Surface and plate waves in phononic crystals

Examples of phononic crystals for surface and plate waves

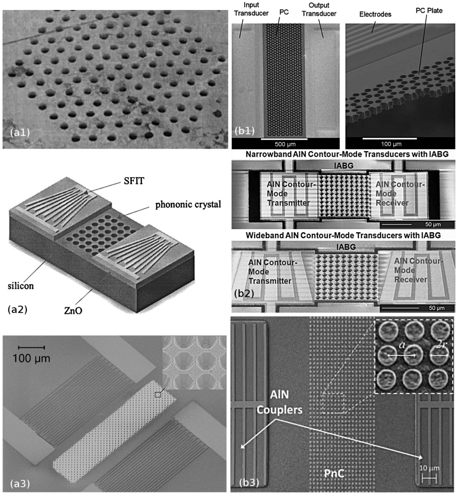

Figure: (a1) Marble [15]. (a2) Si [16]. (a3) $LiNbO_3$ [17]. (b1) Si [18]. (b2) AlN [19]. (b3) W in Si [20].

FEM for phononic crystal slab

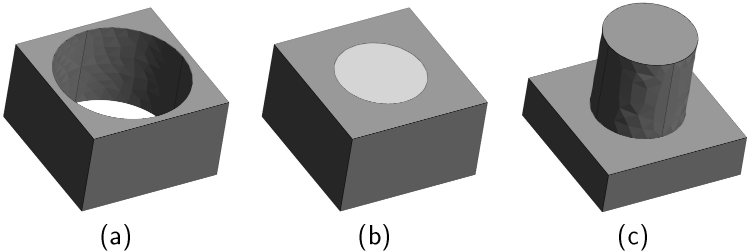

Figure: **Some possible phononic crystal slab unit cells, here shown for a square lattice.** Possible inclusions are (a) holes, (b) filled solid holes, or (c) pillars.

Obtaining the band structure for phononic crystal slabs is similar to the case of 2D phononic crystals: apply periodic boundary conditions on the lateral sides and solve in 3D.

FEM meshes for phononic crystal slab

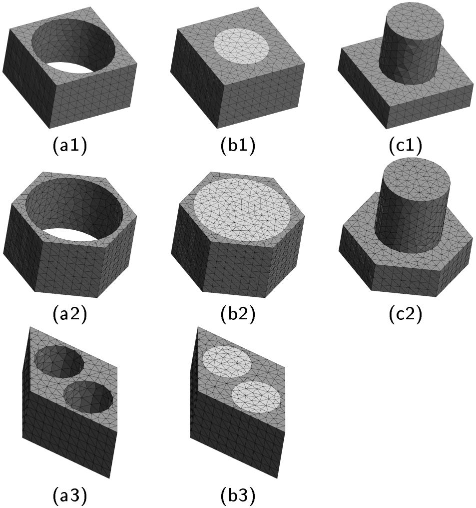

Figure: (1) SQ, (2) HEX, and (3) HC. (a) hollow inclusion, (b) solid inclusion, and (c) pillar sitting on a membrane.

Phononic crystal slab of holes in silicon, HC I

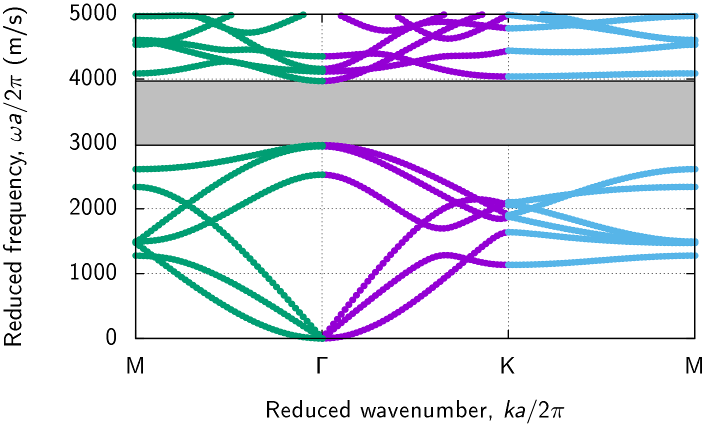

Figure: $h/a = 0.6$ and $d/a = 0.5$.

