Acoustic/elastic coupling

Acoustic/elastic coupling I

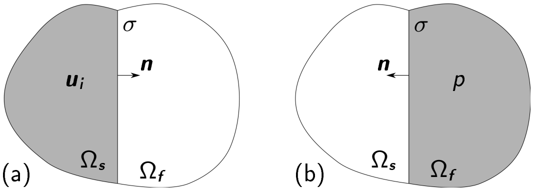

Figure: Schematic representation of the coupling of elastic waves in a solid domain, Ω_s, with the acoustic waves in a fluid domain, Ω_f.

Both domains are a decomposition of the total domain $\Omega = \Omega_s \cup \Omega_f$ and they are separated by boundary $\sigma = \Omega_s \cap \Omega_f$.

1. If the displacement field $\boldsymbol{u}_i$ is known inside Ω_s, then the normal acceleration at the interface is also known and becomes a boundary condition on σ for the fluid domain: $\omega^2 u_n = \frac{1}{\rho_f}\frac{\partial p}{\partial n}$.
2. Reciprocally, if the pressure field p is known inside Ω_f, then the normal traction on interface σ is known and becomes a boundary condition for the solid domain: $T_{ij}n_j = -pn_i$.

Acoustic/elastic coupling II

- The weak forms of pressure and displacements becomes coupled at the interface:

$$\begin{aligned} \omega^2 \left(\int_{\Omega_f} p' \frac{1}{B} p + \int_\sigma p' u_n \right) &= \int_{\Omega_f} p'_{,i} \frac{1}{\rho_f} p_{,i}, \\ \omega^2 \int_{\Omega_s} u'_i \rho_s u_i &= \int_\sigma p u'_n + \int_{\Omega_s} u'_{i,j} c_{ijkl} u_{k,l}. \end{aligned}$$

- The band structure can be obtained from a generalized eigenvalue problem

$$A(k)\mathbf{y} = \omega^2 B \mathbf{y}$$

where vector $\mathbf{y}$ is composed of the values of p and u_i, $i = 1, 2, 3$ at the nodes of the meshes of the domains Ω_f and Ω_s.

$$\begin{aligned} A(k) &= \begin{bmatrix} A_{11}(k) & 0 \\ A_{21} & A_{22}(k) \end{bmatrix}, \\ B &= \begin{bmatrix} B_{11} & B_{12} \\ 0 & B_{22} \end{bmatrix}. \end{aligned}$$

Steel rods in water, revisited

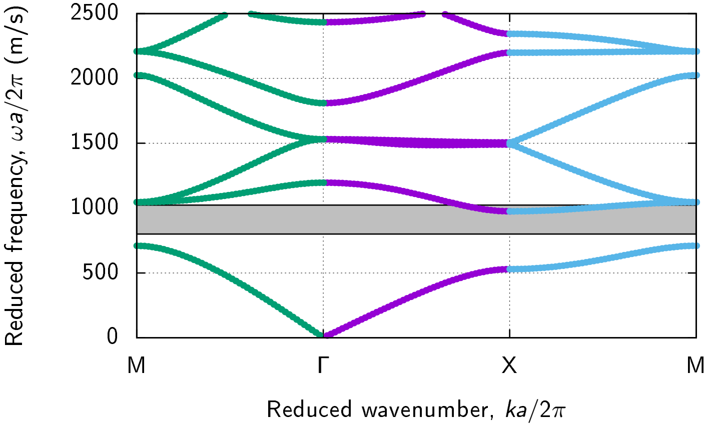

Figure: **2D SQ sonic crystal of steel rods in water.** $d/a = 0.833$. Gray area: full band gap predicted with the pressure wave model.

2D SQ sonic crystal of nylon rods in water

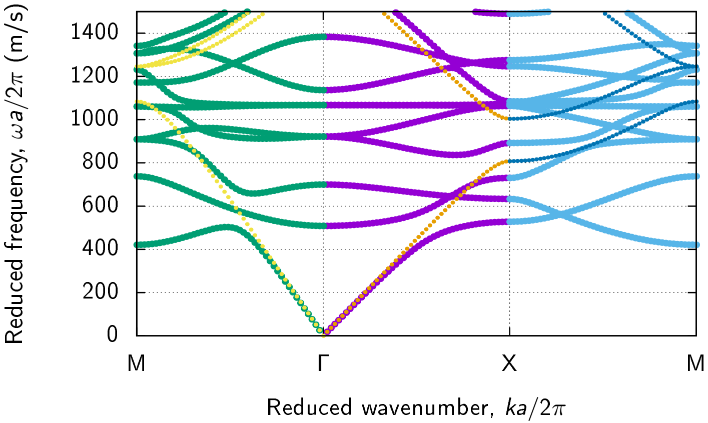

Figure: $d/a = 0.8$. For comparison, the FEM computation for pressure waves is shown with small dots.

