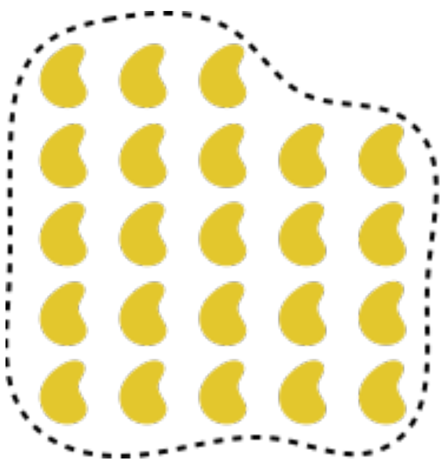
Asymptotic homogenization



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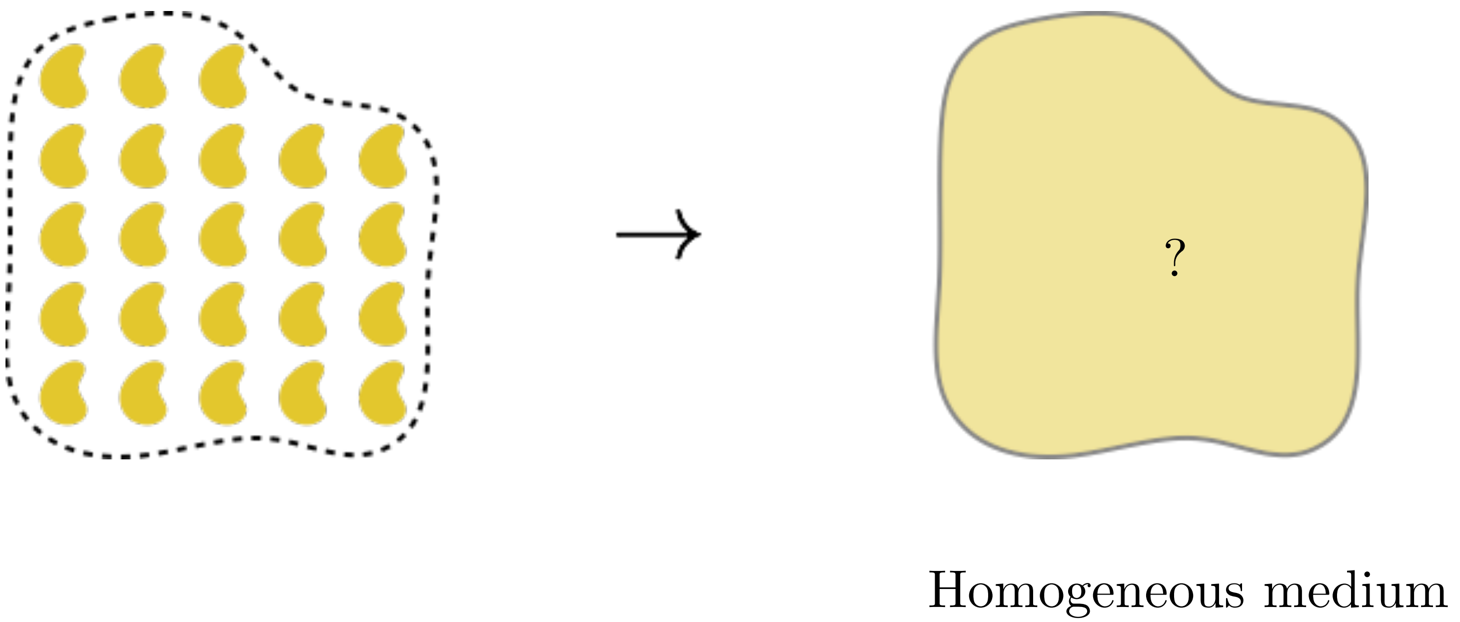

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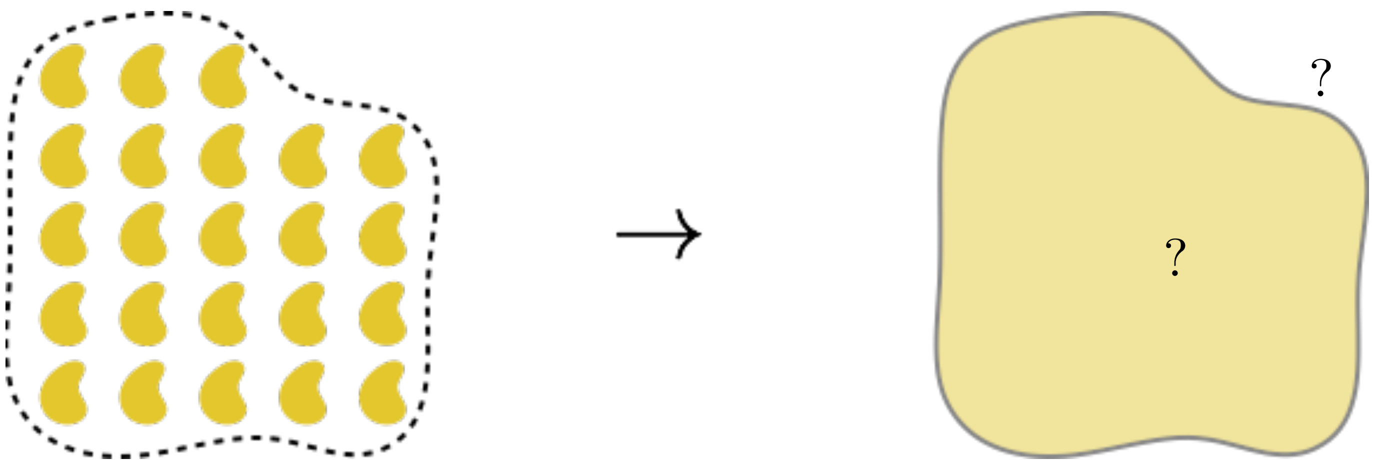
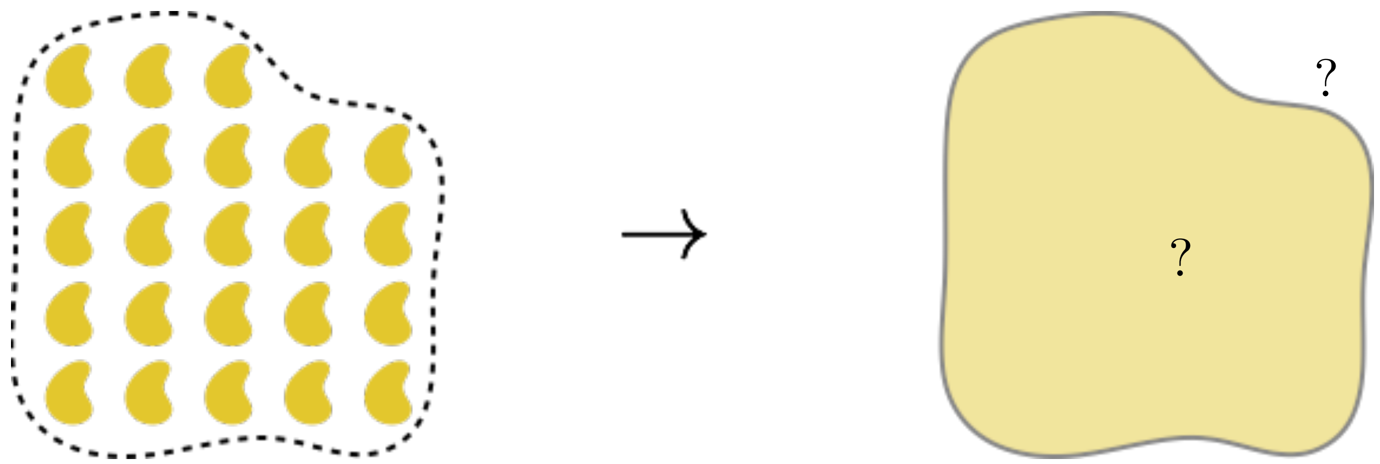


