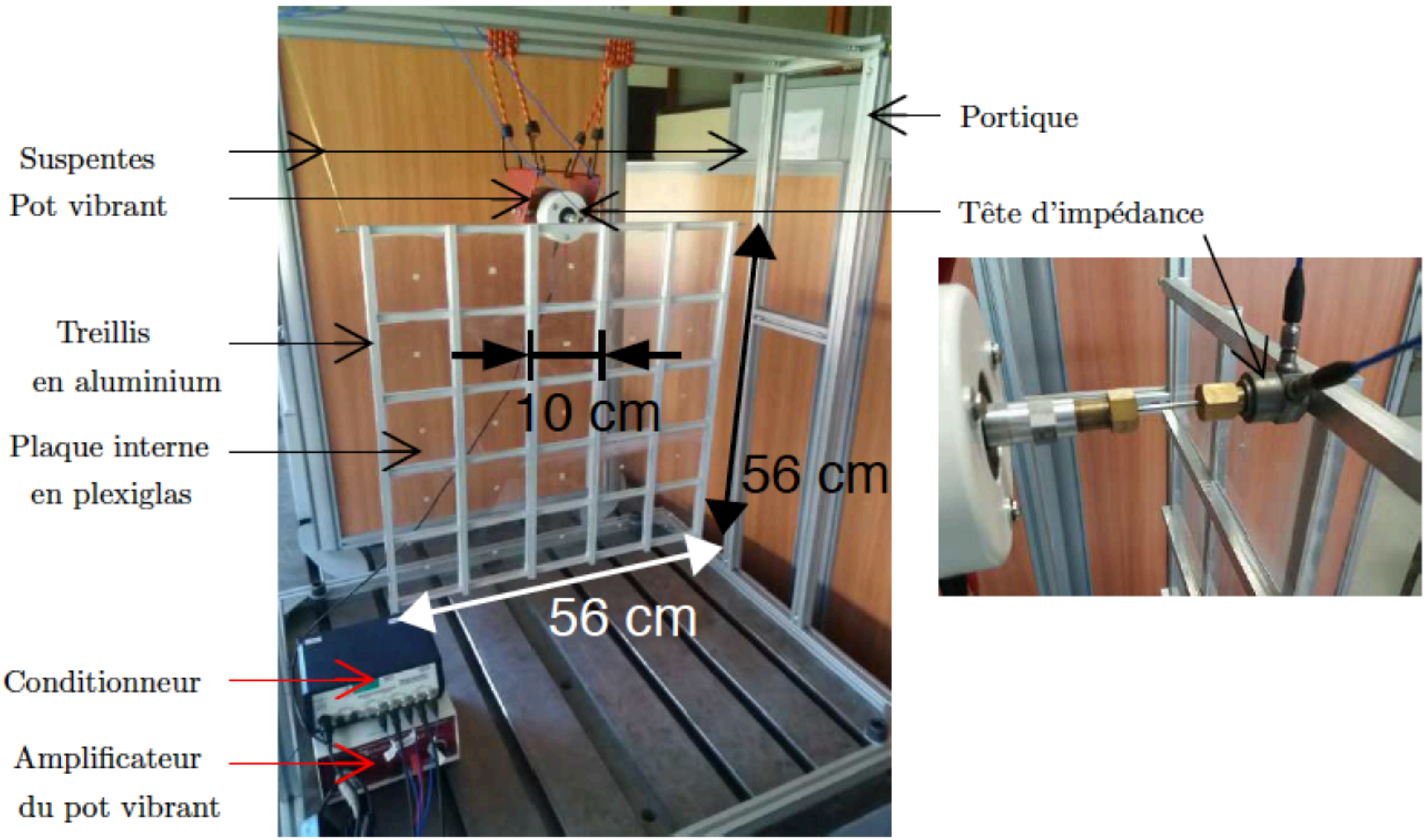
Theory versus direct WFEM computation



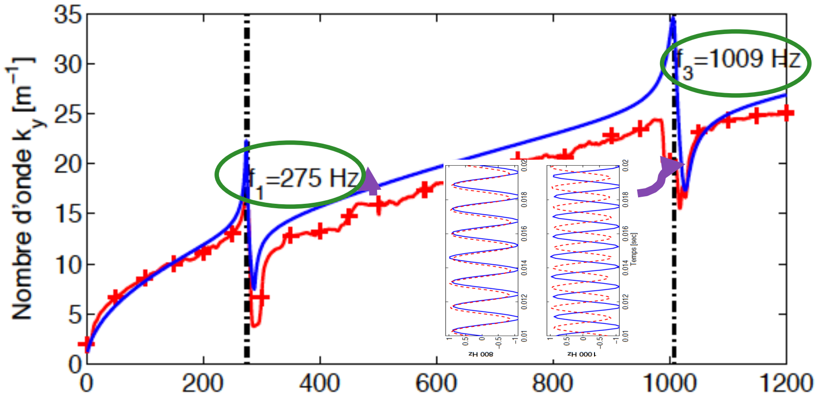
Theory versus direct WFEM computation

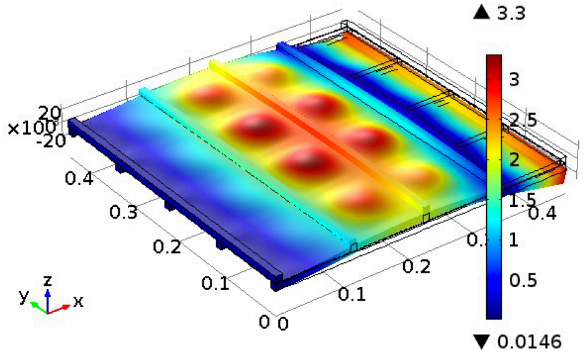


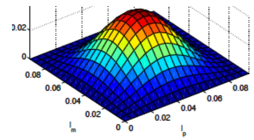
Experiments

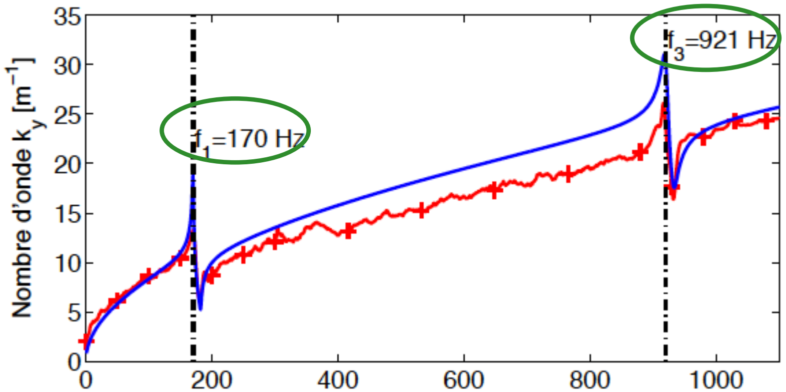


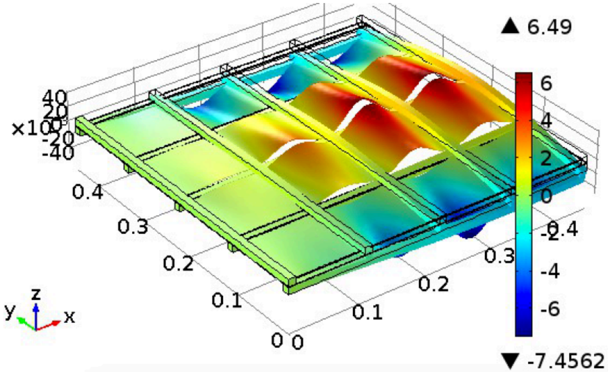
Dispersion for ≠ inner boundary conditions

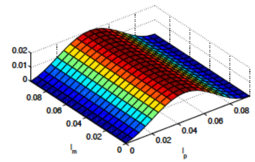


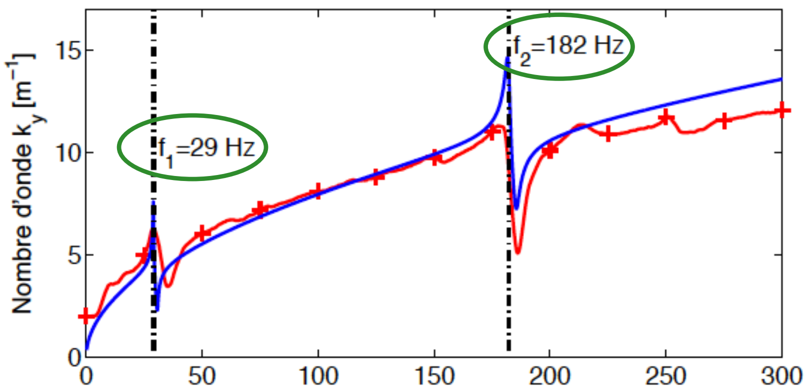


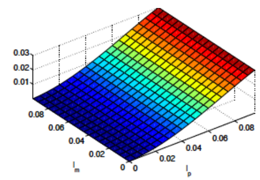












Learnings

Same general principles $\varepsilon = 2\pi\, l/\Lambda \approx \lambda_R / \Lambda_C$

Carrying constituent (connected) & Resonant constituent

Forcing motion Forced regime

High contrast

Codynamic regime and **asymetric coupling**

Effect of the resonator

Source term on the **macroscopic balance**

Momentum balance **Unconventional mass**

1D specificity

Existence of guided slow waves

Stop band in torsion with unconventional mass and **spring**