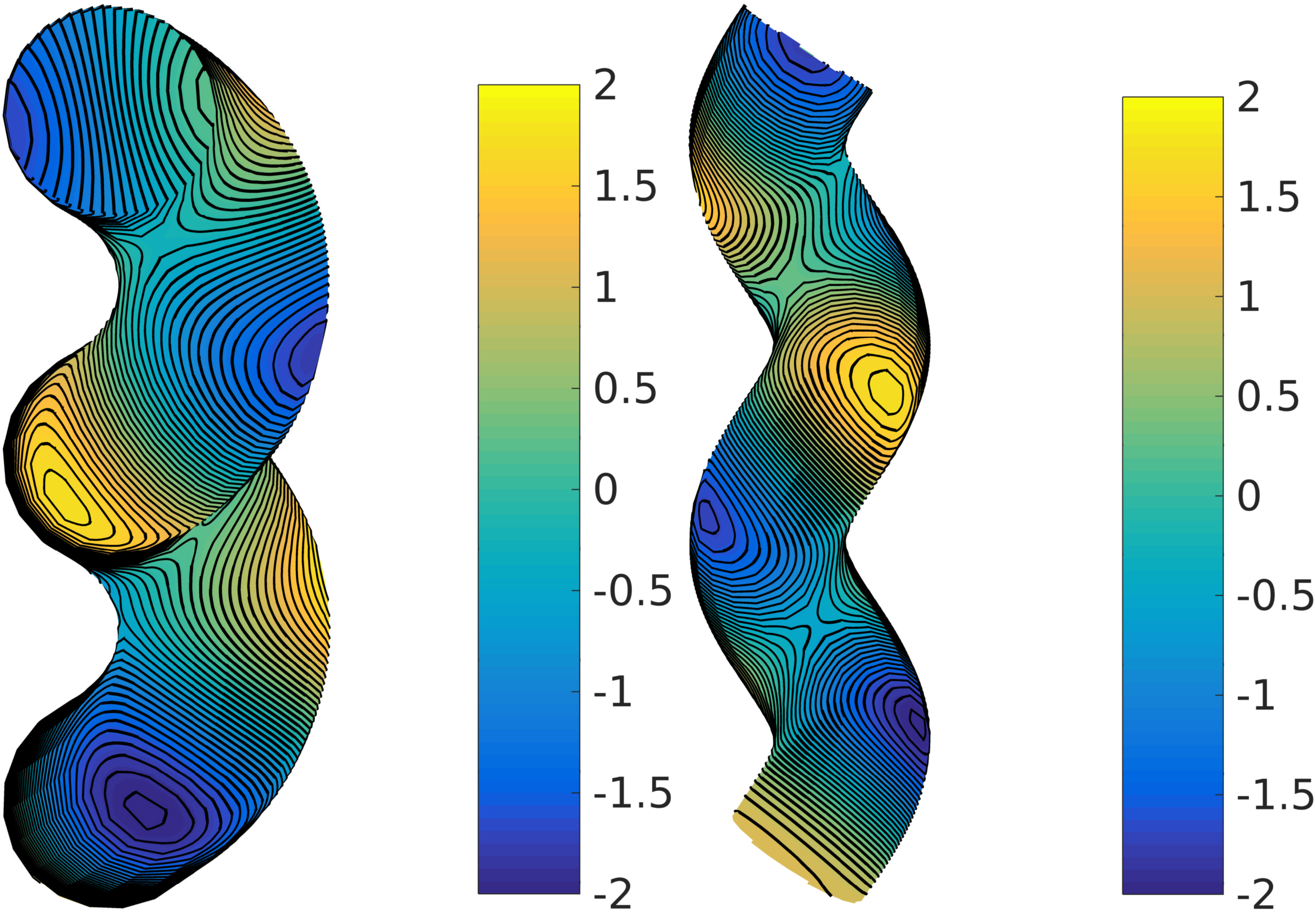