Homogeneous medium

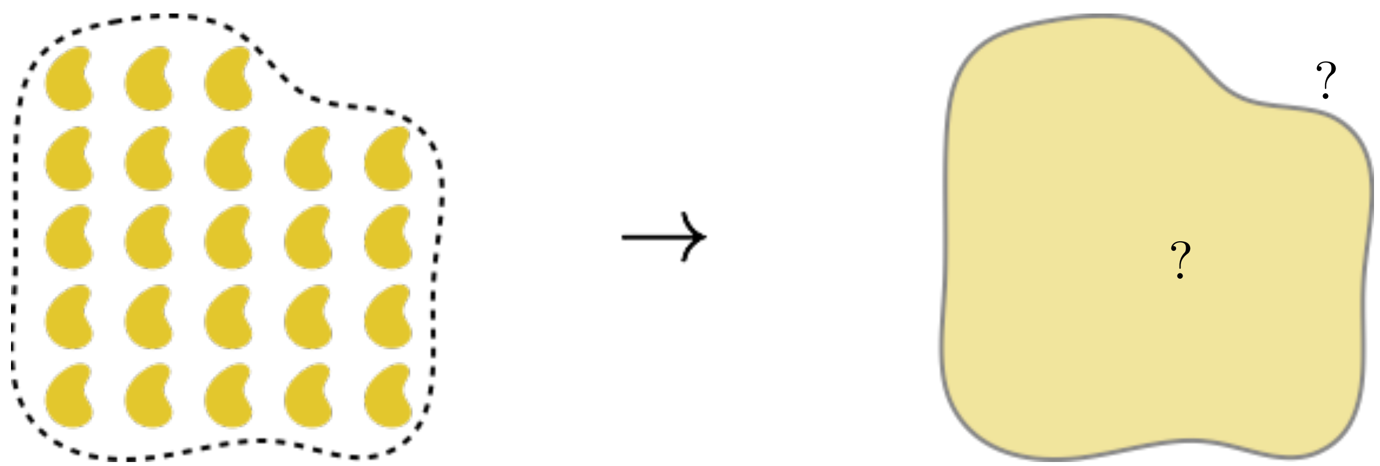
Asymptotic homogenization



Homogeneous medium

interface

Asymptotic homogenization



Homogeneous medium

interface

1) For which kind of structures ?

2) Propagation and boundary layer effects

3) Examples (local resonance or not)

Asymptotic homogenization

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Asymptotic homogenization

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Asymptotic homogenization

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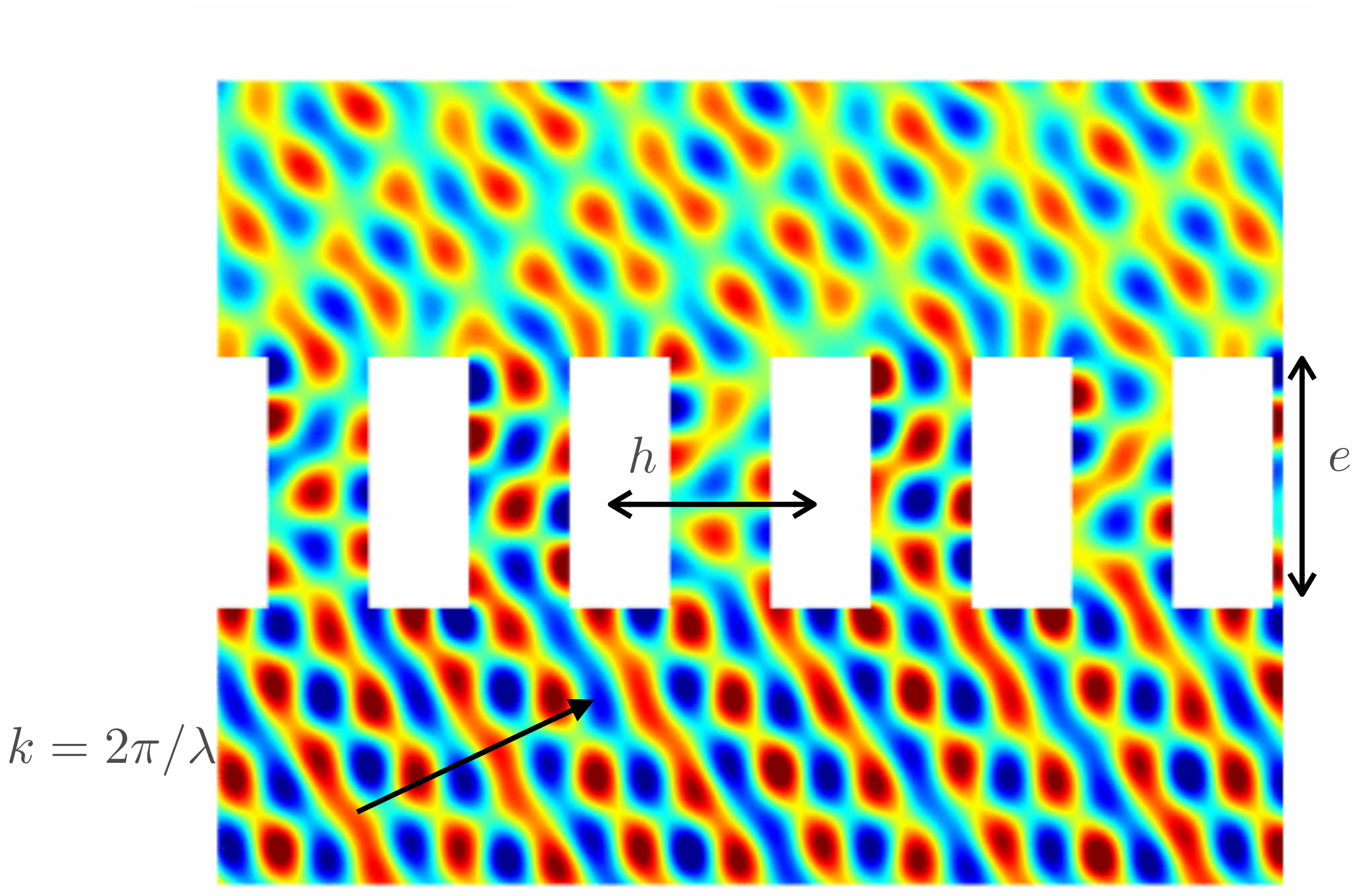

