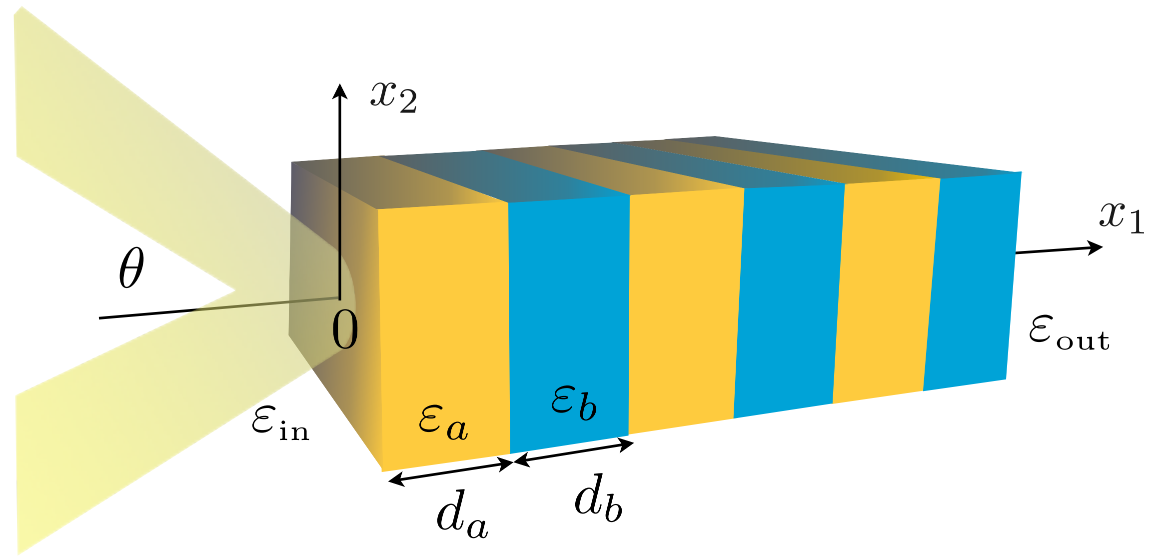
Multilayer structure
High order homogenization



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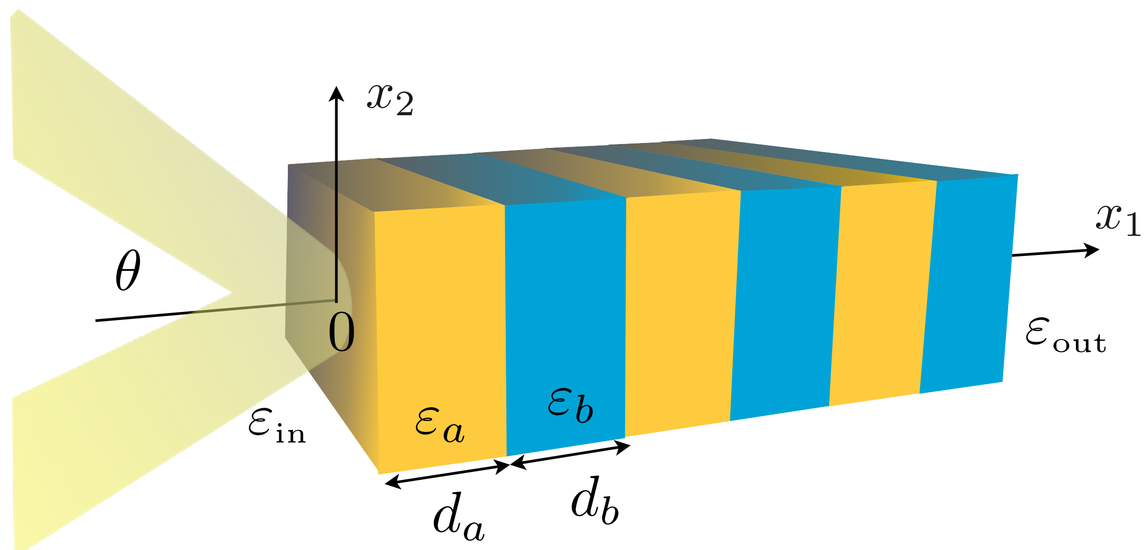

PRL **113,** 243901 (2014) PHYSICAL REVIEW LETTERS week ending 12 DECEMBER 2014

Subwavelength Multilayer Dielectrics: Ultrasensitive Transmission and Breakdown of Effective-Medium Theory

Hanan Herzig Sheinfux, Ido Kaminer, Yonatan Plotnik, Guy Bartal, and Mordechai Segev
Technion–Israel Institute of Technology, Haifa 32000, Israel
(Received 26 May 2014; published 11 December 2014)

We show that a purely dielectric structure made of alternating layers of deep subwavelength thicknesses exhibits novel transmission effects which completely contradict conventional effective medium theories exactly in the regime in which those theories are commonly used. We study waves incident at the vicinity of the effective medium's critical angle for total internal reflection and show that the transmission through the multilayer structure depends strongly on nanoscale variations even at layer thicknesses smaller than $\lambda/50$. In such deep subwavelength structures, we demonstrate dramatic changes in the transmission for variations in properties such as periodicity, order of the layers, and their parity. In addition to its conceptual importance, such sensitivity has important potential applications in sensing and switching.

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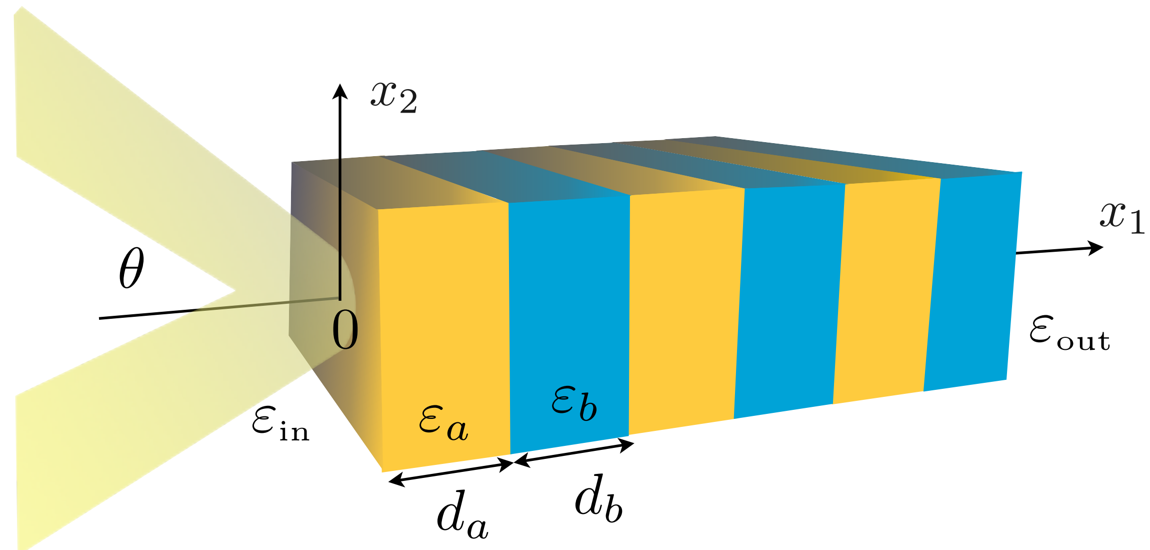

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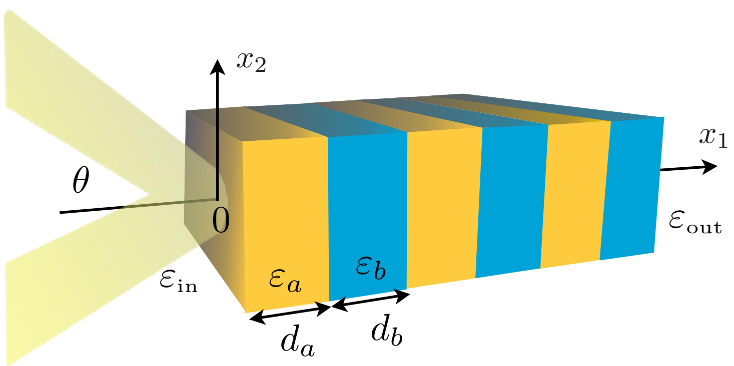

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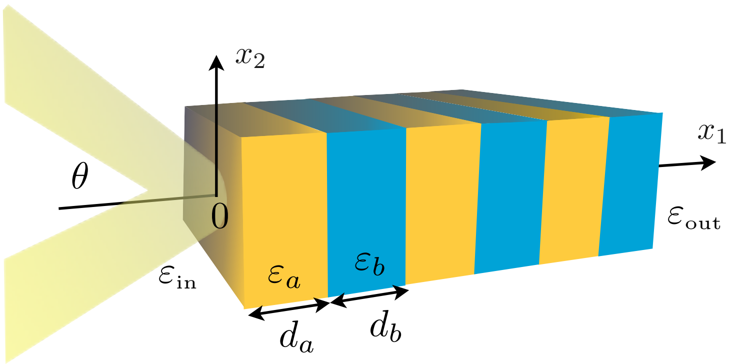

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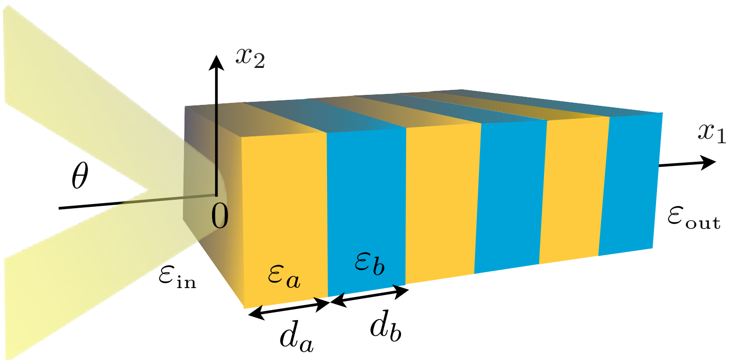

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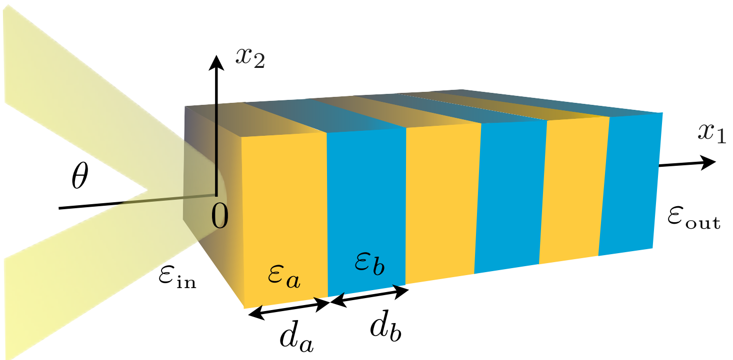
Multilayer structure

High order homogenization



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$$\langle\epsilon\rangle = \frac{d_a\epsilon_a + d_b\epsilon_b}{d_a + d_b}$$

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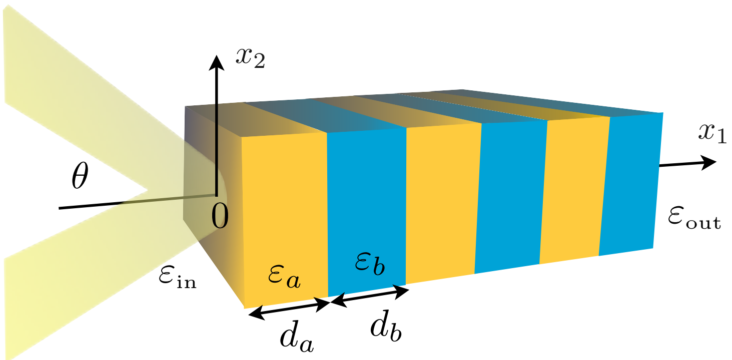