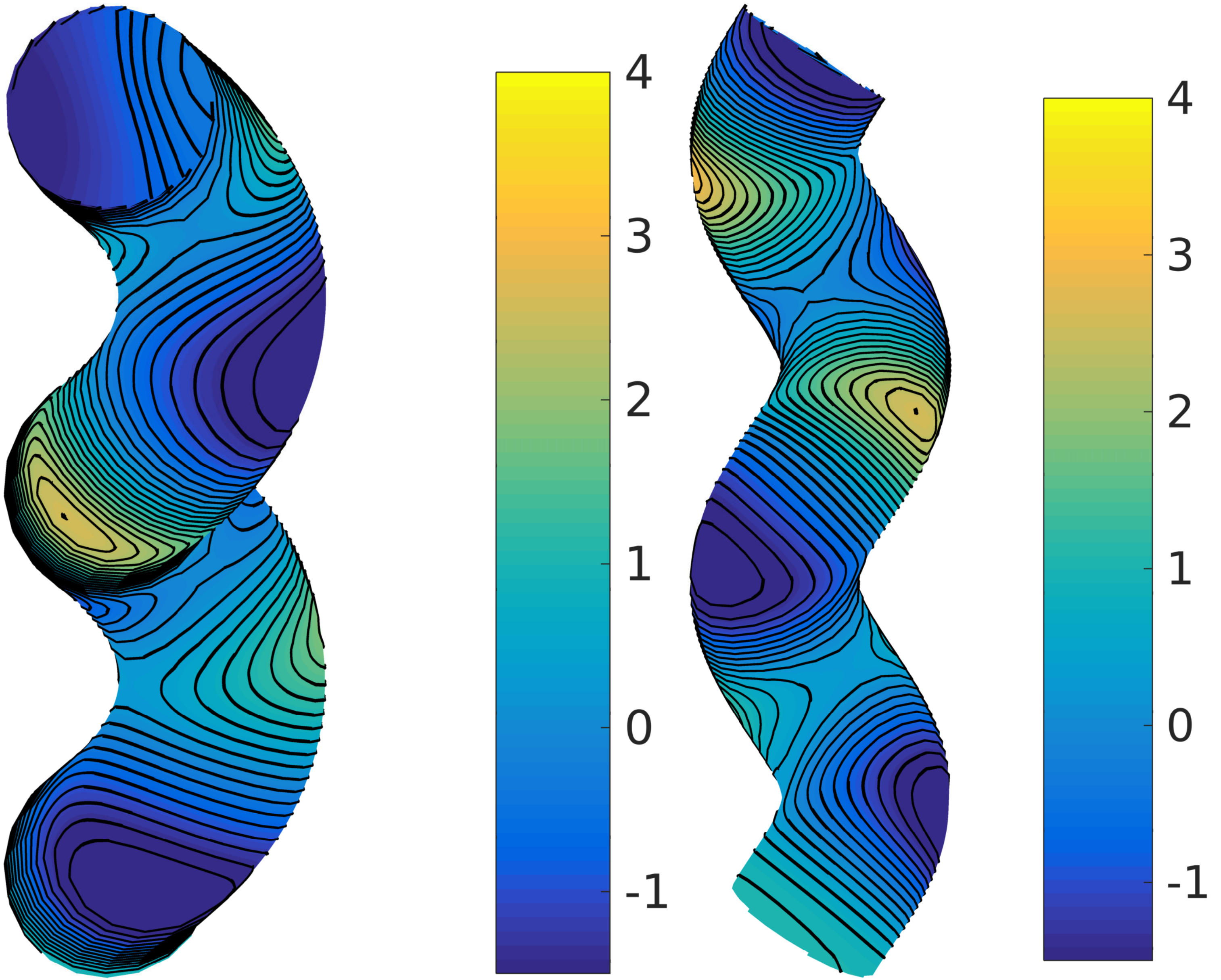
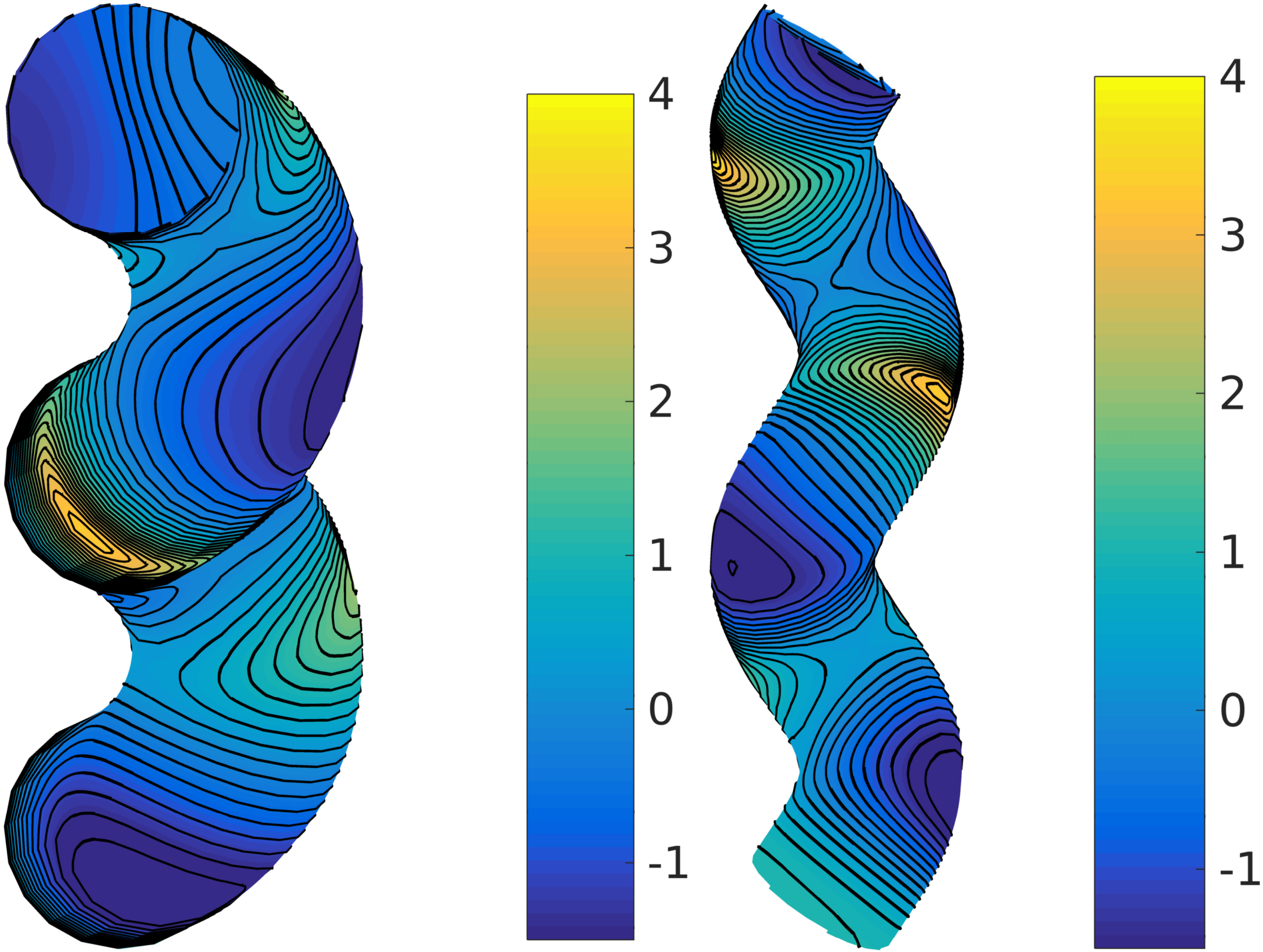
Helix Linear $\tau h = 1.00$