Asymptotic homogenization

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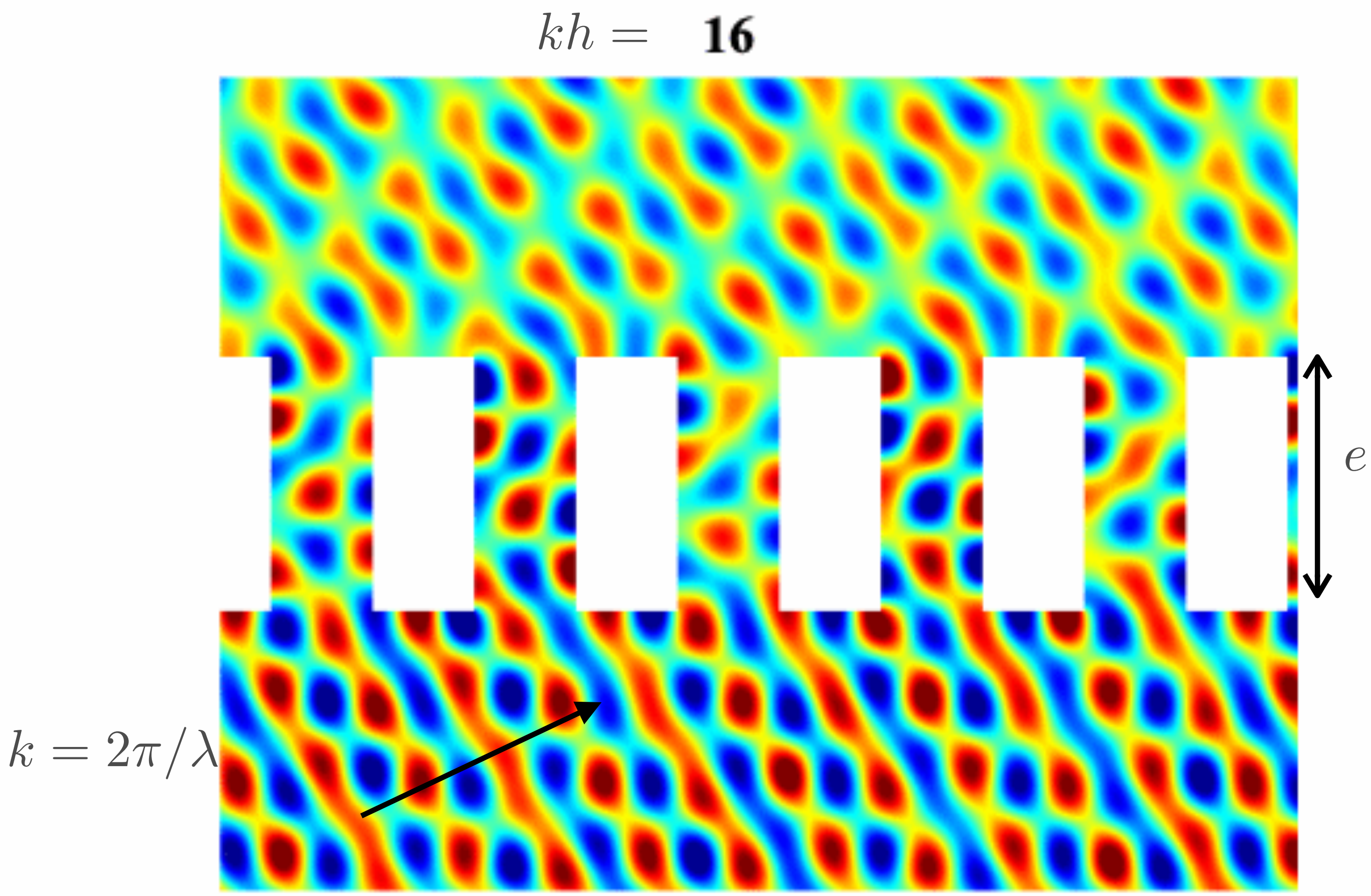

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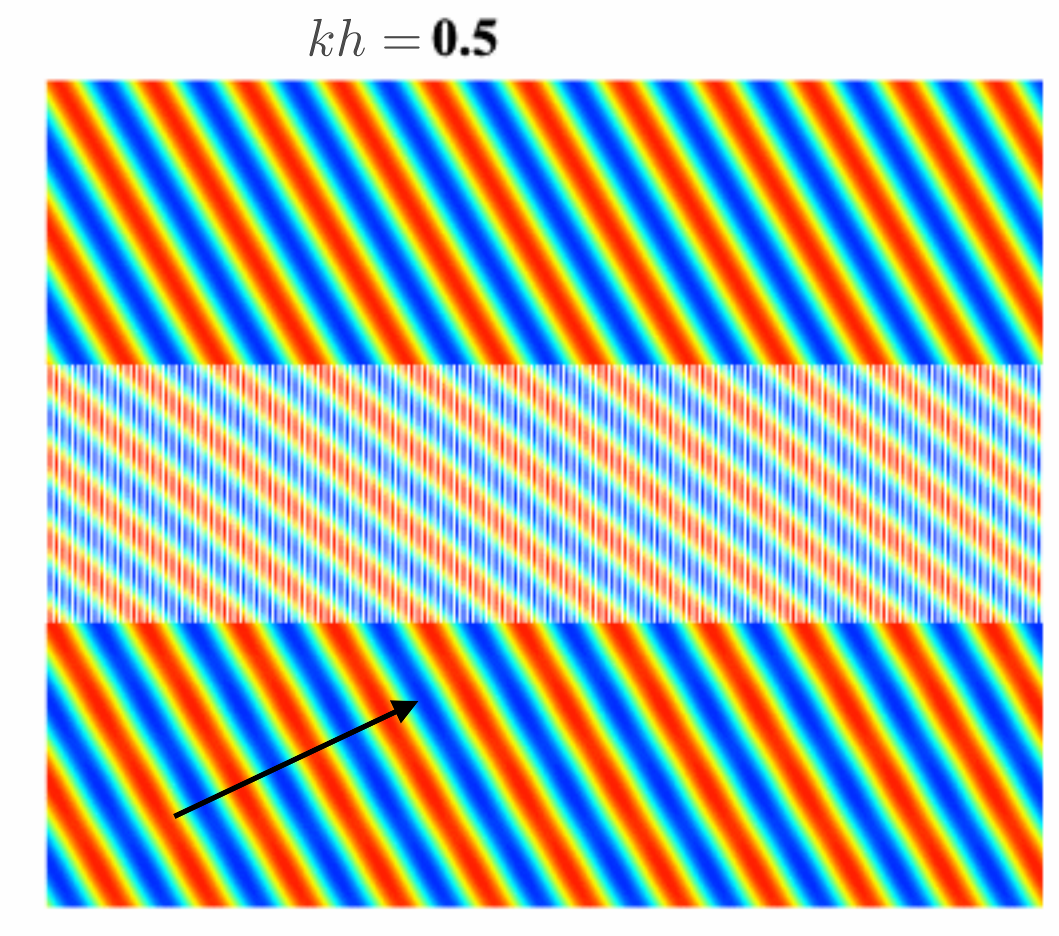