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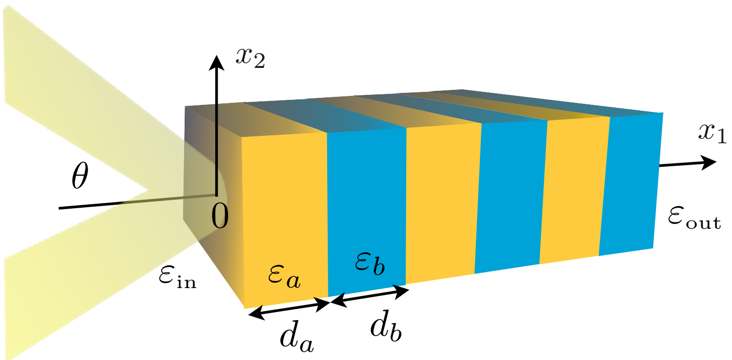
unusual scattering properties

(a) regular order

(b) reversed order

Multilayer structure
High order homogenization



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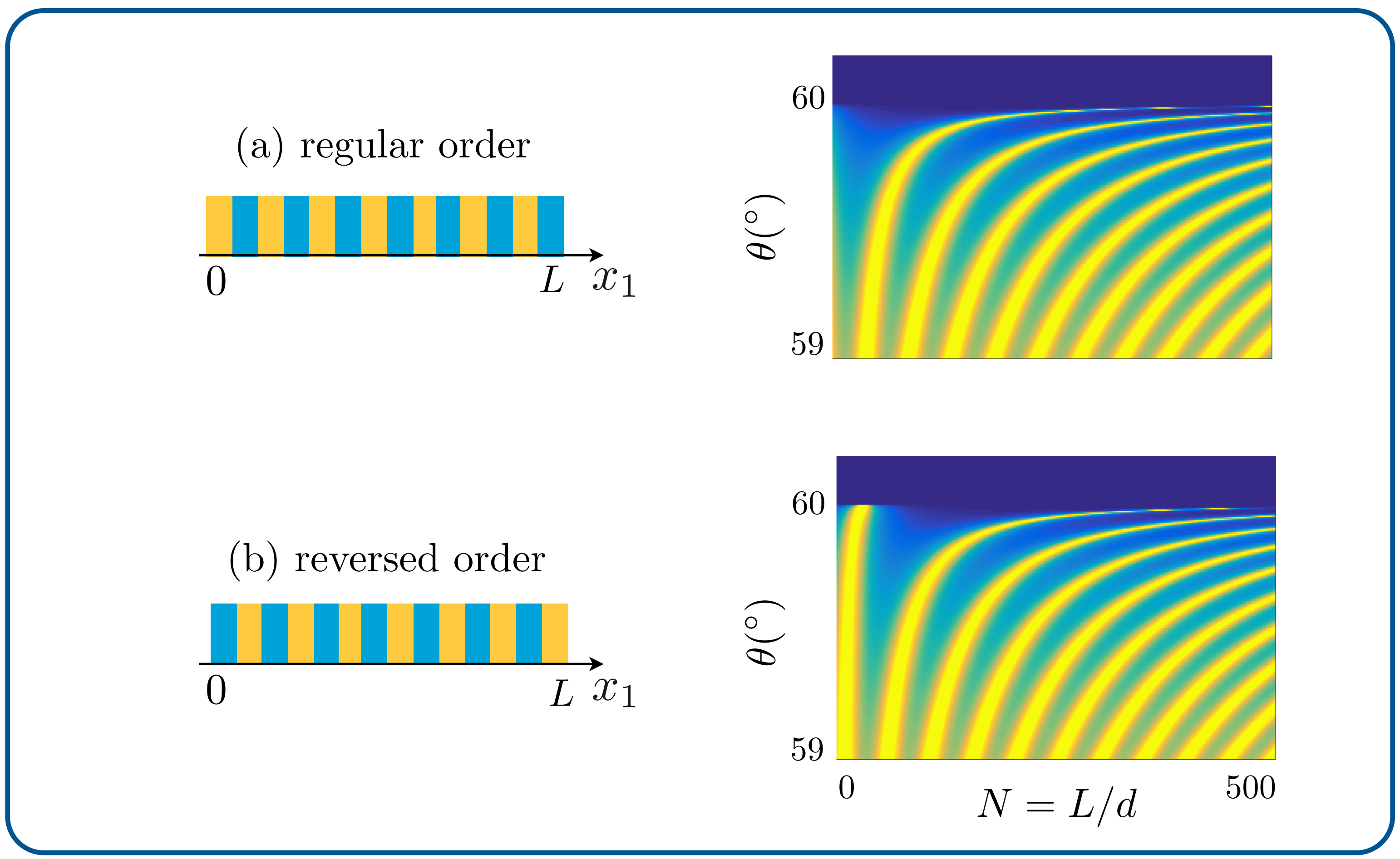
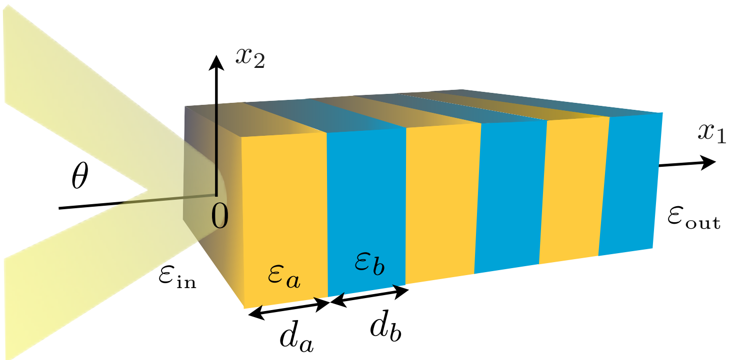


Multilayer structure
High order homogenization



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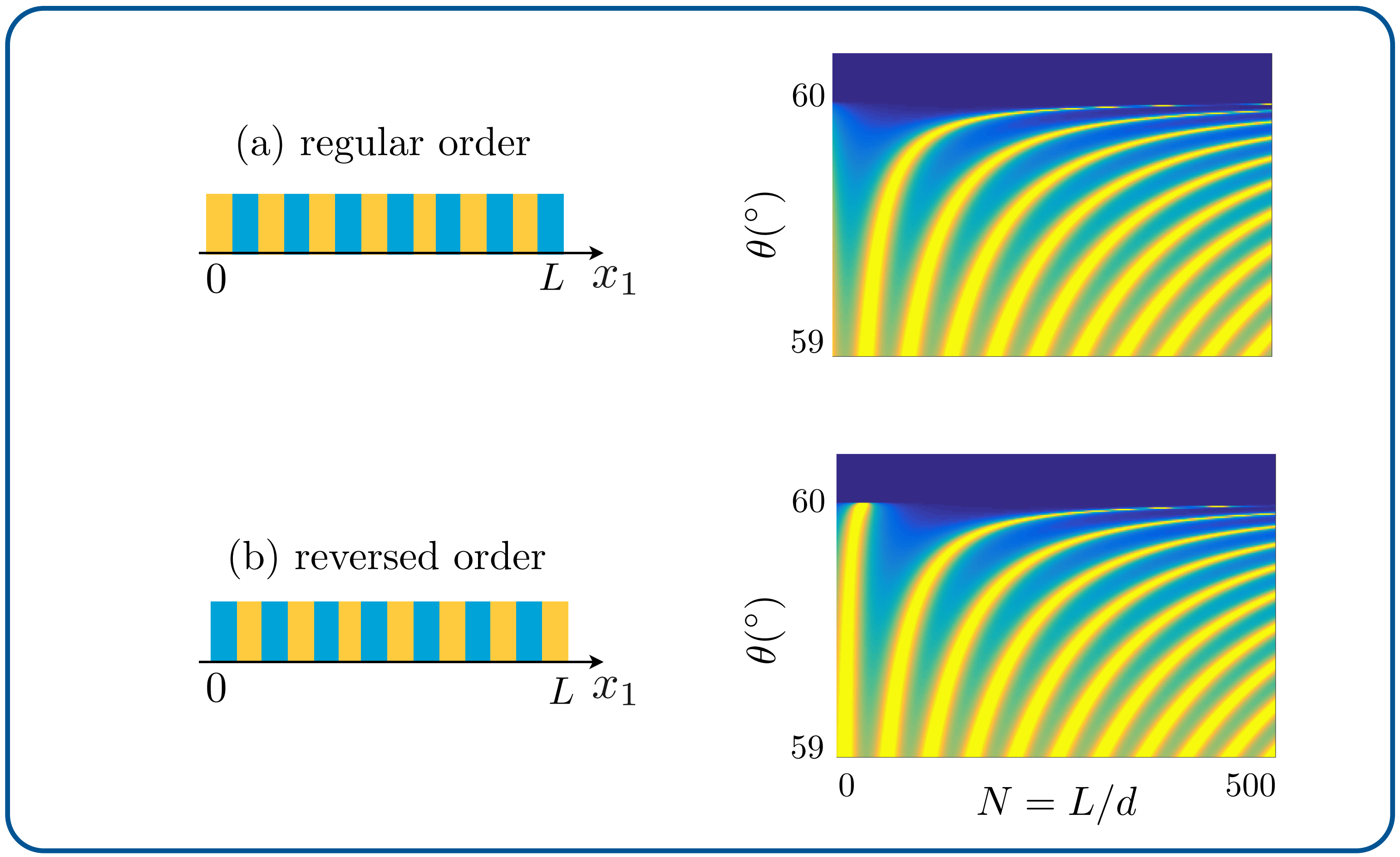
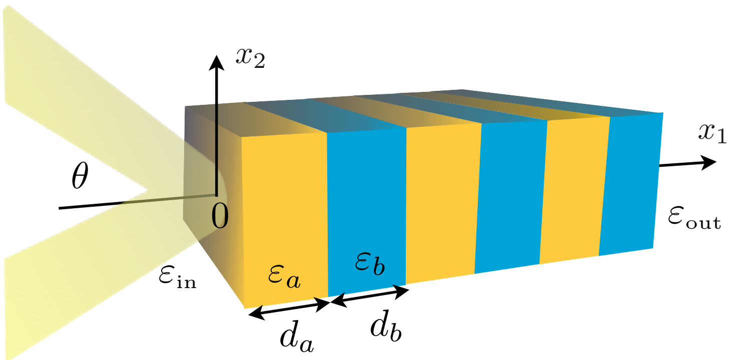


not predicted by the hom. 0

Multilayer structure
High order homogenization



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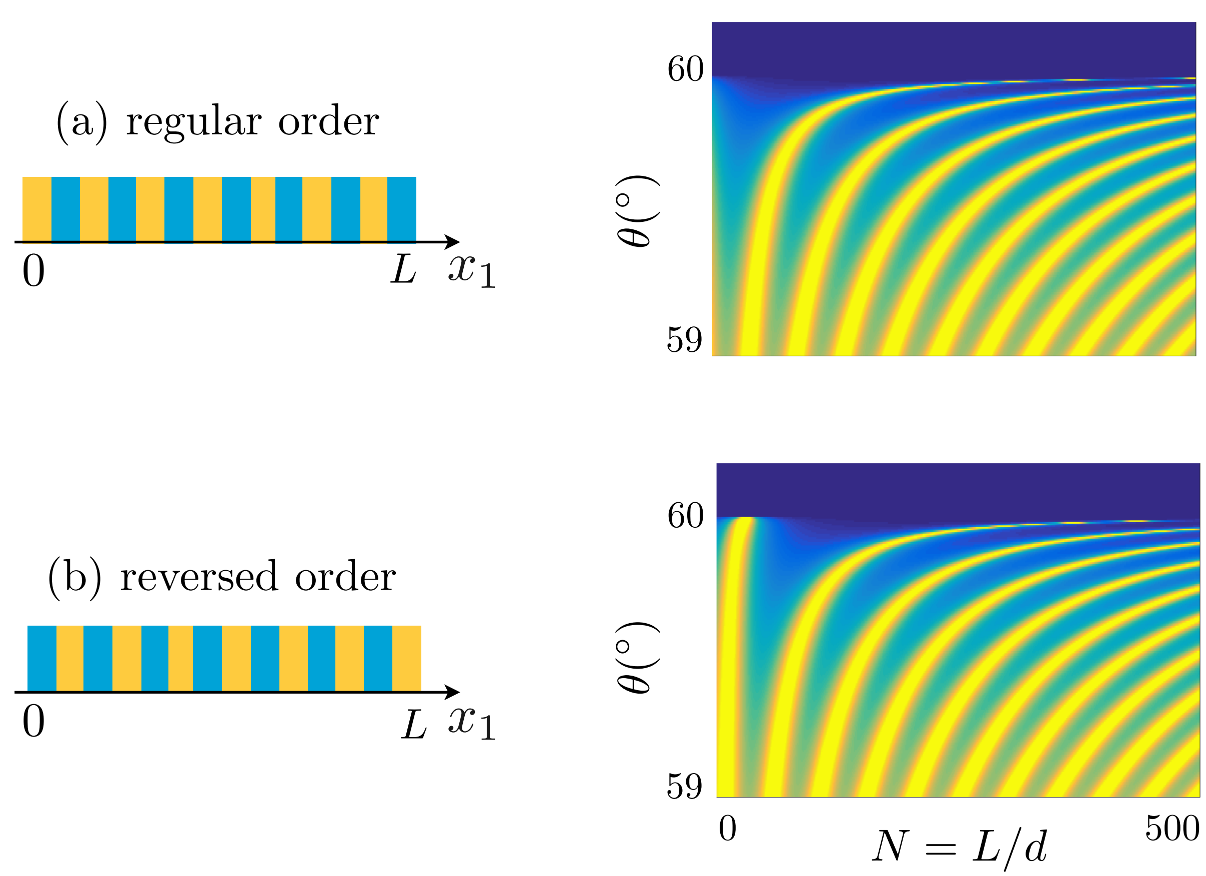