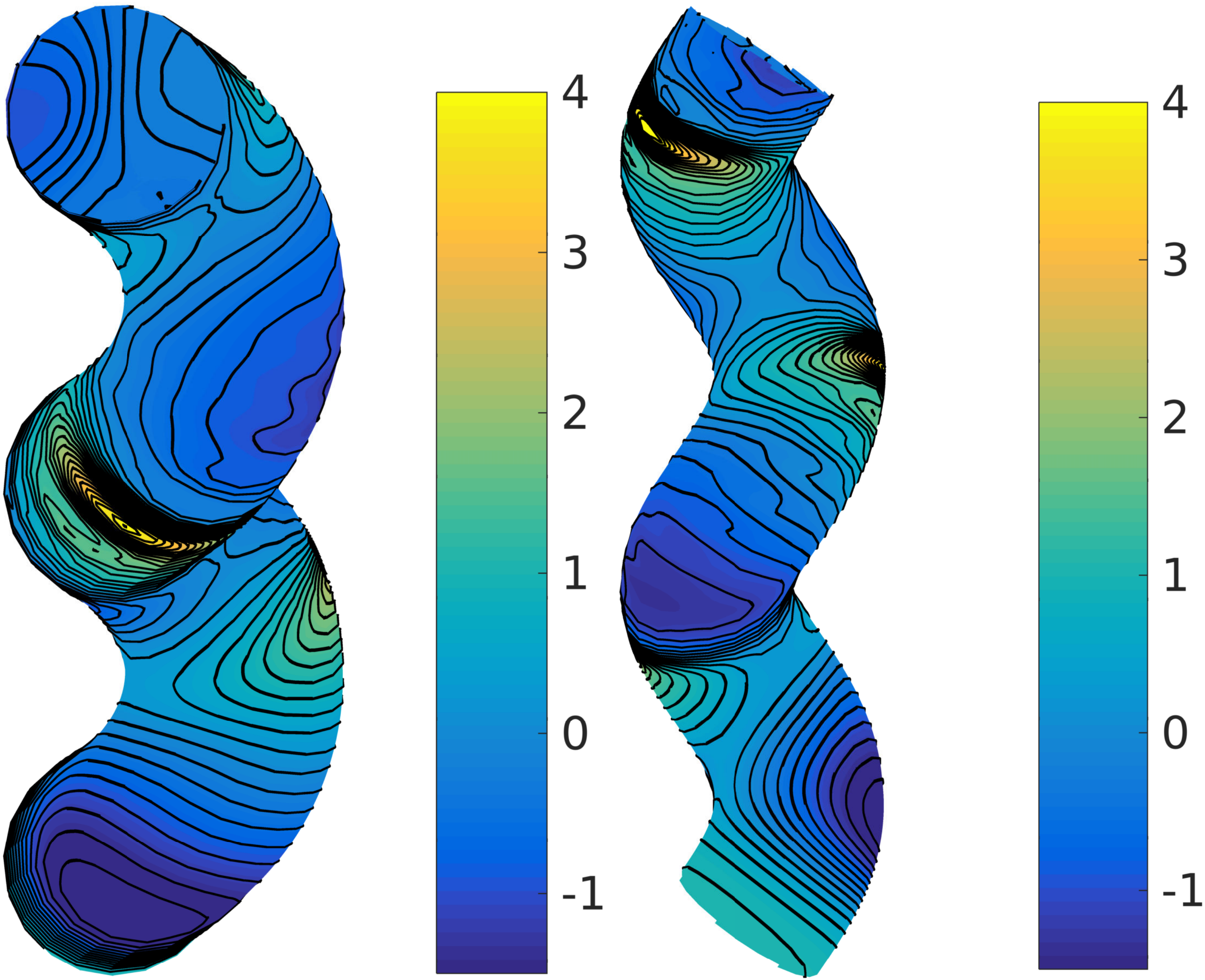
Helix $M = 0.025$ $\tau h = 1.00$



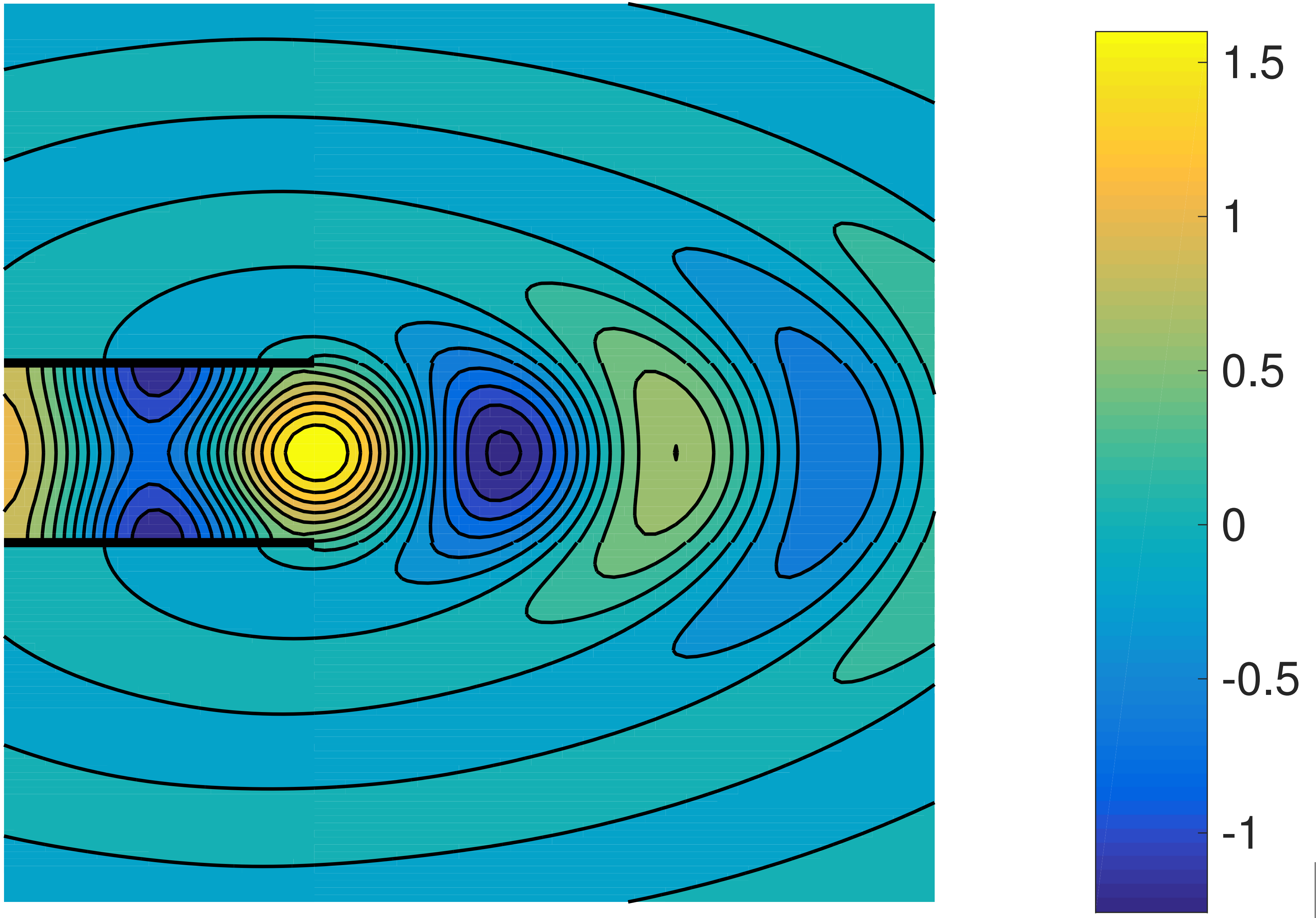
Helix $M = 0.05\ \tau h = 1.00$