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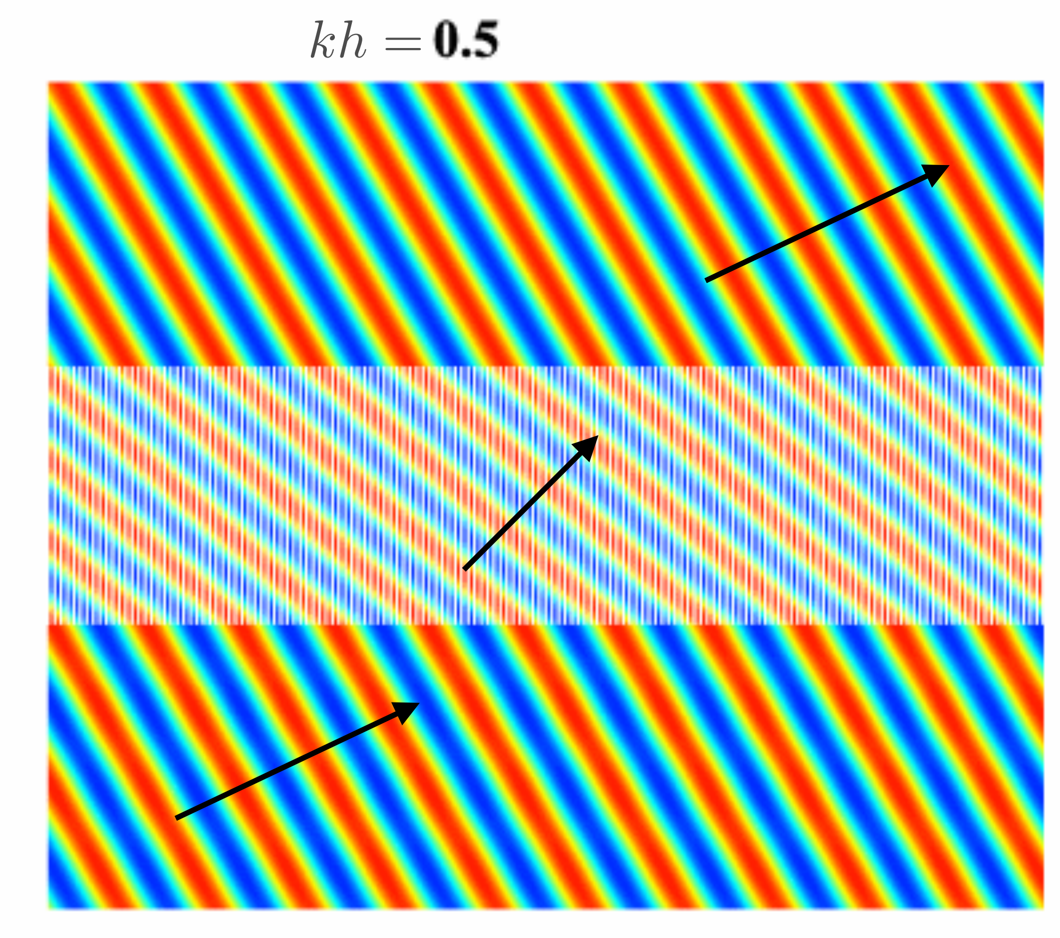

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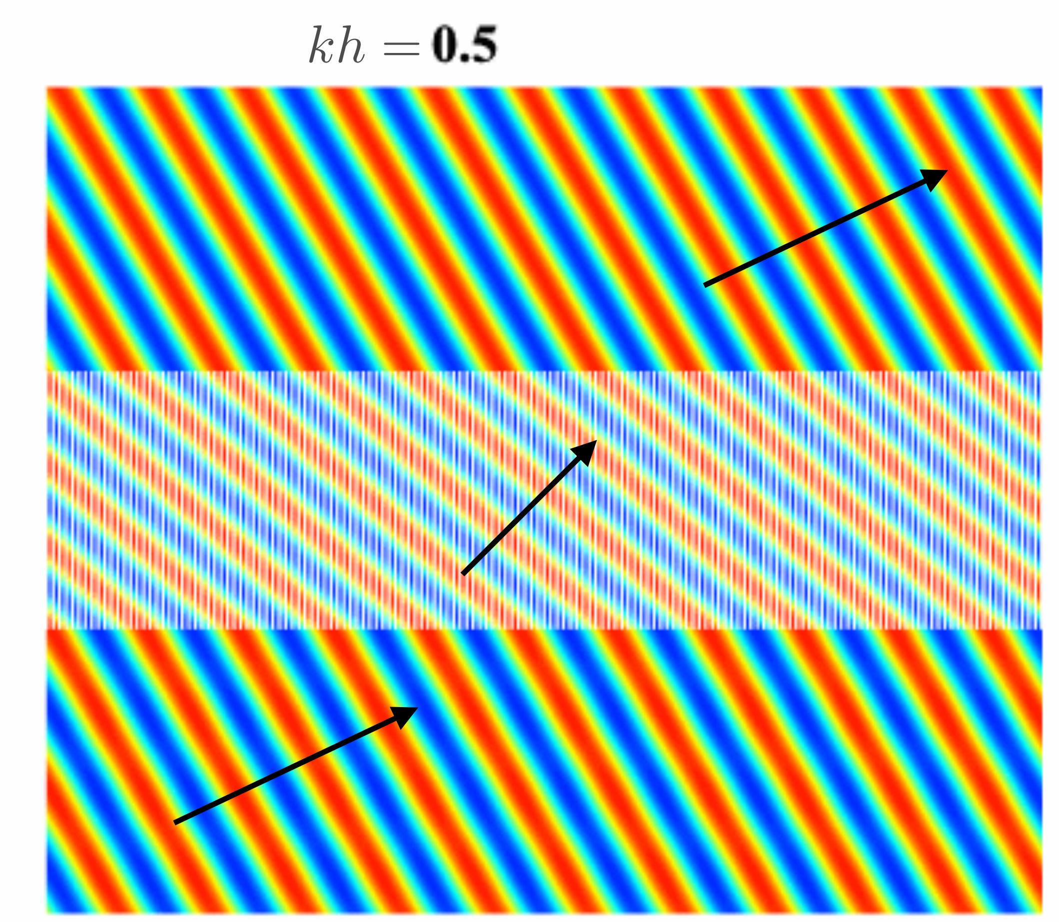

typical wavelength $\gg$ spacing of the structure

$\lambda = 2\pi/k$ $\qquad$ h

Asymptotic homogenization

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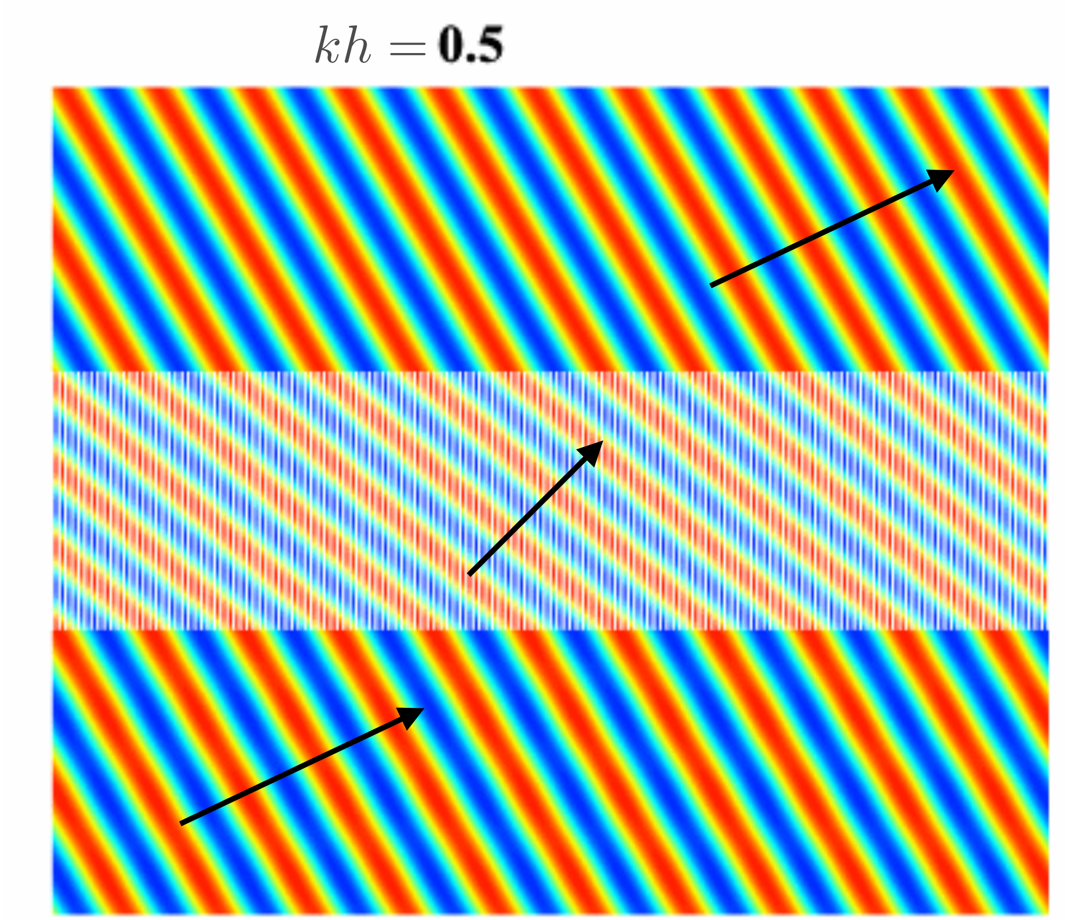