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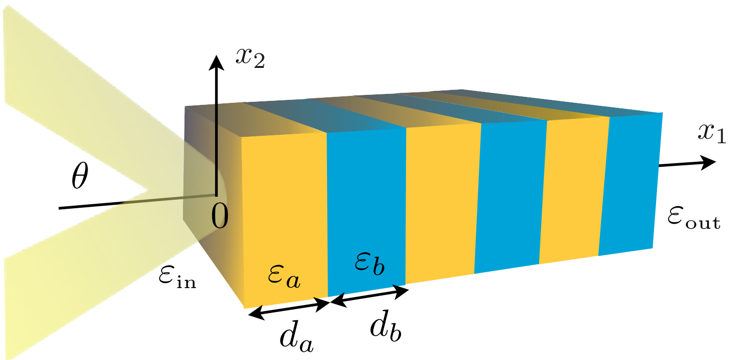

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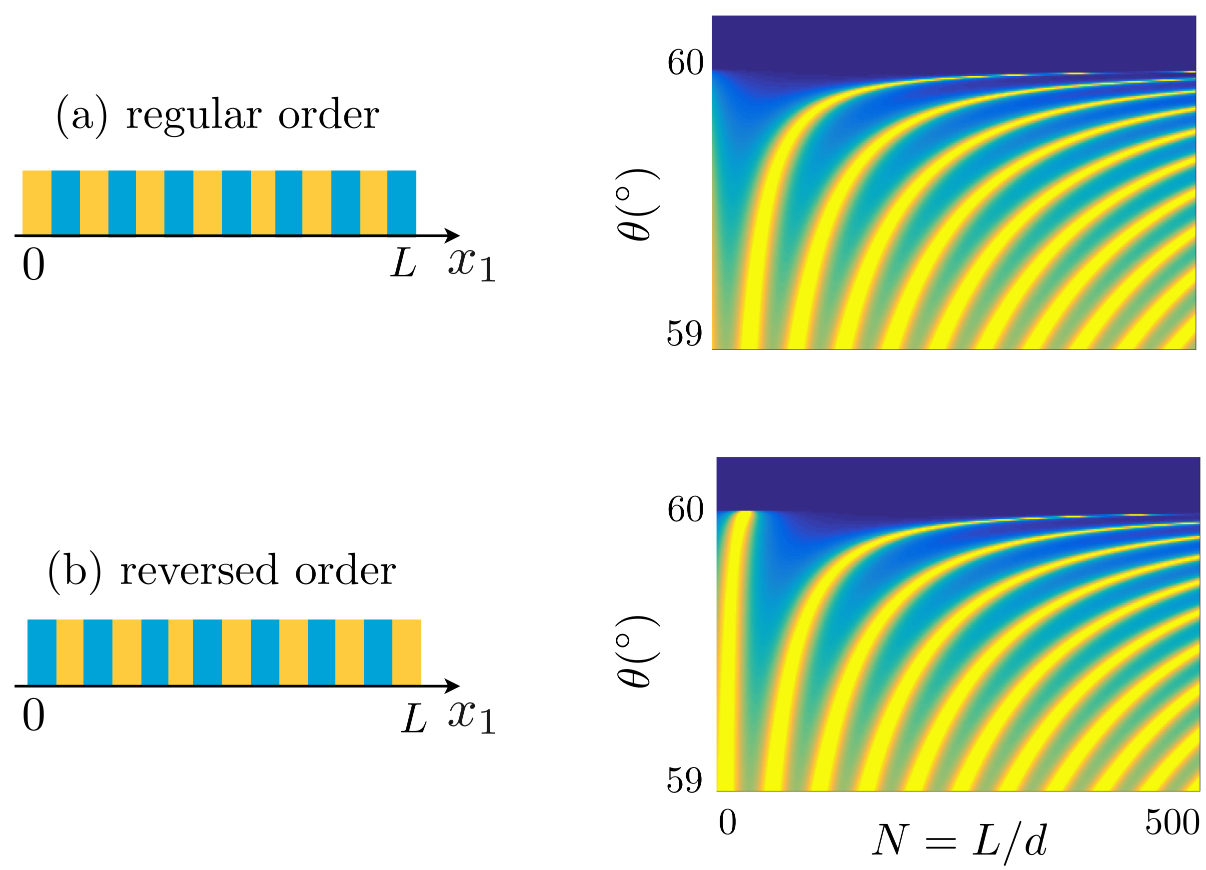

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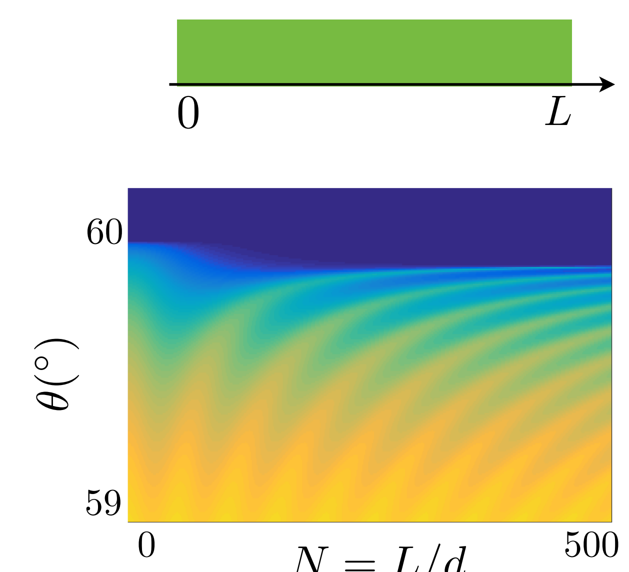

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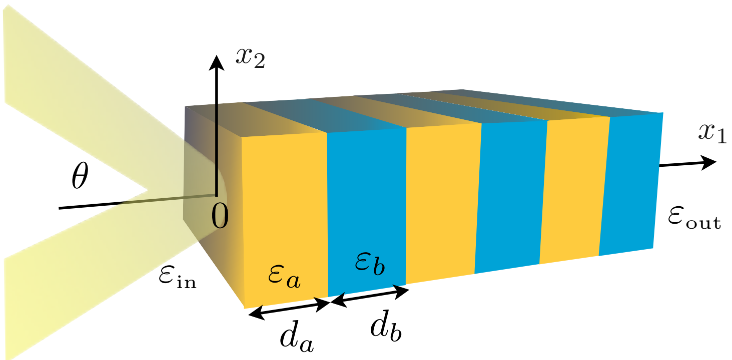

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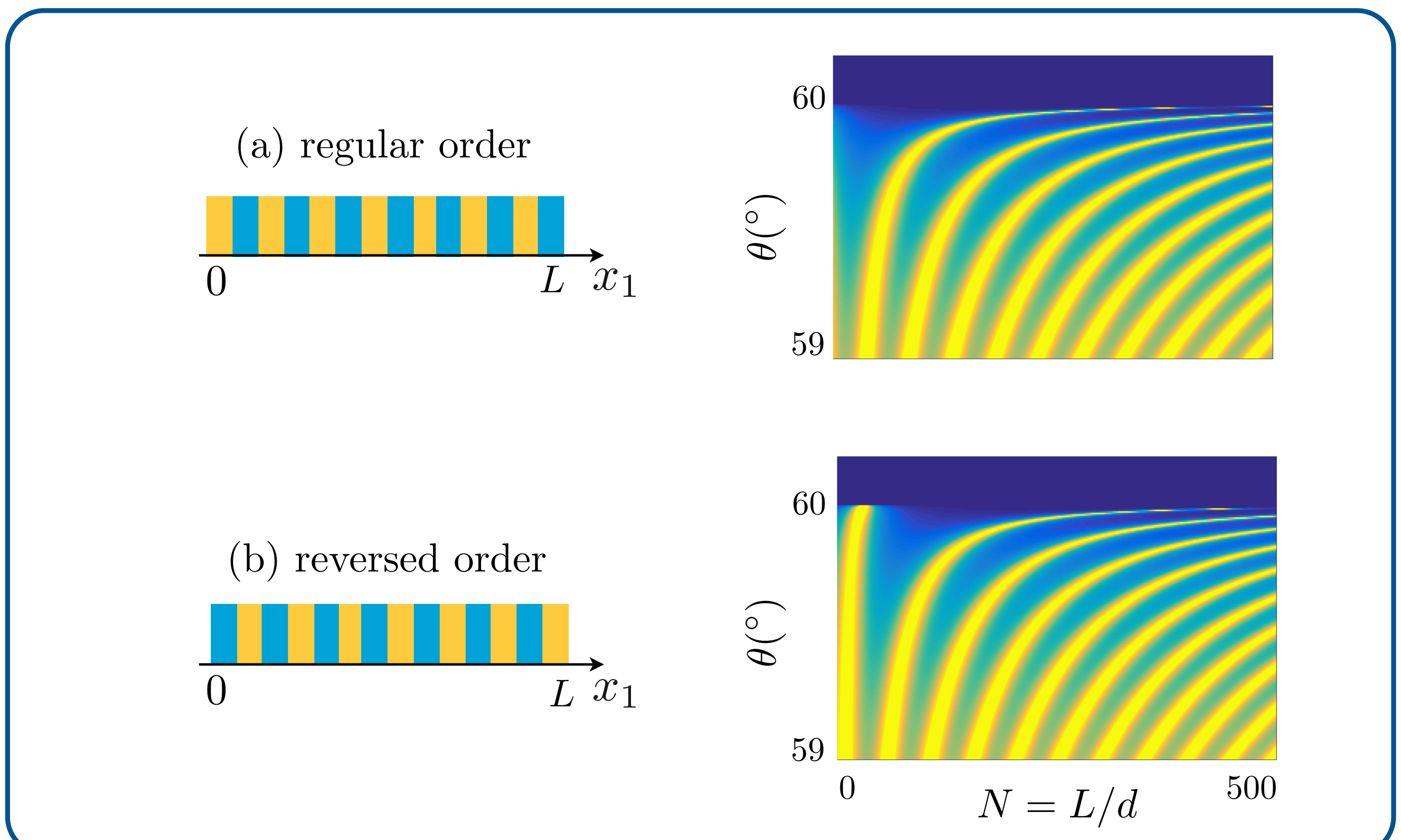

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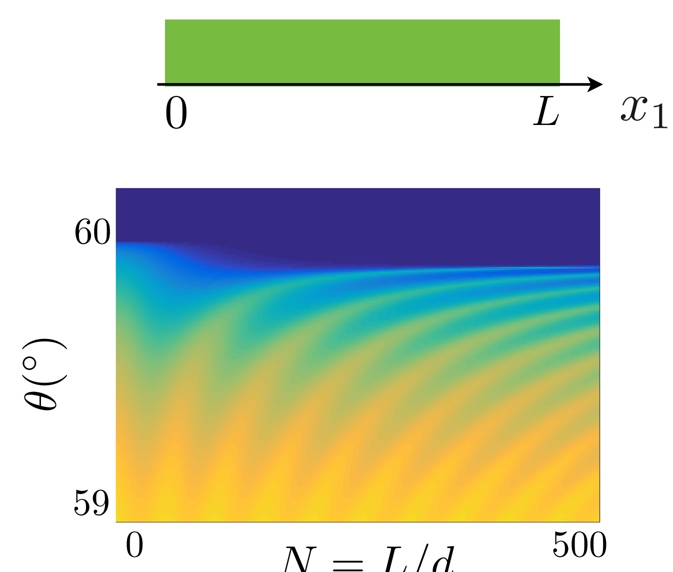