Phononic crystal slab of holes in silicon, HC II

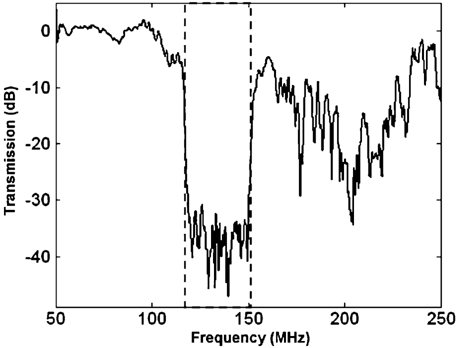

Figure: Geometrical parameters are $a = 26$ μm, $d = 12.8$ μm, and $h = 15$ μm. Transmission is obtained with a pair of interdigital transducers and normalized to the value without the phononic crystal [18].

Phononic crystal slab of holes in AlN/SiO_2, SQ

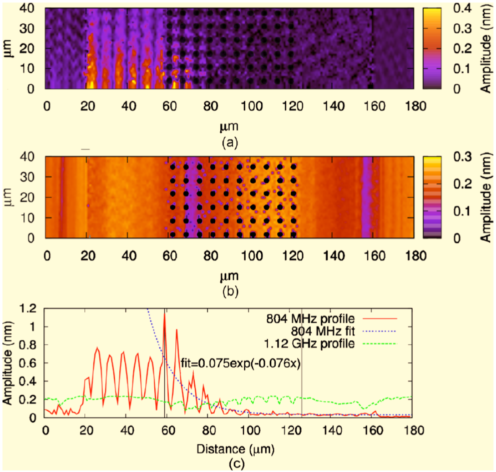

Figure: Optical measurements at (a) 804 MHz and (b) 1120 MHz. (c) Fitted against an exponentially decreasing law [21].

Solid-solid phononic crystal slab of steel beads in epoxy, SQ

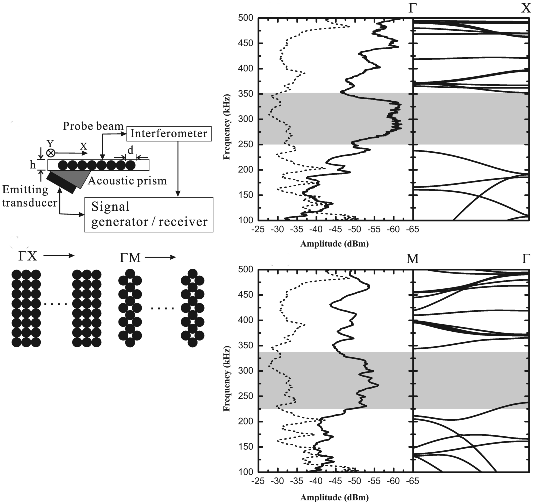

Figure: $d = a = 4$ mm [22].

Phononic crystal slab of quartz in epoxy, SQ

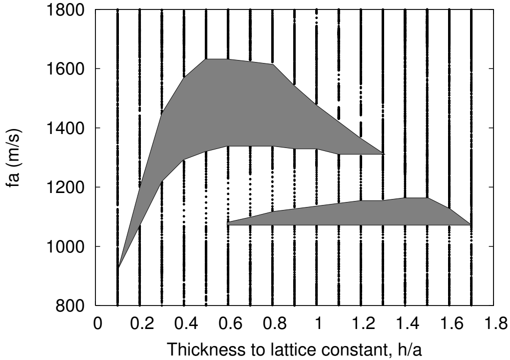

Figure: **Gap map for the square-lattice phononic crystal slab composed of quartz inclusions in an epoxy matrix.** Filling fraction $F = 0.5$. $fa = \omega a/(2\pi)$ [23]