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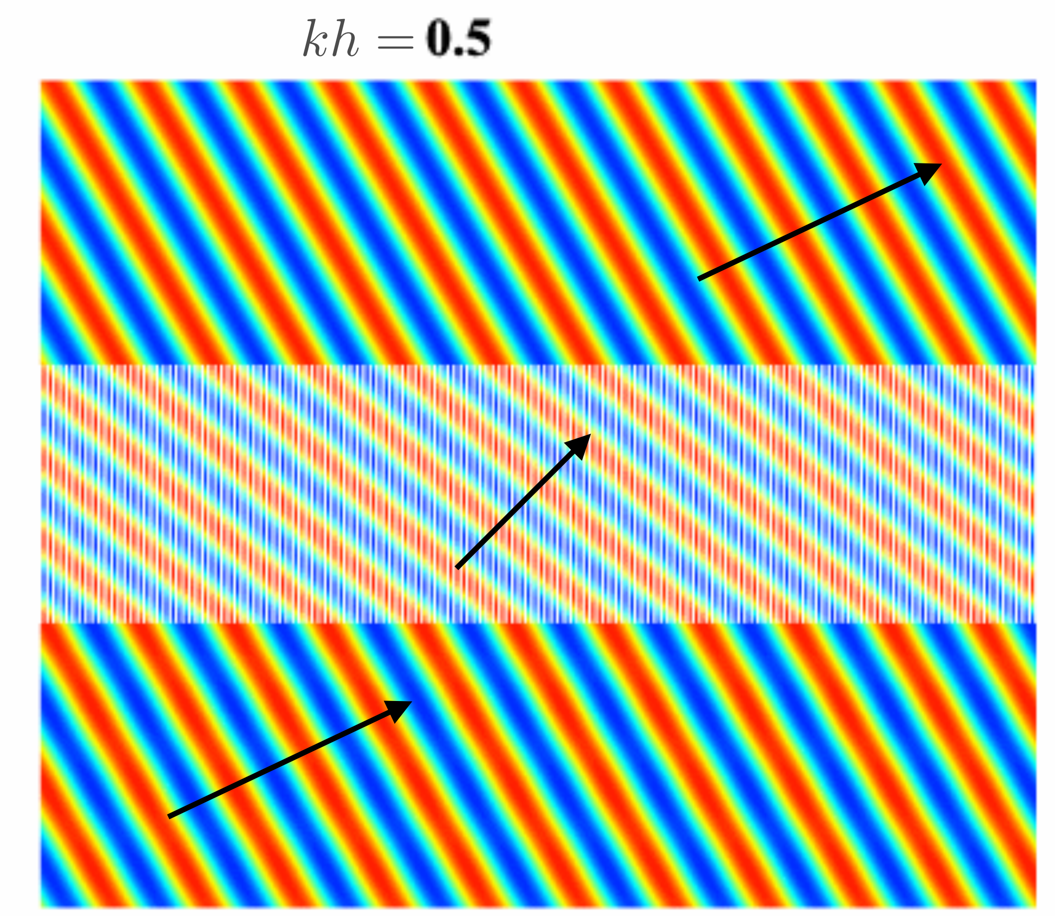

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Other strategies for finite wavelengths $\rightarrow$ Course of Bojan Guzina

Asymptotic homogenization

1) For which kind of structures ?

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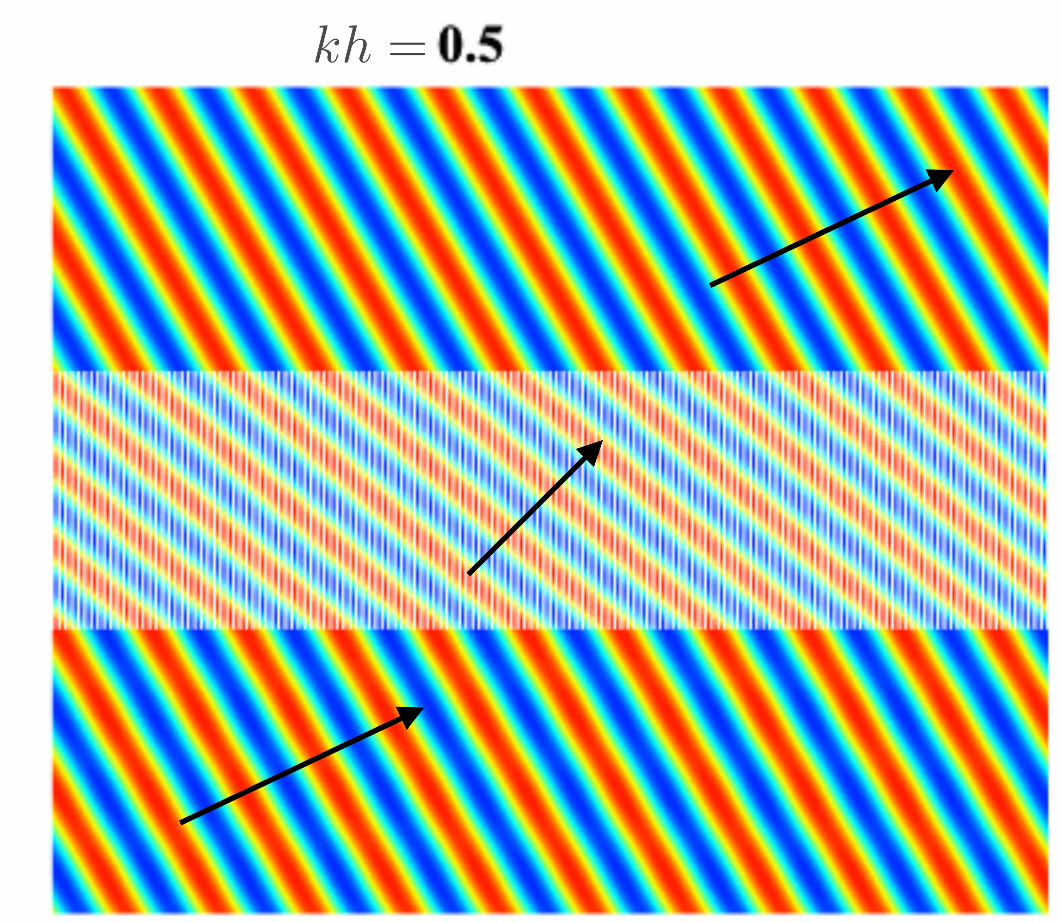

typical wavelength $\gg$ spacing of the structure

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Other strategies for finite wavelengths $\rightarrow$ Course of Bojan Guzina

Other strategies for quasiperiodic structures $\rightarrow$ Course of Sébastien Guenneau

Asymptotic homogenization

1) For which kind of structures ?

periodic structures

for $kh \ll 1$

Asymptotic homogenization

1) For which kind of structures ?