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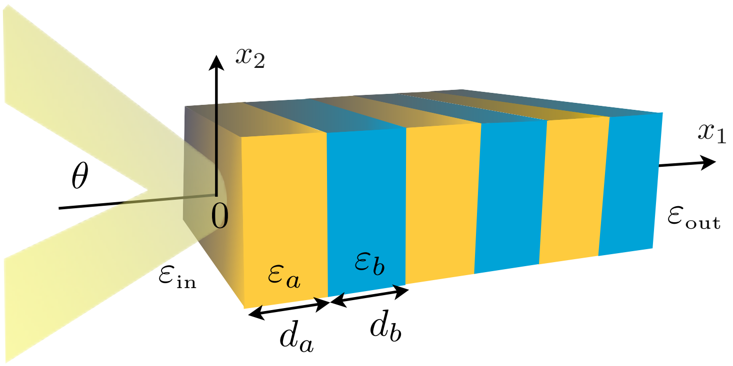

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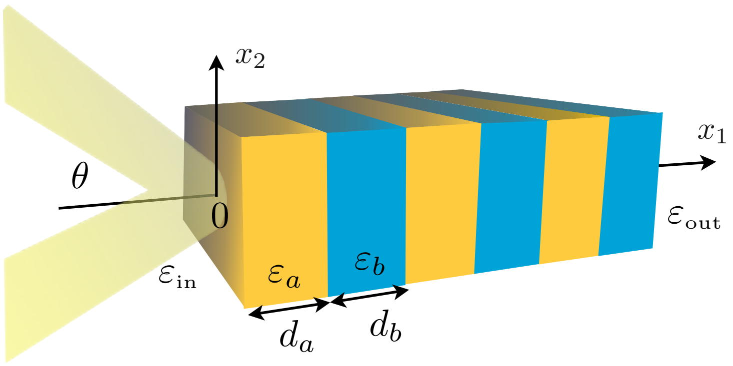

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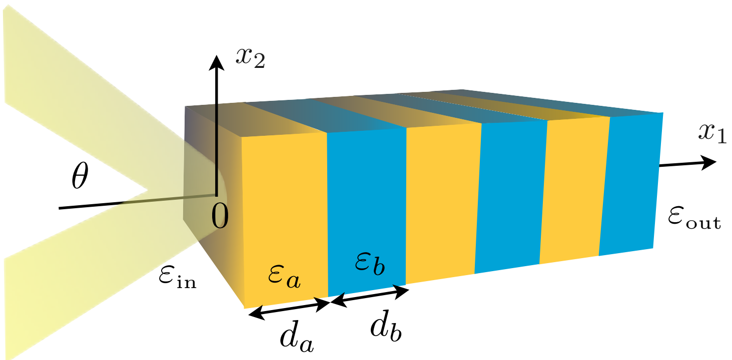
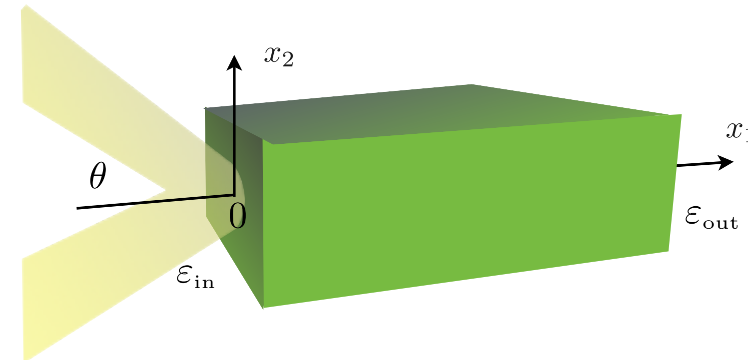


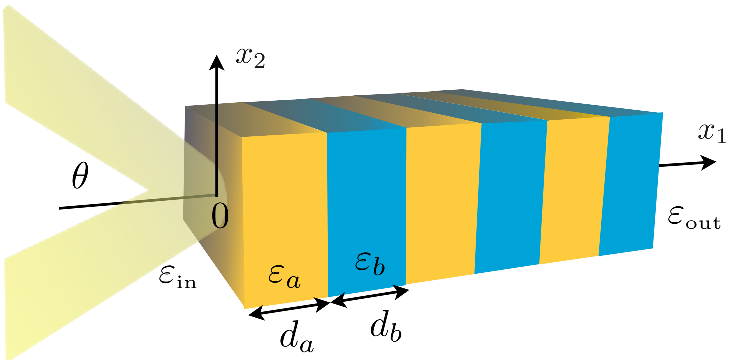


$$\Delta E(\mathbf{x}) + \varepsilon(x_1)k^2 E(\mathbf{x}) = 0$$

continuity of E, $\partial_n E$ at each interface

?

Multilayer structure



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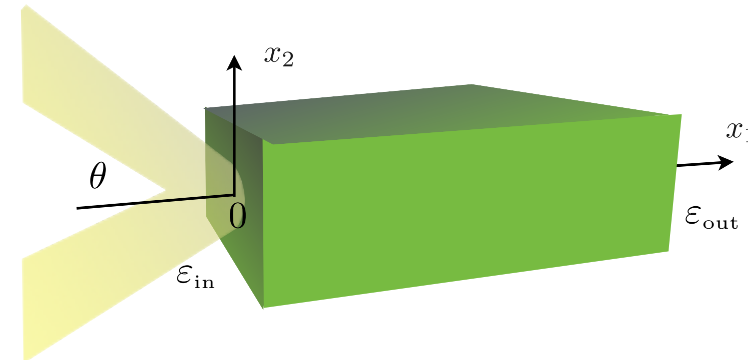

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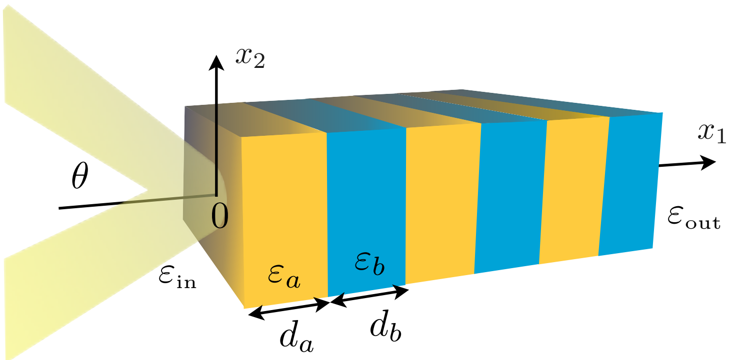

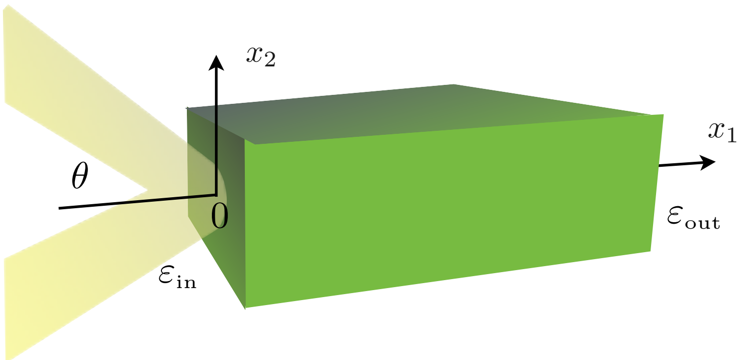

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hom. 0-0

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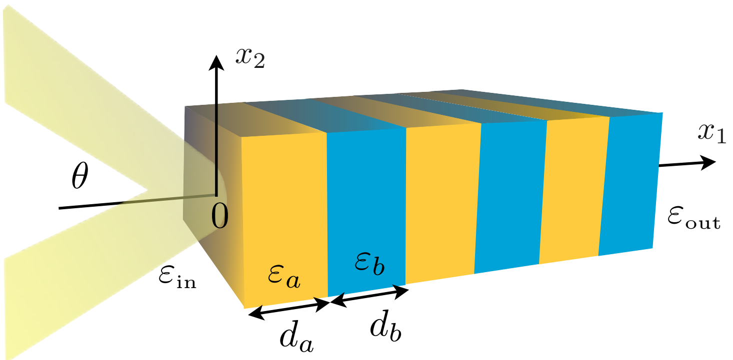
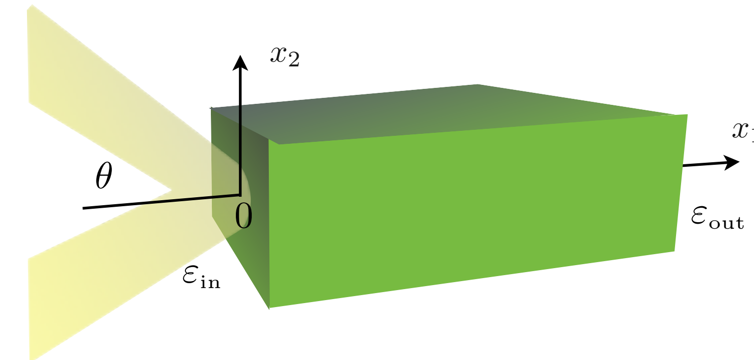
Multilayer structure

High order homogenization





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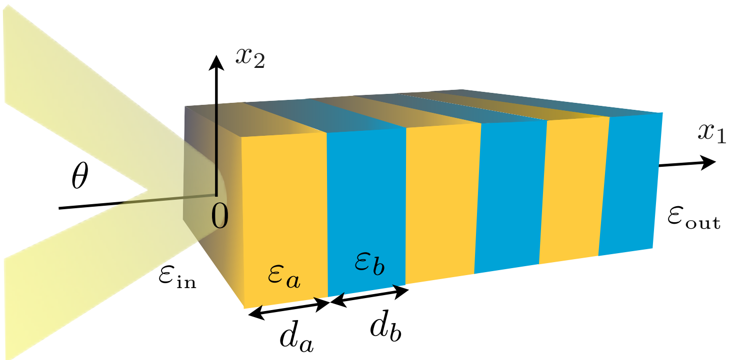

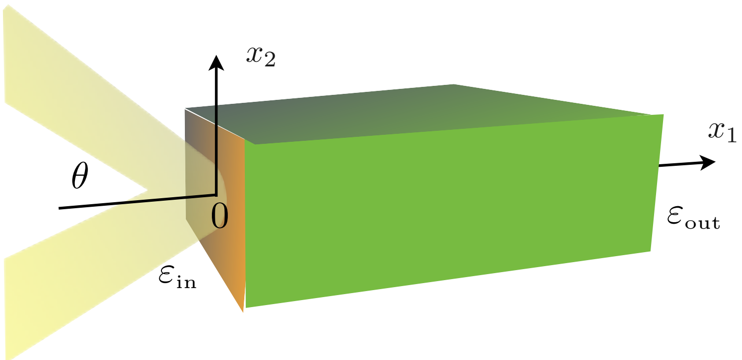

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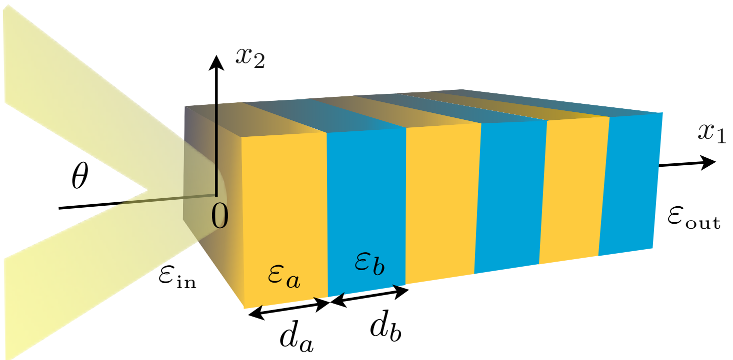