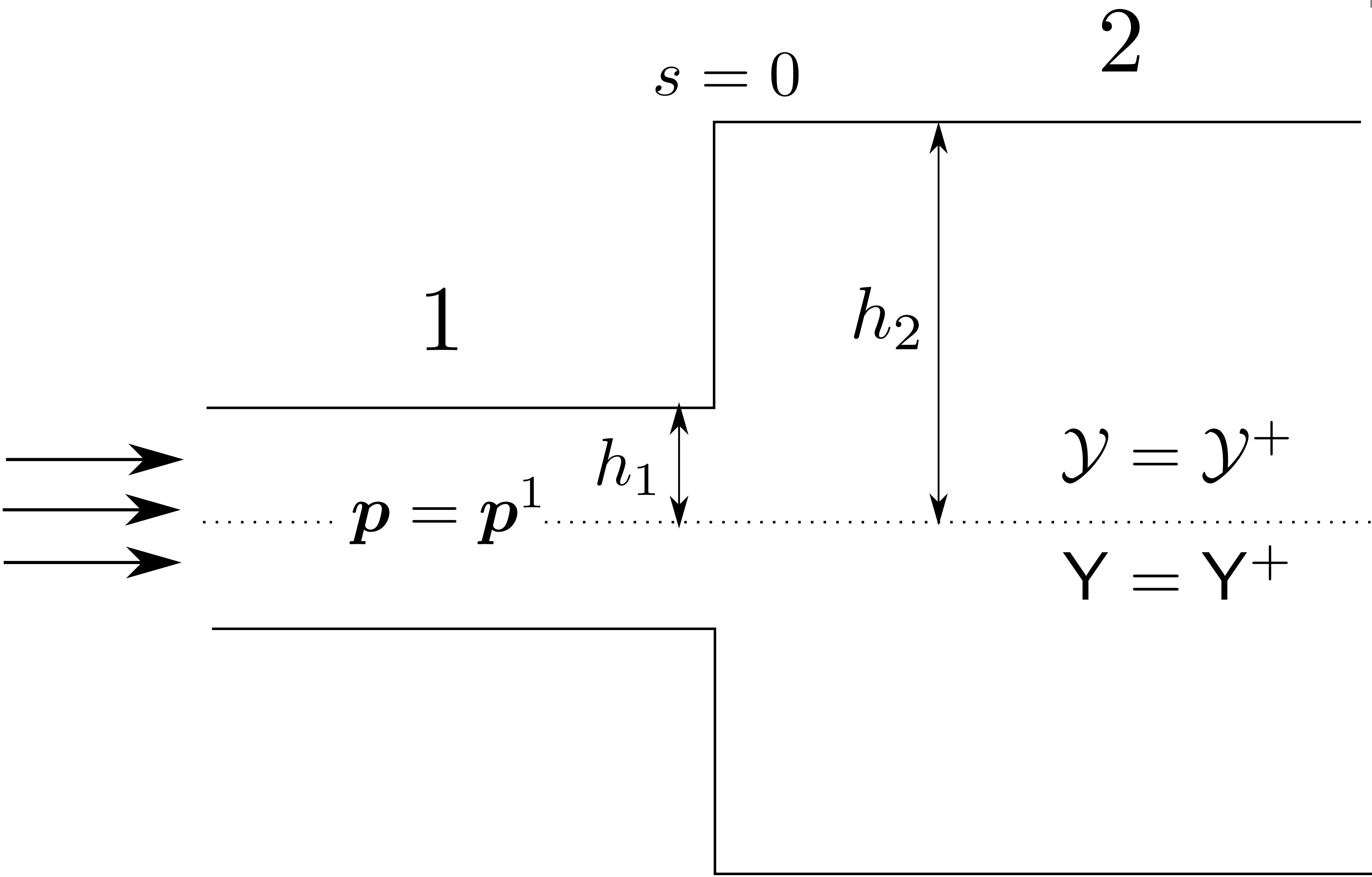
Helix $M = 0.10\ \tau h = 1.00$



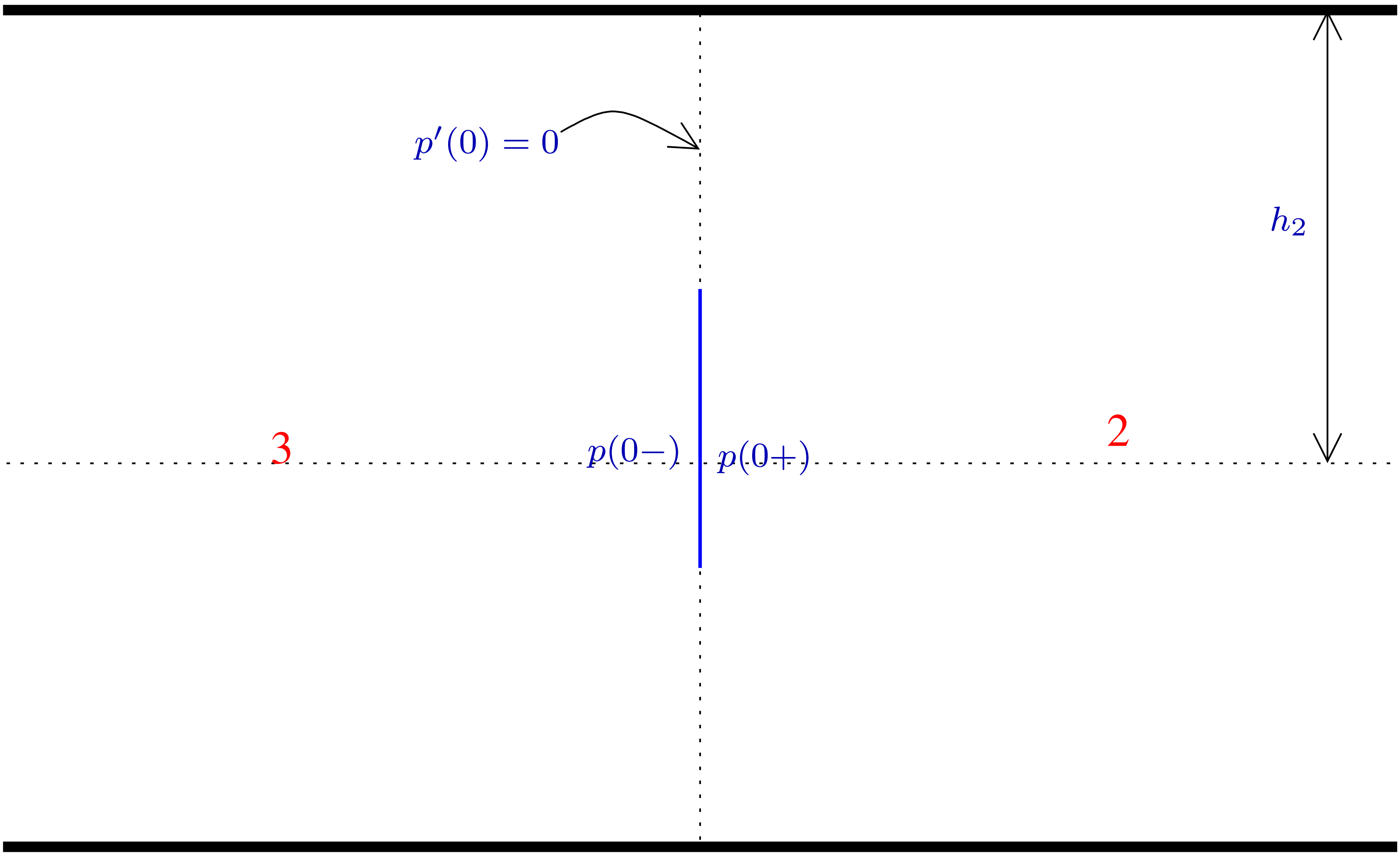
What we don't yet so far



Admittance at a baffled opening I



Admittance at a baffled opening I



Admittance at a baffled opening II

Follow Pagneux, Amir & Kergomard (1996 JASA).