Asymptotic homogenization

2) Propagation and boundary layer effects

Asymptotic homogenization

2) Propagation and boundary layer effects

In its most classical form, the homogenization interrogates effective propagation

Asymptotic homogenization

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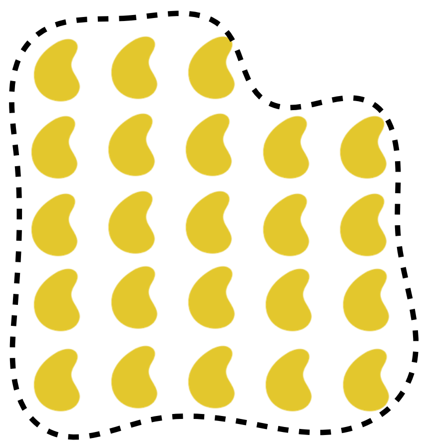
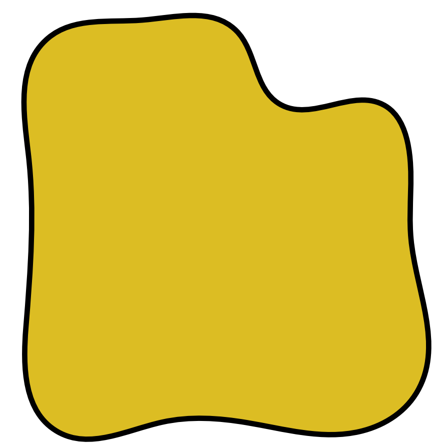


$$\mathrm{div}(a\nabla p) - b\,\frac{\partial^2 p}{\partial t^2} = 0$$

a and b vary in space



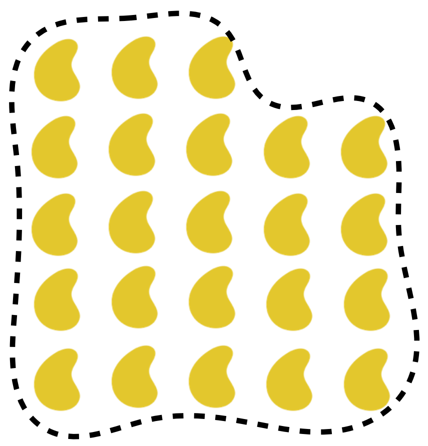
$$\mathrm{div}(\mathsf{a}_{\mathrm{eff}}\nabla p) - b_{\mathrm{eff}}\,\frac{\partial^2 p}{\partial t^2} = 0$$

effective parameters $\mathsf{a}_{\mathrm{eff}}$ (tensor) and b_{eff} (scalar) constant in space

Asymptotic homogenization

2) Propagation and boundary layer effects

In its most classical form, the homogenization interrogates effective propagation



Asymptotic homogenization

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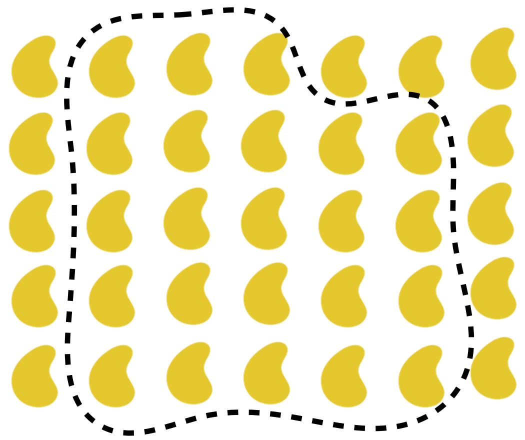

Asymptotic homogenization

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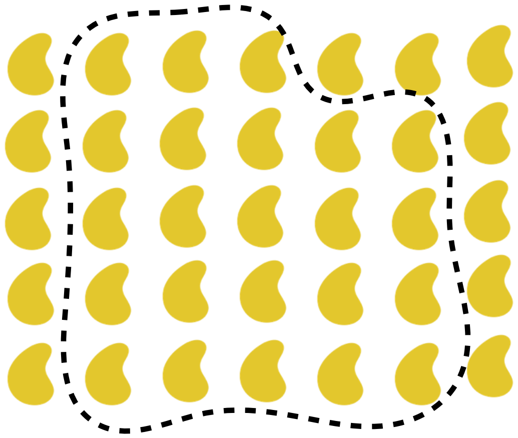
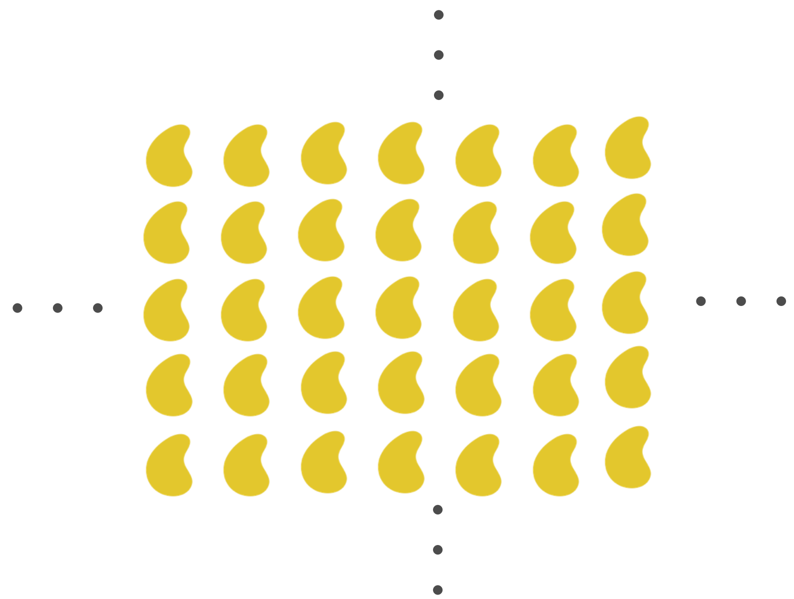


The (classical) homogenization in the bulk assumes an infinite medium

Asymptotic homogenization

2) Propagation and boundary layer effects

In its most classical form, the homogenization interrogates effective propagation



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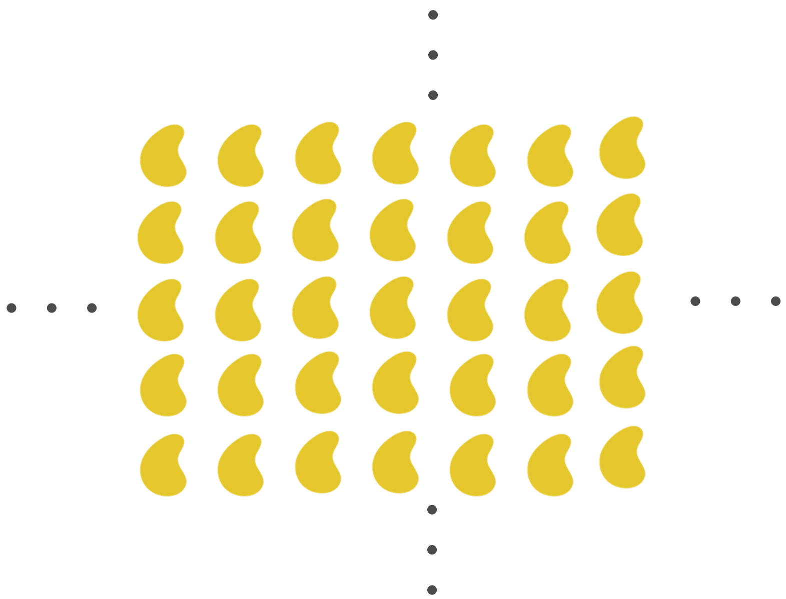