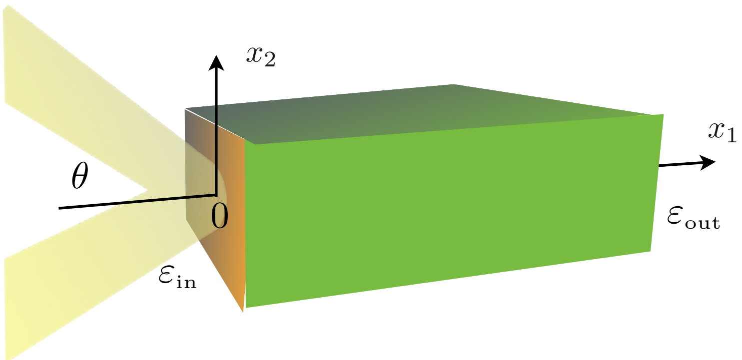

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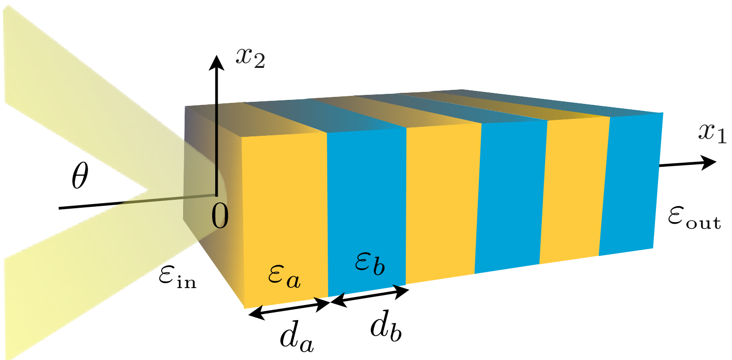
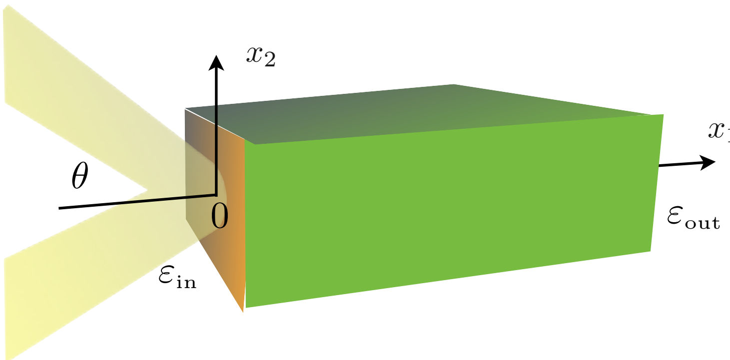
2-2 $\quad \Delta E + \varepsilon_{\|}(k) k^2 E = 0 \qquad ?$

Multilayer structure

High order homogenization





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↓

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Multilayer structure

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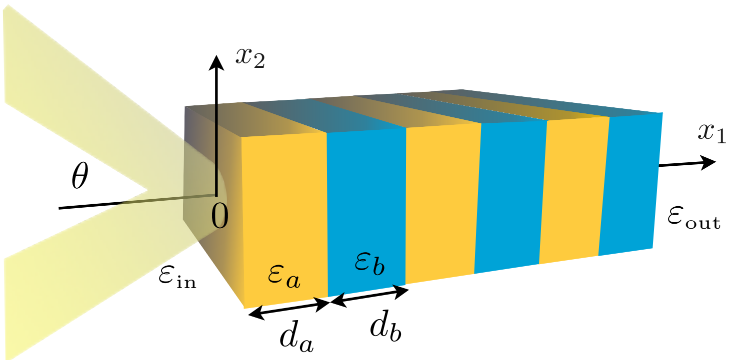

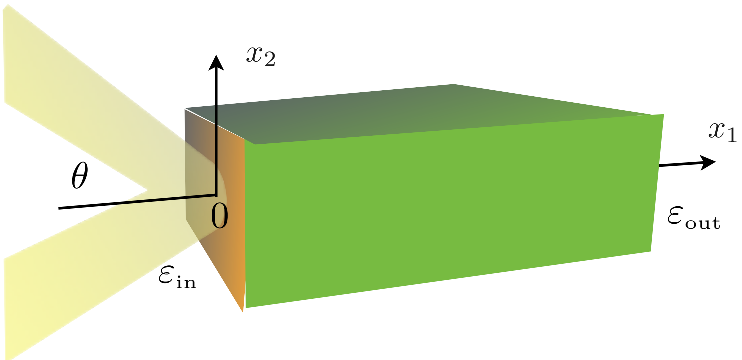

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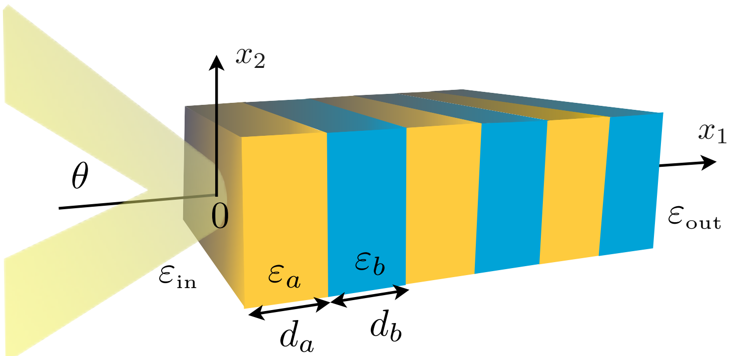
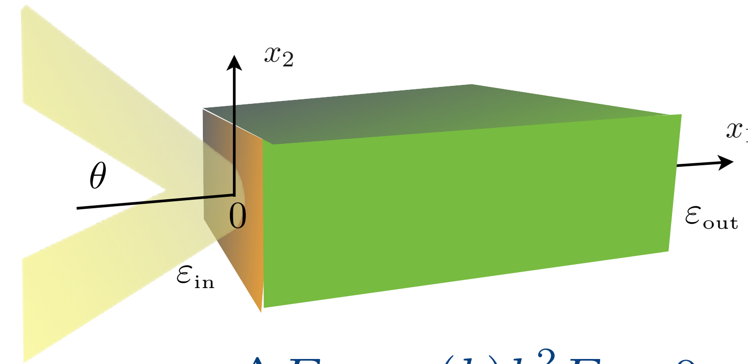
hybrid problem (hom 2-1)

first non trivial correction to the leading order model
in the bulk and at the boundaries

Multilayer structure

High order homogenization





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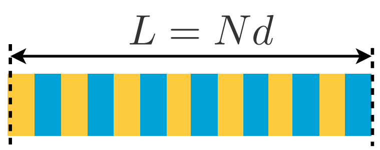
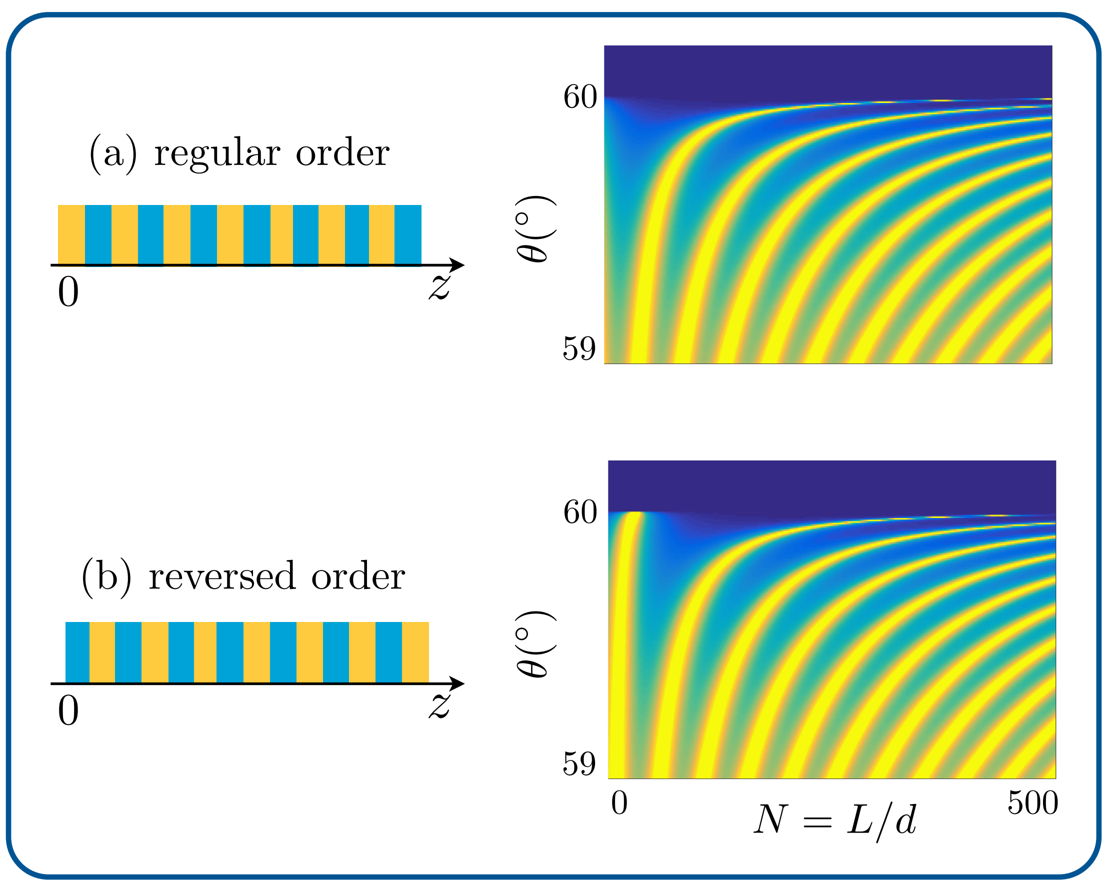
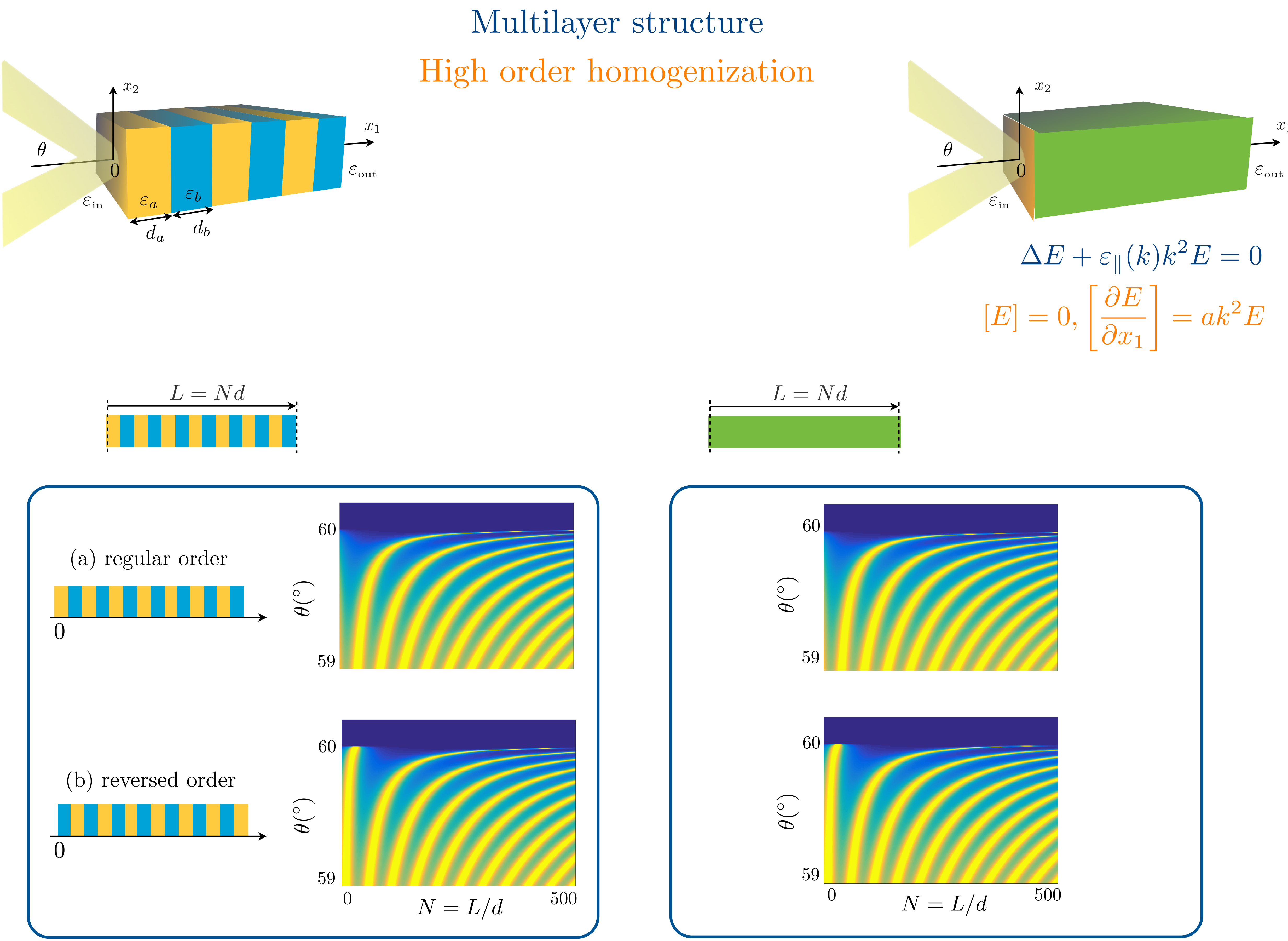
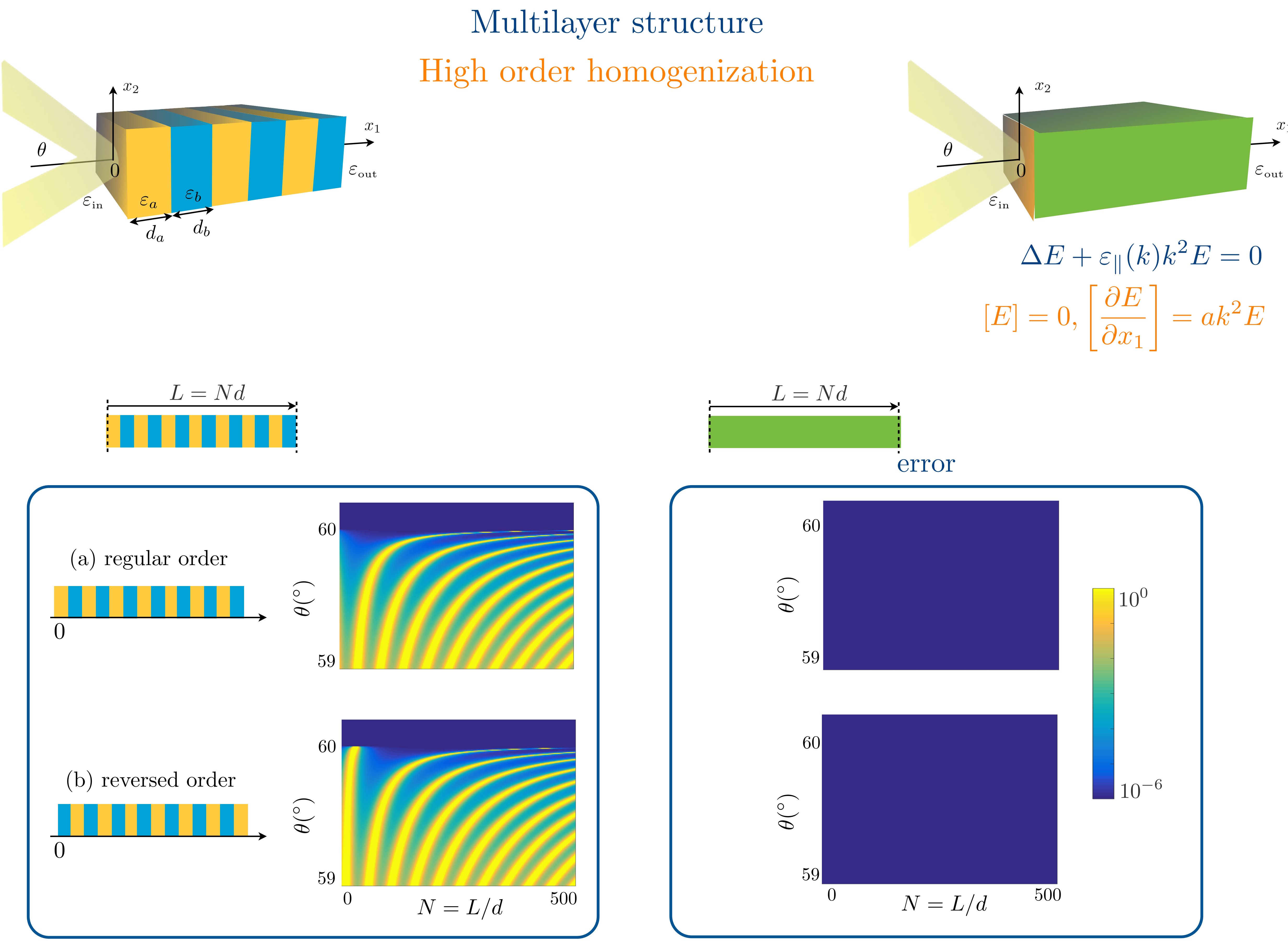