Phononic crystal slab of tungsten in silica, SQ I

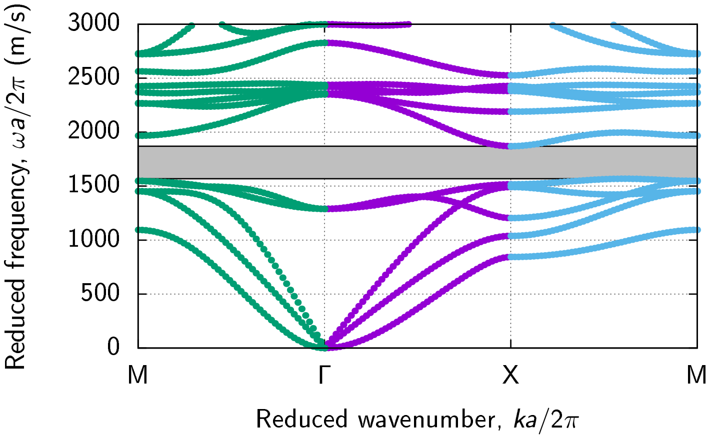

Figure: $h/a = 1$ and $d/a = 0.48$.

Phononic crystal slab of tungsten in silica, SQ II

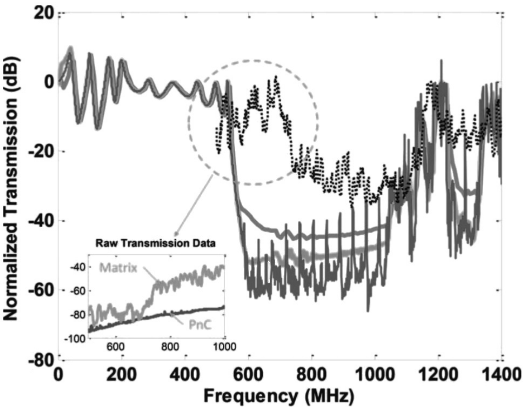

Figure: **Transmission measurements of a phononic crystal slab composed of tungsten (W) rods in a silicon dioxide (SiO_2) matrix.** $a = 2.5$ µm, $d = 1.4$ µm, and $h = 1.85$ µm [24].

Comparison of holey and solid-solid PC slabs

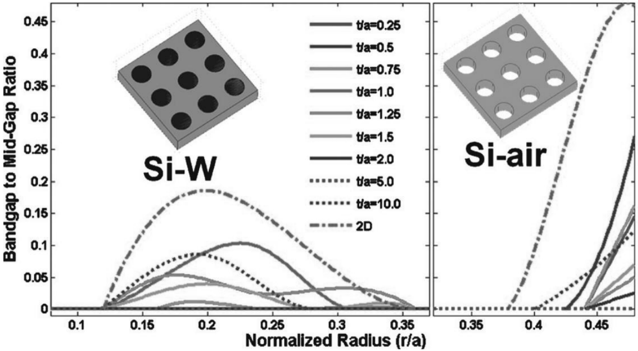

Figure: Phononic band gap versus inclusion radius for a range of slab thicknesses for a phononic crystal formed in a silicon matrix with a square lattice array of cylindrical air holes (right panel) and tungsten rods (left). The full 2D band gaps are shown by the dotted curve [25].

Semi-infinite surface phononic crystals I

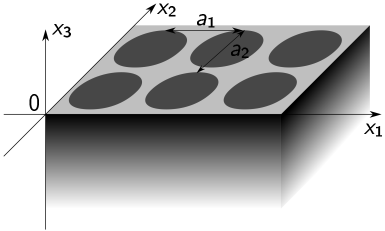

Figure: **Schematic geometry of elastic wave propagation on the surface of a 2D phononic crystal.**

- Surface Bloch waves are 2D-periodic and satisfy surface boundary conditions (e.g., $T_n = 0$) $\longrightarrow$ use a 2D mesh in the plane (x_1, x_2)
- Bloch's theorem must be expressed for both displacements and stresses

$$u_i(t, \boldsymbol{x}) = \tilde{u}_i(\boldsymbol{x}) \exp(\imath(\omega t - \boldsymbol{k} \cdot \boldsymbol{x})),$$
$$T_{ij}(t, \boldsymbol{x}) = \tilde{T}_{ij}(\boldsymbol{x}) \exp(\imath(\omega t - \boldsymbol{k} \cdot \boldsymbol{x}))$$