The (classical) homogenization in the bulk assumes an infinite medium
and the effective parameters are deduced from so-called "cell problems"

Asymptotic homogenization

2) Propagation and boundary layer effects

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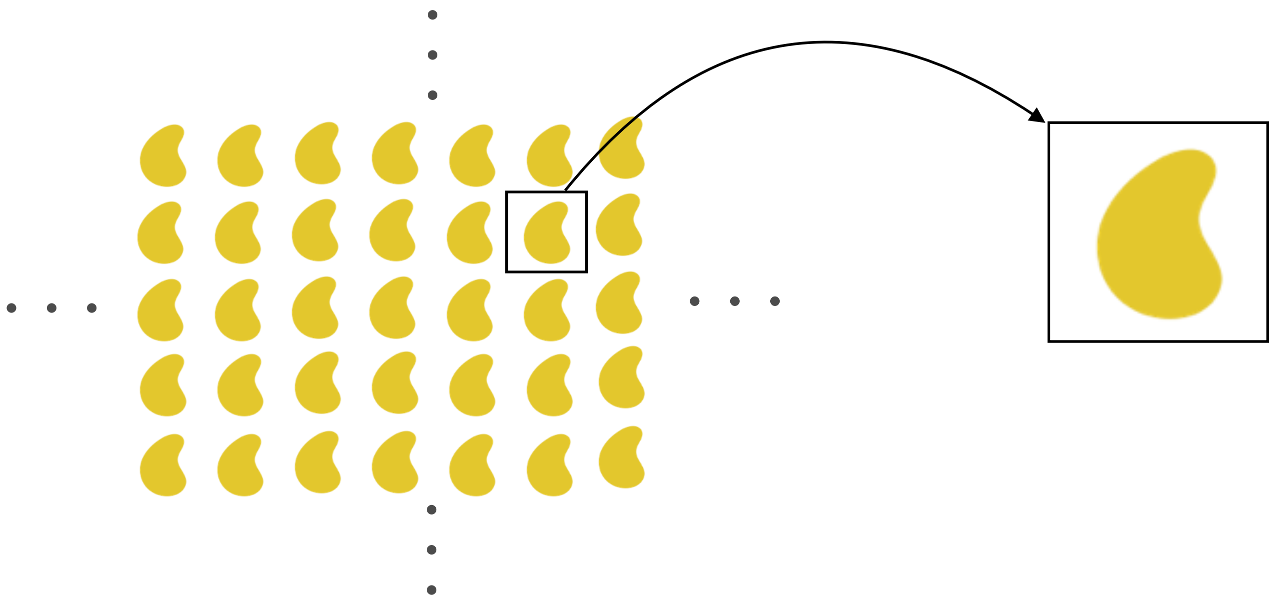

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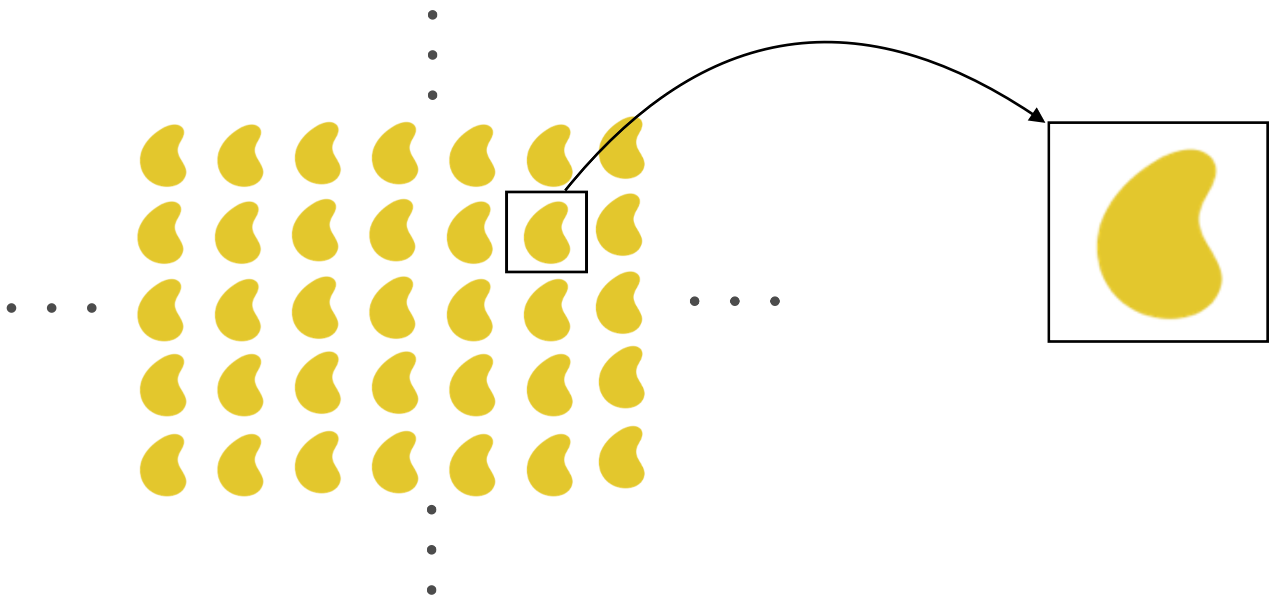

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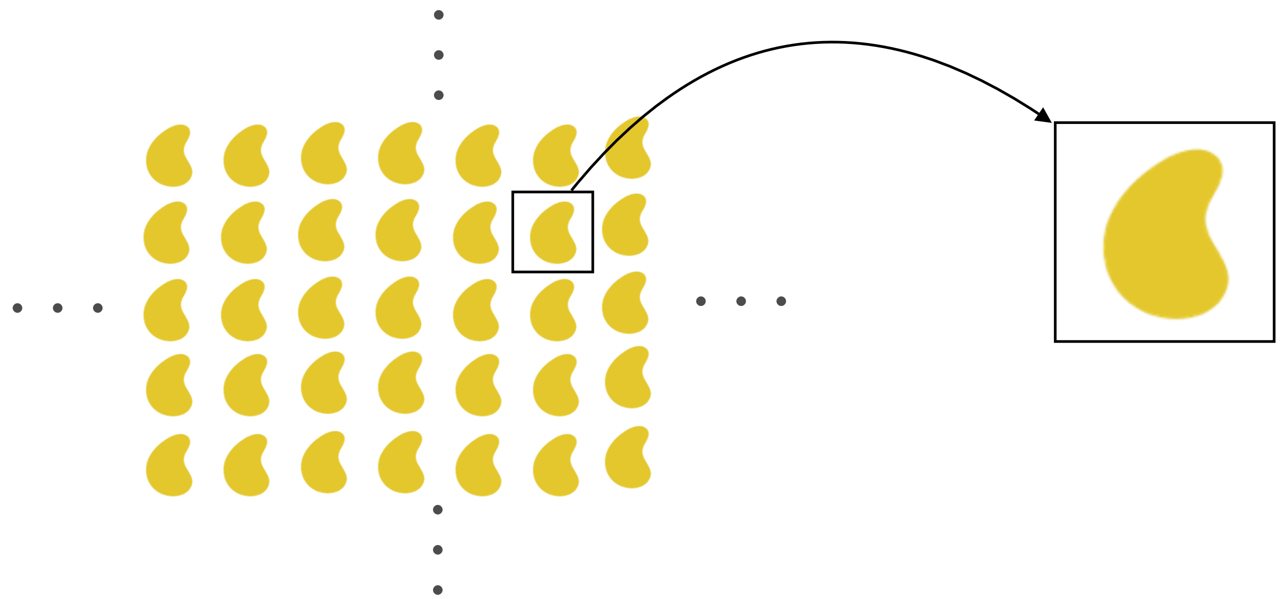