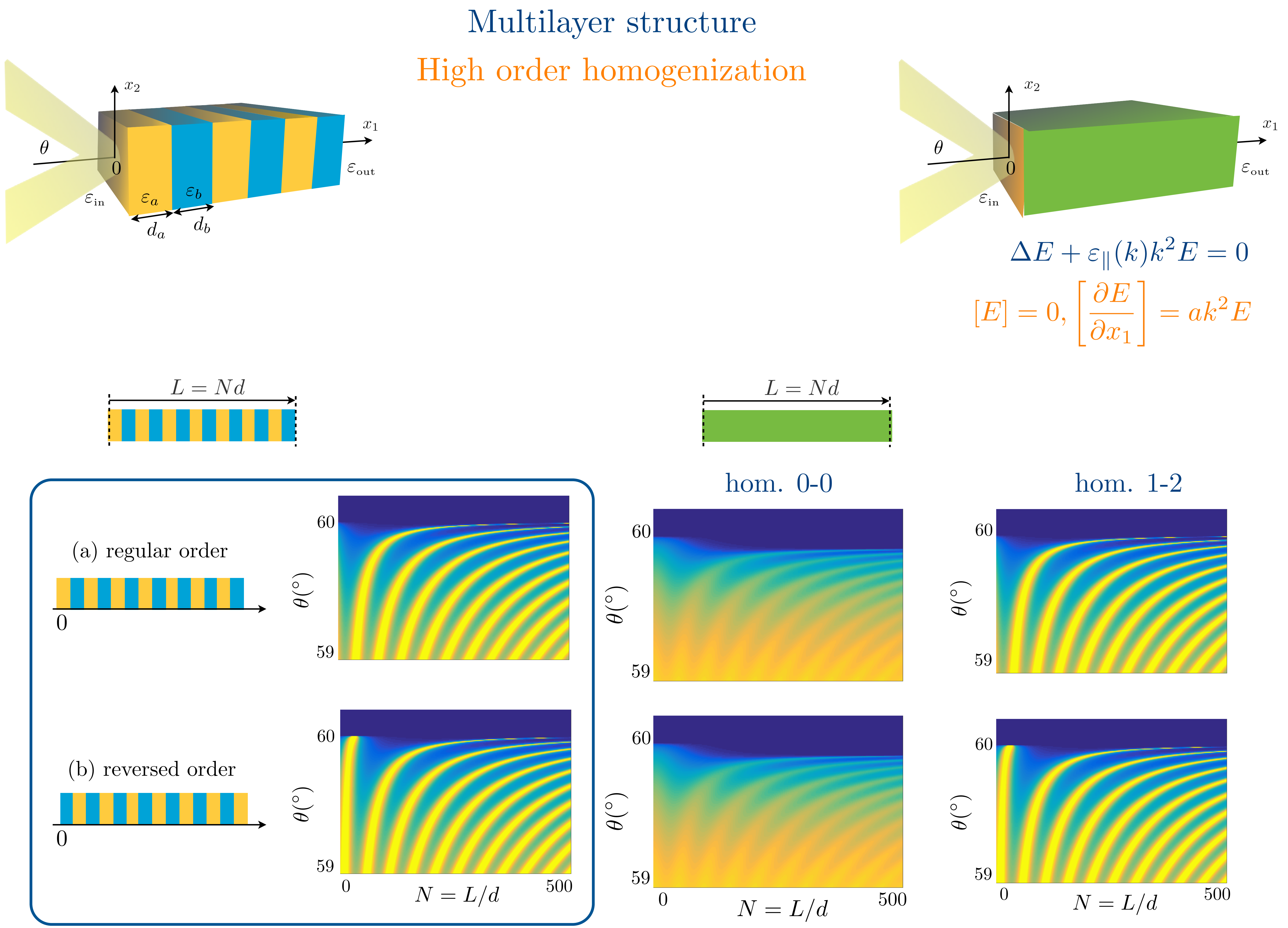




Multilayer structure
High order homogenization
x_2
x_1
θ
0
$\varepsilon_{\mathrm{out}}$
$\varepsilon_{\mathrm{in}}$
ε_a
ε_b
d_a
d_b
$\Delta E + \varepsilon_{\parallel}(k) k^2 E = 0$
$[E] = 0, \left[\dfrac{\partial E}{\partial x_1}\right] = a k^2 E$
$L = Nd$
$L = Nd$
(a) regular order
(b) reversed order
0
0
$\theta(°)$
60
59
$N = L/d$
500