Matching pressure and velocity at $s = 0$ is linear, so take only one Fourier mode.

$$p^i = \sum_{\alpha=0}^{\infty} \psi^i_\alpha P^i_\alpha, \qquad u^i = \sum_{\alpha=0}^{\infty} \psi^i_\alpha U^i_\alpha, \qquad \text{for } i = 1, 2$$

Equating pressure for $r < h_1$,

$$P^1_\alpha = \int_{r<h_1} p^2 \psi^1_\alpha \, \mathrm{d}S = \int_{r<h_1} P^2_\beta \psi^2_\beta \psi^1_\alpha \, \mathrm{d}S = \int_{r<h_1} \psi^1_\alpha \psi^2_\beta \, \mathrm{d}S P^2_\beta = \mathsf{F}_{\alpha\beta} P^2_\beta$$

Equating normal velocity for $r < h_1$,

$$U^2_\alpha = \int_{r<h_2} u^2 \psi^2_\alpha \, \mathrm{d}S = \int_{r<h_1} u^2 \psi^2_\alpha \, \mathrm{d}S = \int_{r<h_1} u^1 \psi^2_\alpha \, \mathrm{d}S = \int_{r<h_1} \psi^2_\alpha \psi^1_\beta \, \mathrm{d}S U^1_\beta = (\mathsf{F}^T)_{\beta\alpha}$$

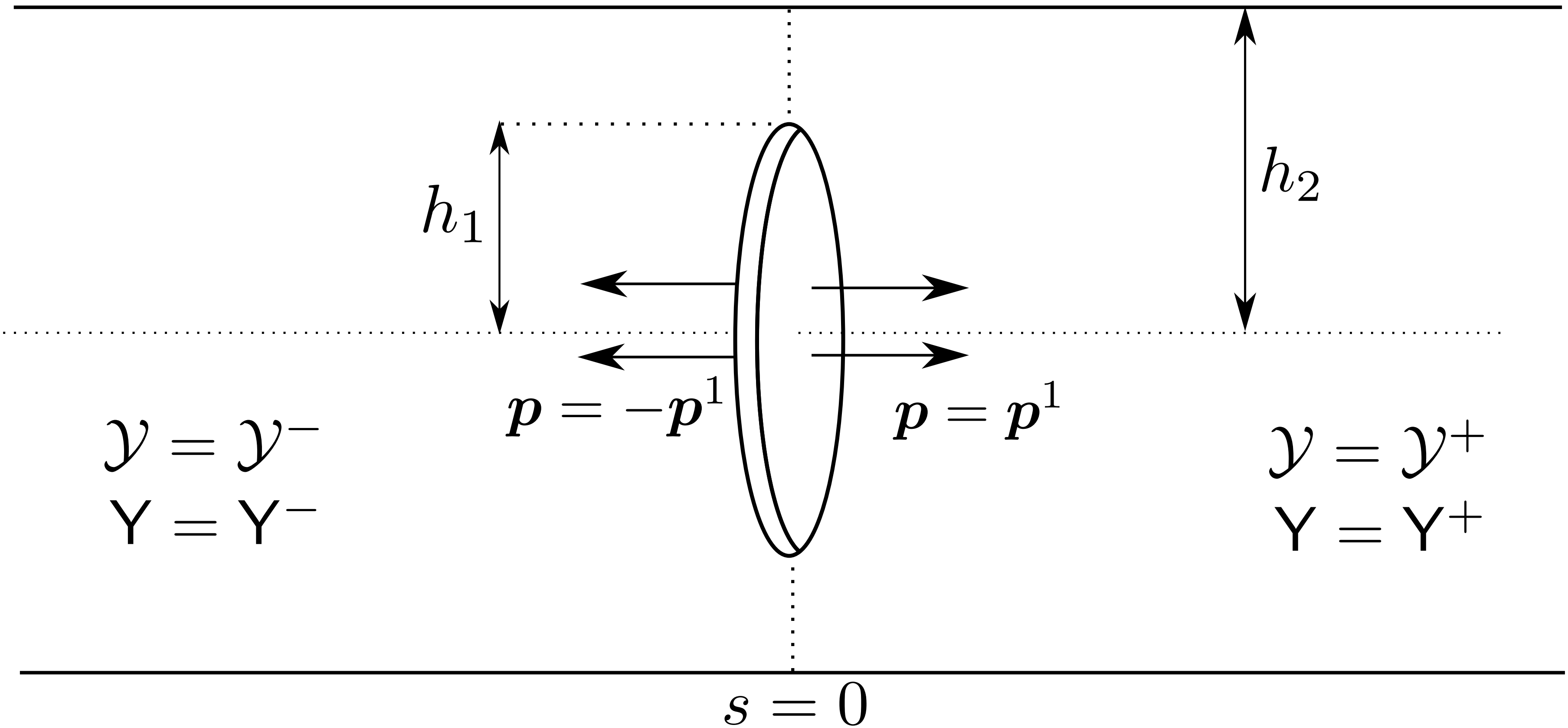
Eliminate $\boldsymbol{p}^2$ and $\boldsymbol{u}^2$ from $\boldsymbol{p}^2 = \mathsf{Z}^+ \boldsymbol{u}^2 + \mathcal{Z}^+[\boldsymbol{u}^2, \boldsymbol{u}^2]$ to get