2D SQ phononic crystal of cylindrical air holes in silicon

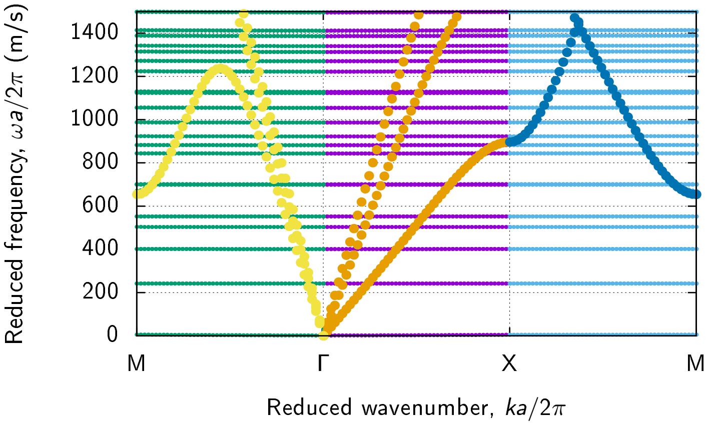

Figure: $d/a = 0.8$. Small dots: acousto-elastic coupling. Large dots: pure elastic case (free hole boundaries). The many flat bands are produced by resonant modes of the air cylinders.

2D SQ PC of holes in tungsten, filled with a fluid

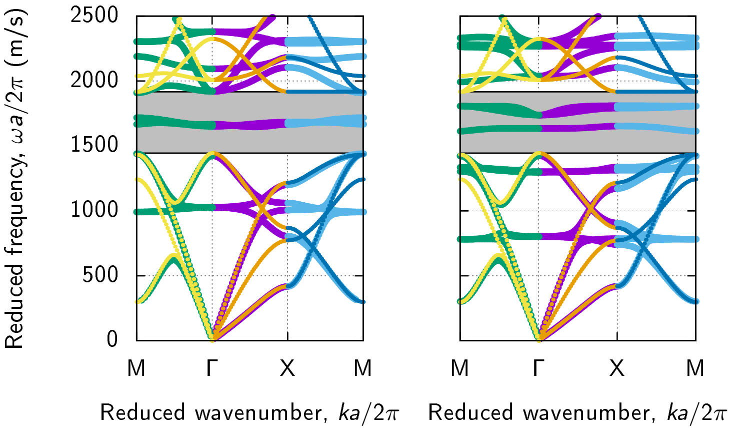

Figure: $d/a = 0.874$. Small dots: hollow phononic crystal. Large dots: acousto-elastic coupling. The fluid is (a) water or (b) Propanol.

2D SQ PC of holes in aluminum, filled with a fluid

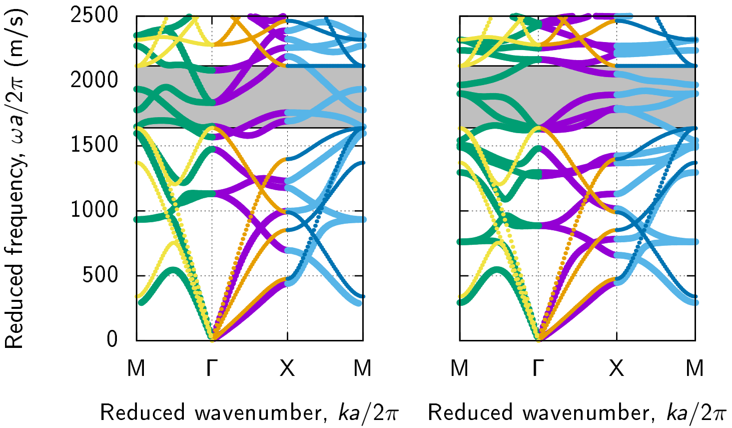

Figure: $d/a = 0.874$. Small dots: hollow phononic crystal. Large dots: acousto-elastic coupling. The fluid is (a) water or (b) Propanol.