Semi-infinite surface phononic crystals II

- The natural variables are displacements and normal stresses:

$$\int_\Omega u_i^{t*}(\omega^2 \rho_{ik} u_k + T_{ij,j}) + \int_\Omega T_{i3}^{t*}(T_{i3} - c_{i3kl} u_{k,l}) = 0.$$

leading to [2]:

$$\omega^2 \int_\Omega \tilde{u}_i^{t*} \rho_{ik} \tilde{u}_k - \sum_{j,l=1}^{2} \int_\Omega \tilde{u}_{i,j}^{t*} c_{ijkl} \tilde{u}_{k,l} - \sum_{l=1}^{2} \int_\Omega \tilde{T}_{i3}^{t*} c_{i3kl} \tilde{u}_{k,l} + \int_\Omega \tilde{T}_{i3}^{t*} \tilde{T}_{i3}$$
$$= \imath k_3 \left(\sum_{j=1}^{2} \int_\Omega \tilde{u}_{i,j}^{t*} c_{ijk3} \tilde{u}_k + u_i^{t*} T_{i3} - \int_\Omega \tilde{T}_{i3}^{t*} c_{i3k3} \tilde{u}_k \right).$$

- Generalized eigenvalue problem:

$$\begin{bmatrix} \omega^2 R - B & 0 \\ -C_2 & I_d \end{bmatrix} \mathbf{h} = \imath k_3 \begin{bmatrix} C_1 & I_d \\ -D & 0 \end{bmatrix} \mathbf{h} \quad ; \boldsymbol{h} = (\tilde{u}_i, \tilde{T}_{i3}), i = 1 \ldots r.$$

Semi-infinite surface phononic crystals III

- Once the Q eigenvalues and eigenvectors are found, form expansion in partial waves:

$$\mathbf{h}(\mathbf{x}, t) = \sum_{q=1}^{Q} A_q \tilde{\mathbf{h}}_q(\boldsymbol{x}) \exp(\jmath(\omega t - \mathbf{k}_q \cdot \mathbf{x})),$$

- Keep only partial waves that are *inside* the substrate (selection rule):

$$\text{Select partial wave } q \text{ if } \begin{cases} P_{3q} < 0 & \text{if } \Im(k_{3q}) = 0, \\ \Im(k_{3q}) > 0 & \text{otherwise} \end{cases}$$

with the Poynting vector

$$P_{3q} = \frac{1}{2S(\Omega)} \int_{\Omega} \Re(\imath\omega \tilde{T}_{i3q} \tilde{u}_{iq}^{*})$$

- Finally, compute the surface boundary condition determinant $\Delta(\omega, k_1, k_2) = |T_{i3q}| = 0$.

Surface boundary condition determinants

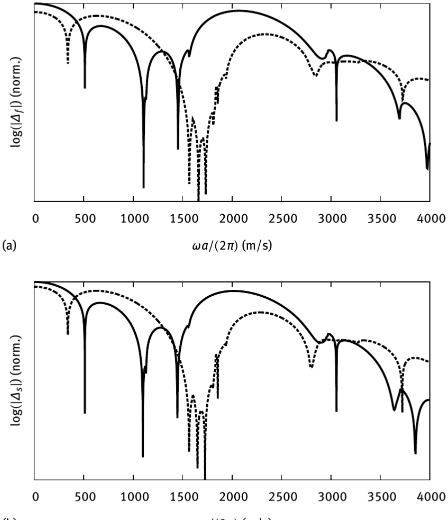

Figure: Free (a) and shorted (b) boundary conditions determinants for the X (solid line) and the M (dotted line) points of the first Brillouin zone, for a Y-cut SQ lithium niobate surface phononic crystal with $d/a = 0.9$ [26].