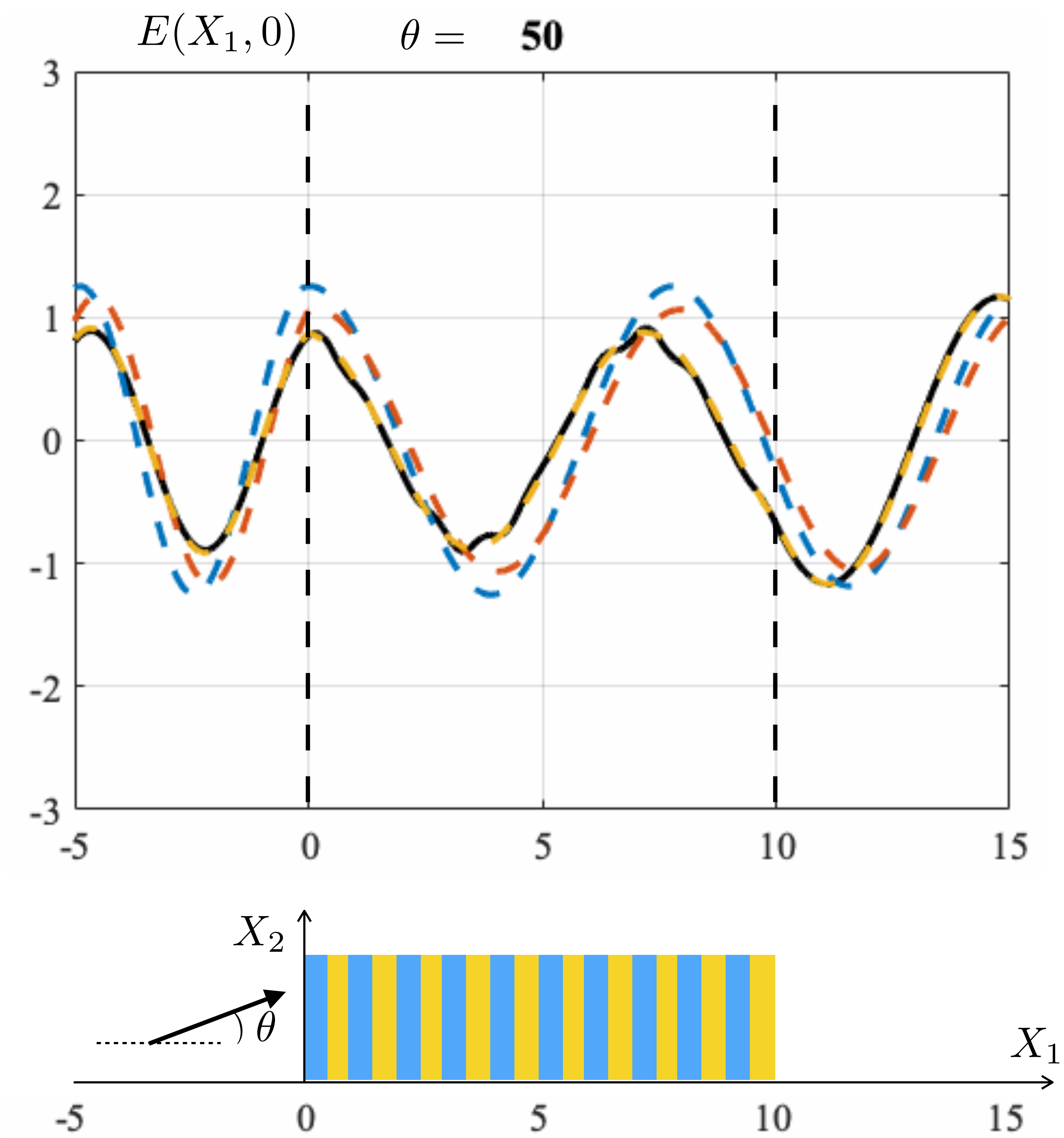


Multilayer structure
High order homogenization
x_2
x_1
θ
0
ε_{out}
ε_{in}
ε_a
ε_b
d_a
d_b
$\Delta E + \varepsilon_{\parallel}(k)k^2 E = 0$
$[E] = 0, \left[\frac{\partial E}{\partial x_1}\right] = ak^2 E$
$L = Nd$
$L = Nd$
error
(a) regular order
(b) reversed order
$\theta(°)$
60
59
0
500
$N = L/d$
10^0
10^{-6}


Multilayer structure
High order homogenization
x_2
x_1
θ
0
ε_{in}
ε_a
ε_b
d_a
d_b
ε_{out}
$\Delta E + \varepsilon_{\parallel}(k) k^2 E = 0$
$[E] = 0, \left[\frac{\partial E}{\partial x_1}\right] = a k^2 E$
$L = Nd$
$L = Nd$
(a) regular order
(b) reversed order
hom. 0-0
hom. 1-2
$\theta(^\circ)$
60
59
0
500
$N = L/d$

Multilayer structure

High order homogenization



direct numerics

hom. 0-0 $\Delta E + \langle \varepsilon \rangle k^2 E = 0$

$[E] = 0, \left[\frac{\partial E}{\partial x_1}\right] = 0$

hom. 1-1 $\Delta E + \langle \varepsilon \rangle k^2 E = 0$

$[E] = 0, \left[\frac{\partial E}{\partial x_1}\right] = ak^2 E$

hom. 2-1 $\Delta E + \varepsilon_{\|}(k) k^2 E = 0$

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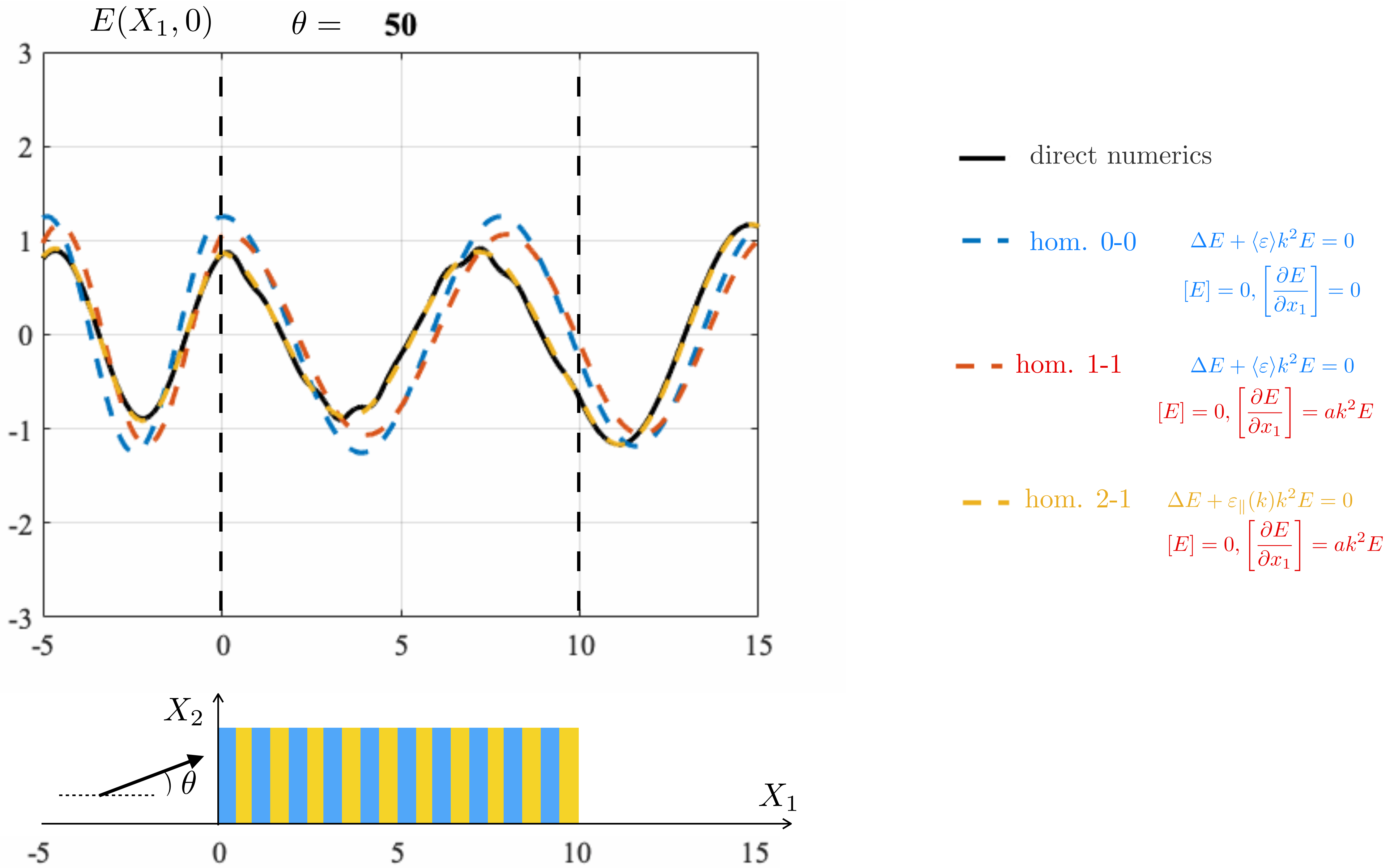
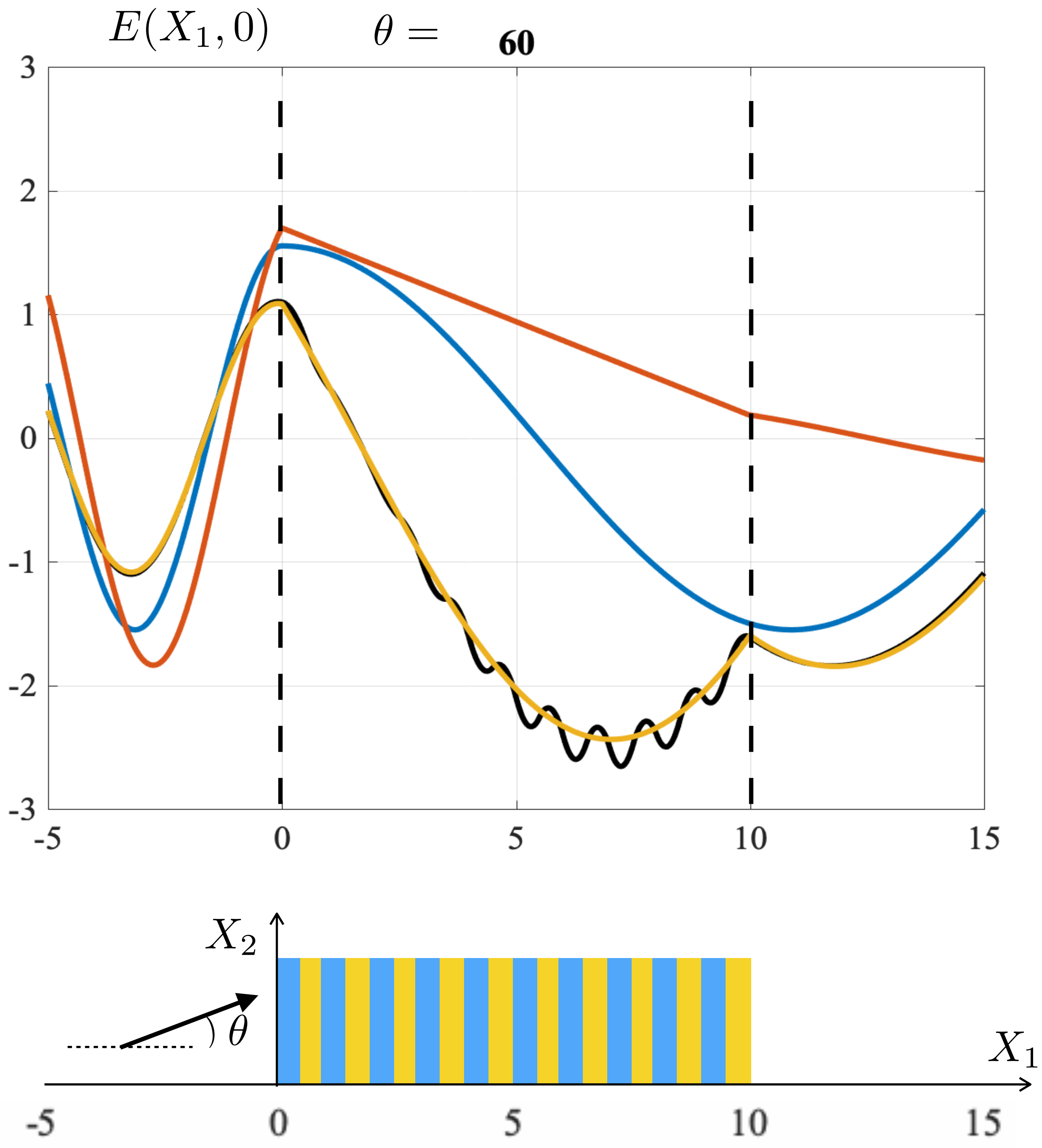
Multilayer structure