$$\boldsymbol{p}^1 = \mathsf{F}\mathsf{Z}^+\mathsf{F}^T \boldsymbol{u}^1 + \mathsf{F}\mathcal{Z}^+[\mathsf{F}^T \boldsymbol{u}^1, \mathsf{F}^T \boldsymbol{u}^1]$$

Compare with $\boldsymbol{p}^1 = \mathsf{Z}^1 \boldsymbol{u}^1 + \mathcal{Z}^1[\boldsymbol{u}^1, \boldsymbol{u}^1]$ and invert impedance to find

$$\mathsf{Y}^1 = \left(\mathsf{F}\mathsf{Z}^+\mathsf{F}^T\right)^{-1} \qquad \mathcal{Y}^1 = -\mathsf{Y}^1 \mathsf{F}\mathcal{Z}^+[\mathsf{F}^T \mathsf{Y}^1, \mathsf{F}^T \mathsf{Y}^1]$$

Dipole pressure source I



Dipole pressure source II

As before:

$$P^2_\alpha = \int_{r<h_2} p^2 \psi^2_\alpha \, \mathrm{d}S = \int_{r<h_1} p^2 \psi^2_\alpha \, \mathrm{d}S = \int_{r<h_1} p^1 \psi^2_\alpha \, \mathrm{d}S = \int_{r<h_1} \psi^1_\beta \psi^2_\alpha \, \mathrm{d}S P^1_\beta = (\mathsf{F}^T)_{\beta\alpha}$$

$$U^1_\alpha = \int_{r<h_1} u^2 \psi^1_\alpha \, \mathrm{d}S = \int_{r<h_1} \psi^1_\alpha \psi^2_\beta \, \mathrm{d}S U^2_\beta = \mathsf{F}_{\alpha\beta} U^2_\beta$$

Eliminate $\boldsymbol{p}^2$ and $\boldsymbol{u}^2$ from $\boldsymbol{u}^2 = \mathsf{Y}^+ \boldsymbol{p}^2 + \mathcal{Y}^+[\boldsymbol{p}^2, \boldsymbol{p}^2]$ to get

$$\boldsymbol{u}^1 = \mathsf{F}\boldsymbol{u}^2 = \mathsf{F}\mathsf{Y}^+ \boldsymbol{p}^2 + \mathsf{F}\mathcal{Y}^+[\boldsymbol{p}^2, \boldsymbol{p}^2] = \mathsf{F}\mathsf{Y}^+\mathsf{F}^T \boldsymbol{p}^1 + \mathsf{F}\mathcal{Y}^+[\mathsf{F}^T\boldsymbol{p}^1, \mathsf{F}^T\boldsymbol{p}^1]$$

Compare with $\boldsymbol{u}^1 = \mathsf{Y}^1 \boldsymbol{p}^1 + \mathcal{Y}^1[\boldsymbol{p}^1, \boldsymbol{p}^1]$ and cancel $\boldsymbol{p}^1$ to get

$$\mathsf{Y}^1 = \mathsf{F}\mathsf{Y}^+\mathsf{F}^T \qquad\qquad \mathcal{Y}^1 = \mathsf{F}\mathcal{Y}^+[\mathsf{F}^T, \mathsf{F}^T]$$