Semi-infinite surface phononic crystal: band structure

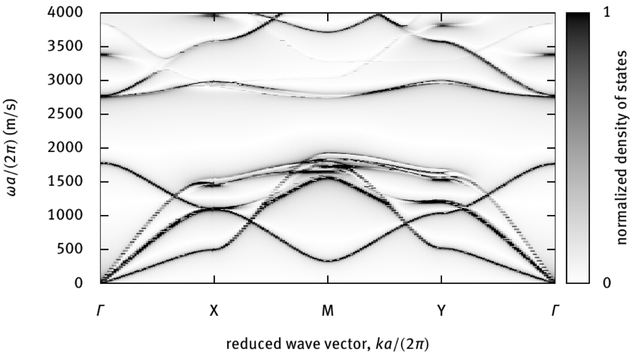

Figure: **Total density of surface states plotted along the irreducible Brillouin zone.** The plot is along the path Γ-X-M-Y-Γ around the irreducible Brillouin zone, for a Y-cut square lattice lithium niobate surface phononic crystal with $d/a = 0.9$ [26].

Finite-depth surface phononic crystals: the crystal layer

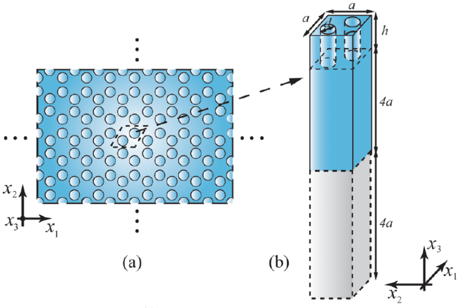

Figure: **The finite depth surface phononic crystal problem.** 3D mesh with truncated substrate [27]

- In practice, one cannot etch infinitely deep holes
- $\longrightarrow$ phononic crystal layer on a (semi-infinite) substrate
- *Sound cone*: radiation limit of bulk waves in the substrate

Phononic band structure for surface guided waves

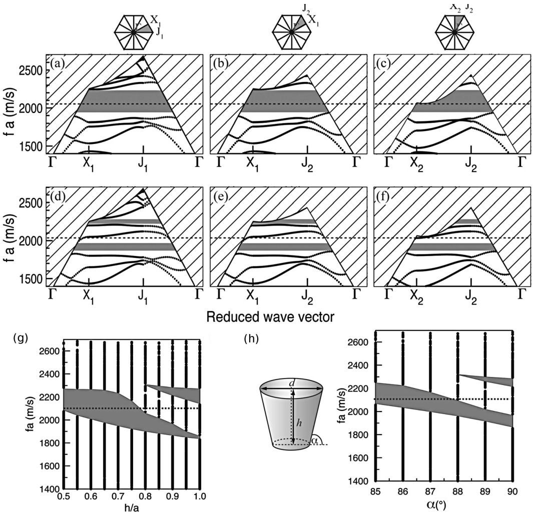

Figure: **Finite-depth HC phononic crystals.** Y-cut lithium niobate. (a)-(c) $h = 0.7a$, $d = 0.5a$, and (d)-(f) $h = 0.9a$, $d = 0.5a$ [27]

The first finite-depth surface phononic crystal

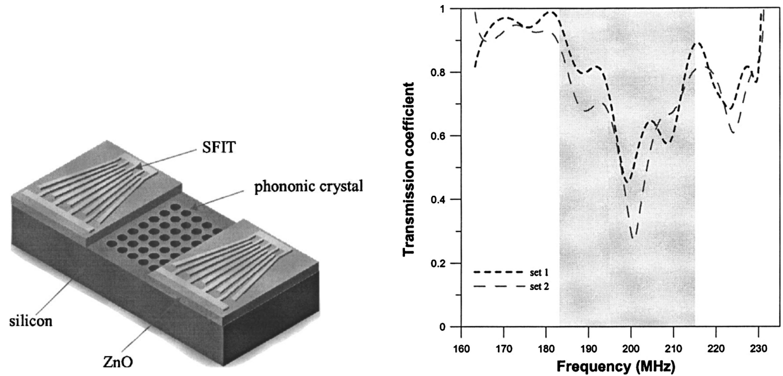

Figure: **Experimental transmission of a surface phononic crystal in silicon** [16]