High order homogenization



Multilayer structure

High order homogenization



direct numerics

hom. 0-0 $\Delta E + \langle \varepsilon \rangle k^2 E = 0$

$[E] = 0, \left[\frac{\partial E}{\partial x_1}\right] = 0$

hom. 1-1 $\Delta E + \langle \varepsilon \rangle k^2 E = 0$

$[E] = 0, \left[\frac{\partial E}{\partial x_1}\right] = a k^2 E$

hom. 2-1 $\Delta E + \varepsilon_{\|}(k) k^2 E = 0$

$[E] = 0, \left[\frac{\partial E}{\partial x_1}\right] = a k^2 E$

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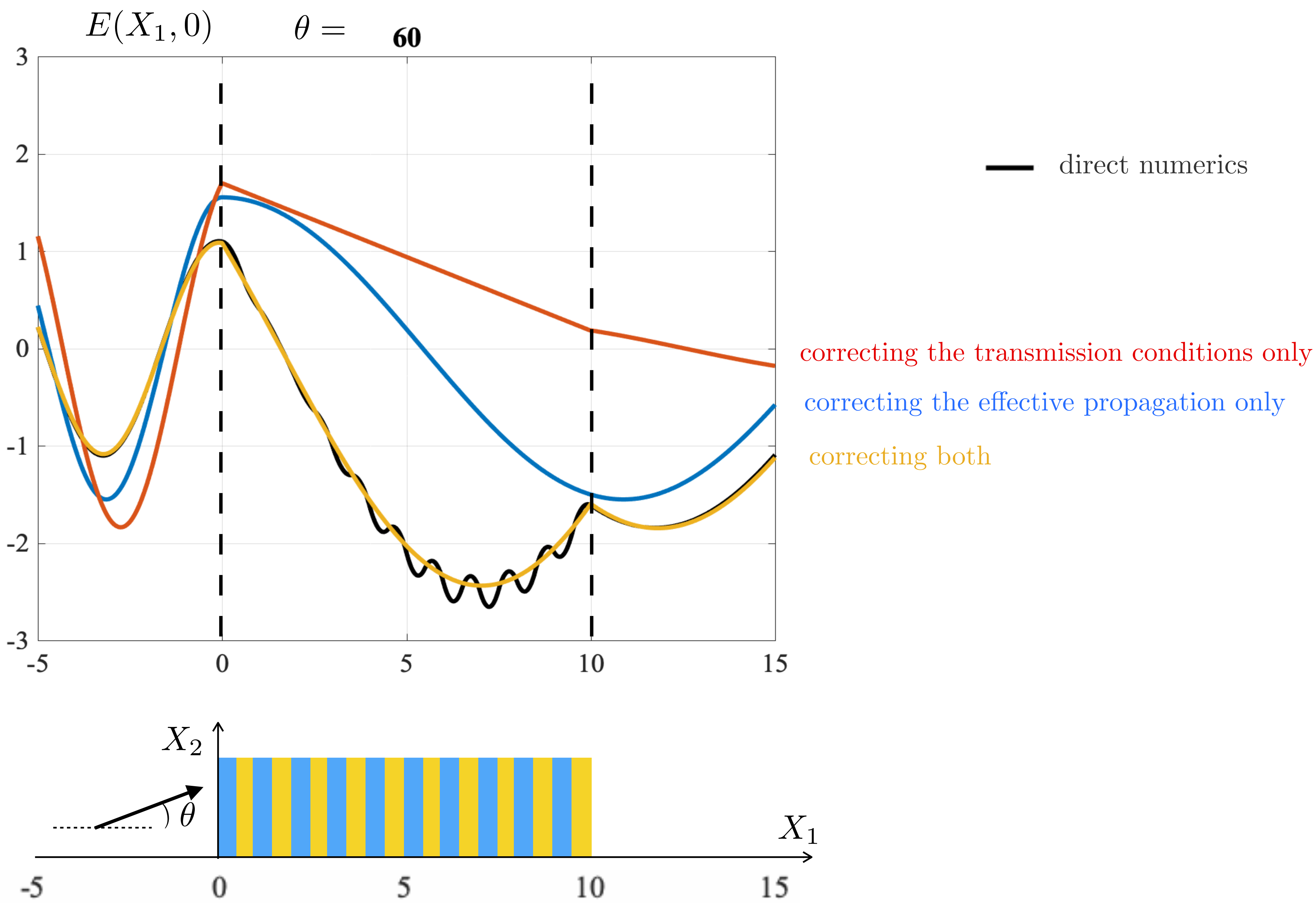

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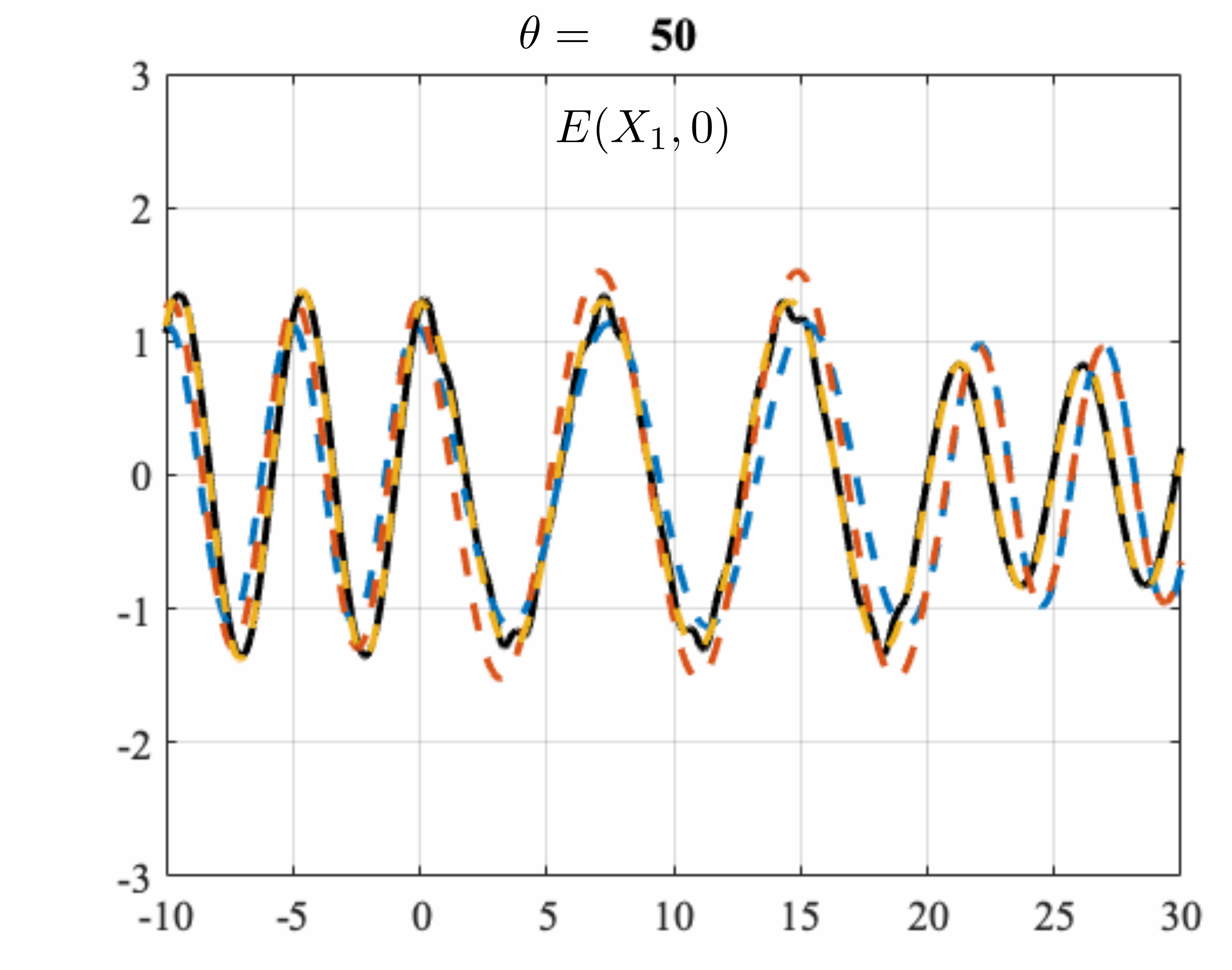

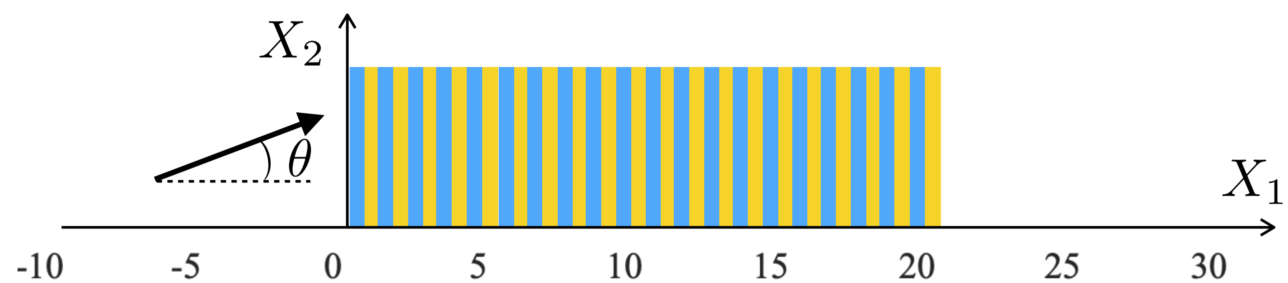

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$[E] = 0, \left[\frac{\partial E}{\partial x_1}\right] = ak^2 E$

$$[E] = 0, \left[\frac{\partial E}{\partial x_1}\right] = ak^2 E$$

$$\Delta E + \langle \varepsilon \rangle k^2 E = 0$$

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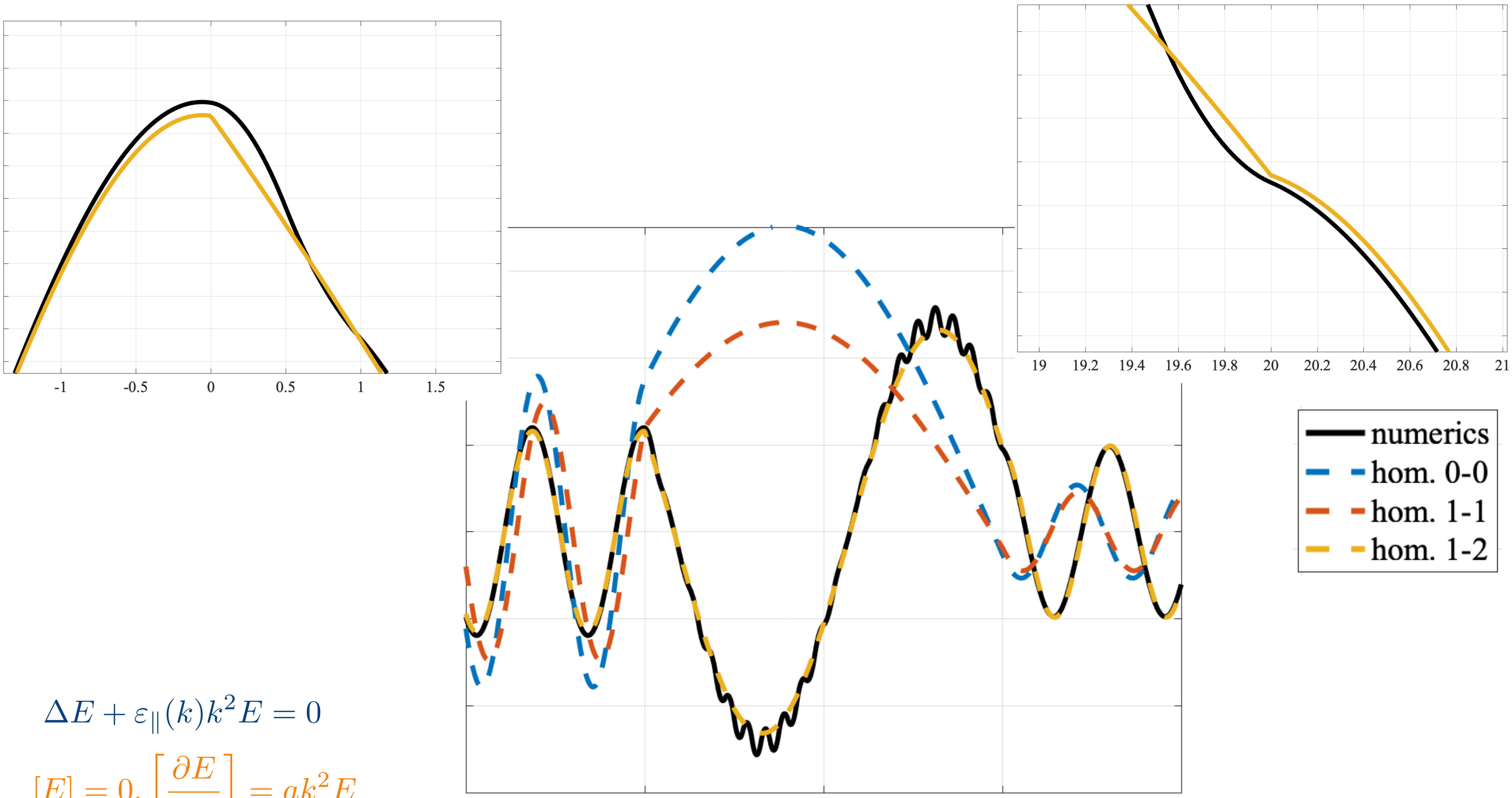

$$\Delta E + \varepsilon_{\parallel}(k) k^2 E = 0$$

$$[E] = 0, \left[\frac{\partial E}{\partial x_1} \right] = a k^2 E$$