Optical probing by short pulse generation

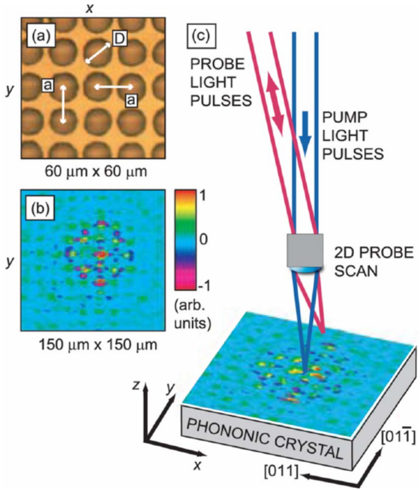

Figure: **Optical pump-probe experiments with a silicon surface phononic crystal.** (a) Si phononic crystal coated with a 40 nm gold layer. $d = 12$ µm and $a = 15$ µm. (b) SAW image at delay time $\tau = 7.4$ ns. (c) Outline of experimental setup with SAW image at $\tau = 10.3$ ns [28]

The first complete-band-gap surface phononic crystal

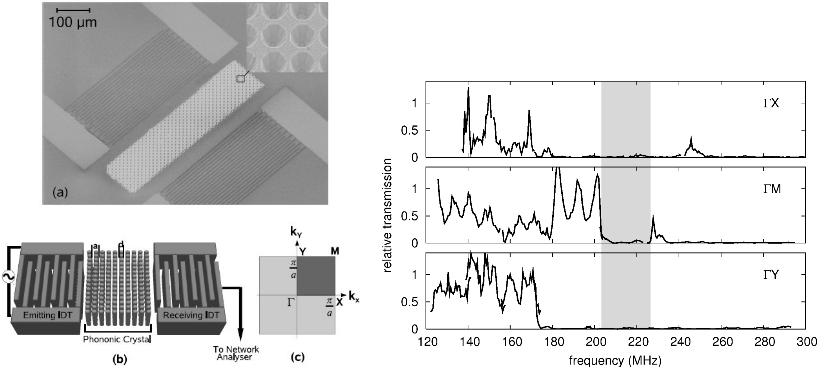

Figure: **Full phononic band gap for a square-lattice Y-cut lithium niobate surface phononic crystal.** $d/a = 0.9$, $a = 10\ \mu\text{m}$ [17]

Optical interferometric observation of SAW phononic crystal

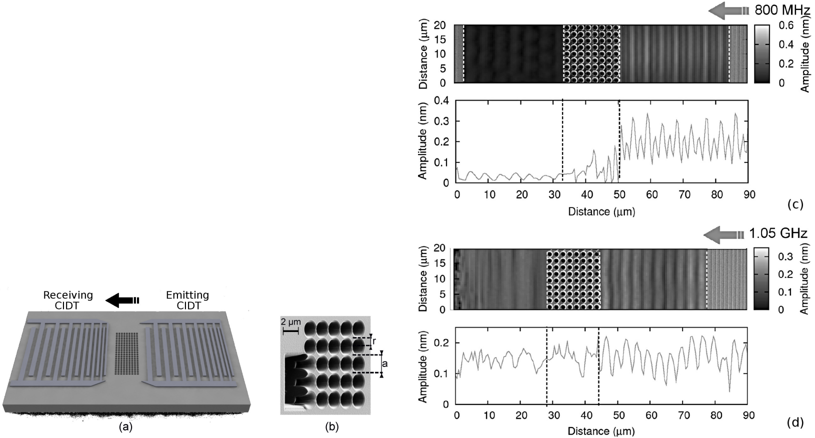

Figure: **Heterodyne optical interferometer measurement.** $a = 2.2$ µm, $d = 2$ µm [29]