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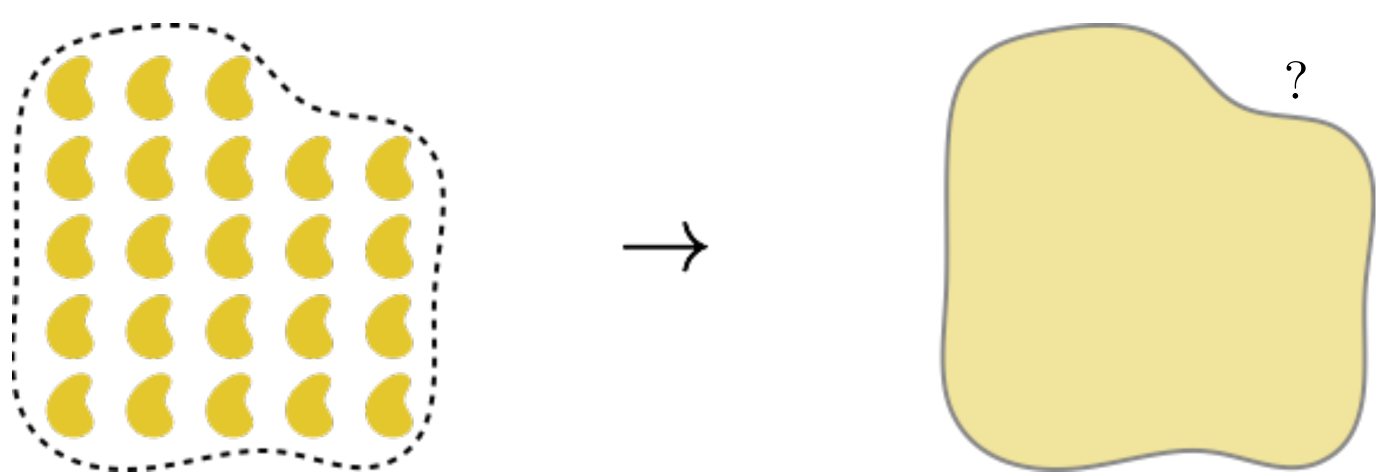
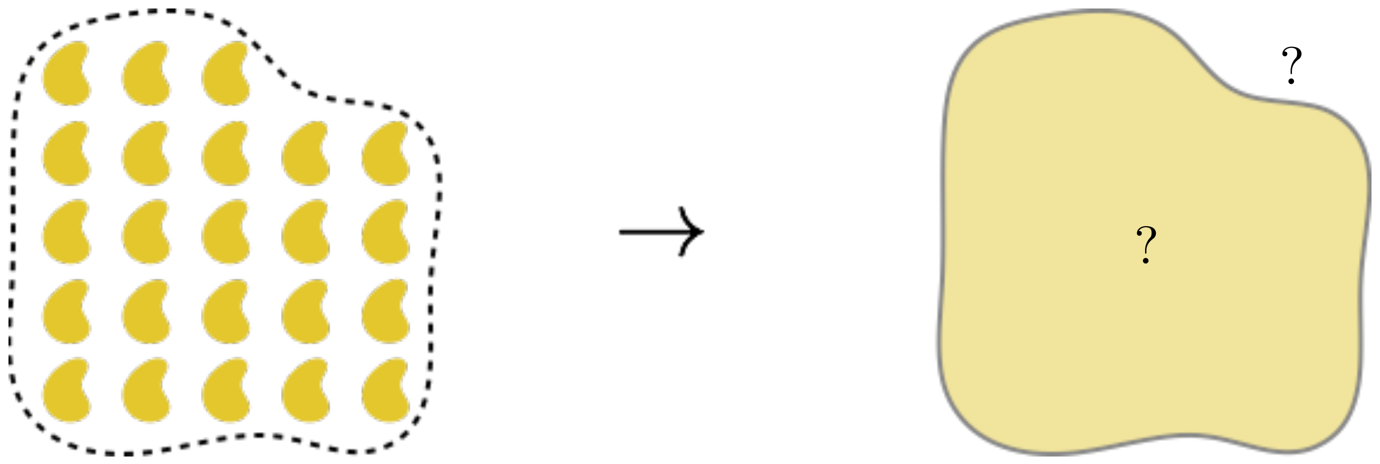


Asymptotic homogenization

2) Propagation and boundary layer effects



Asymptotic homogenization

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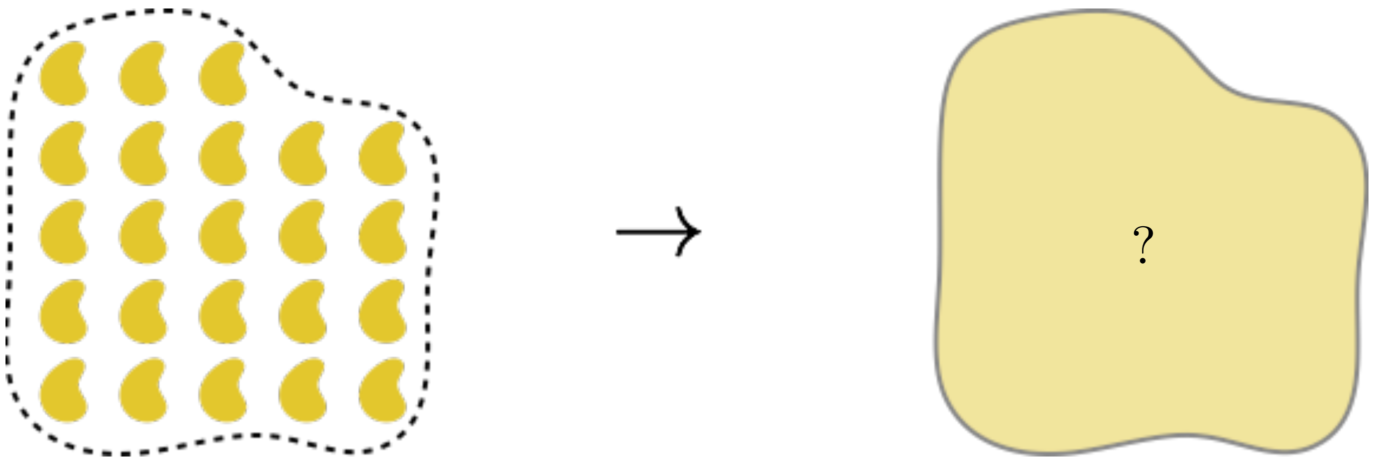

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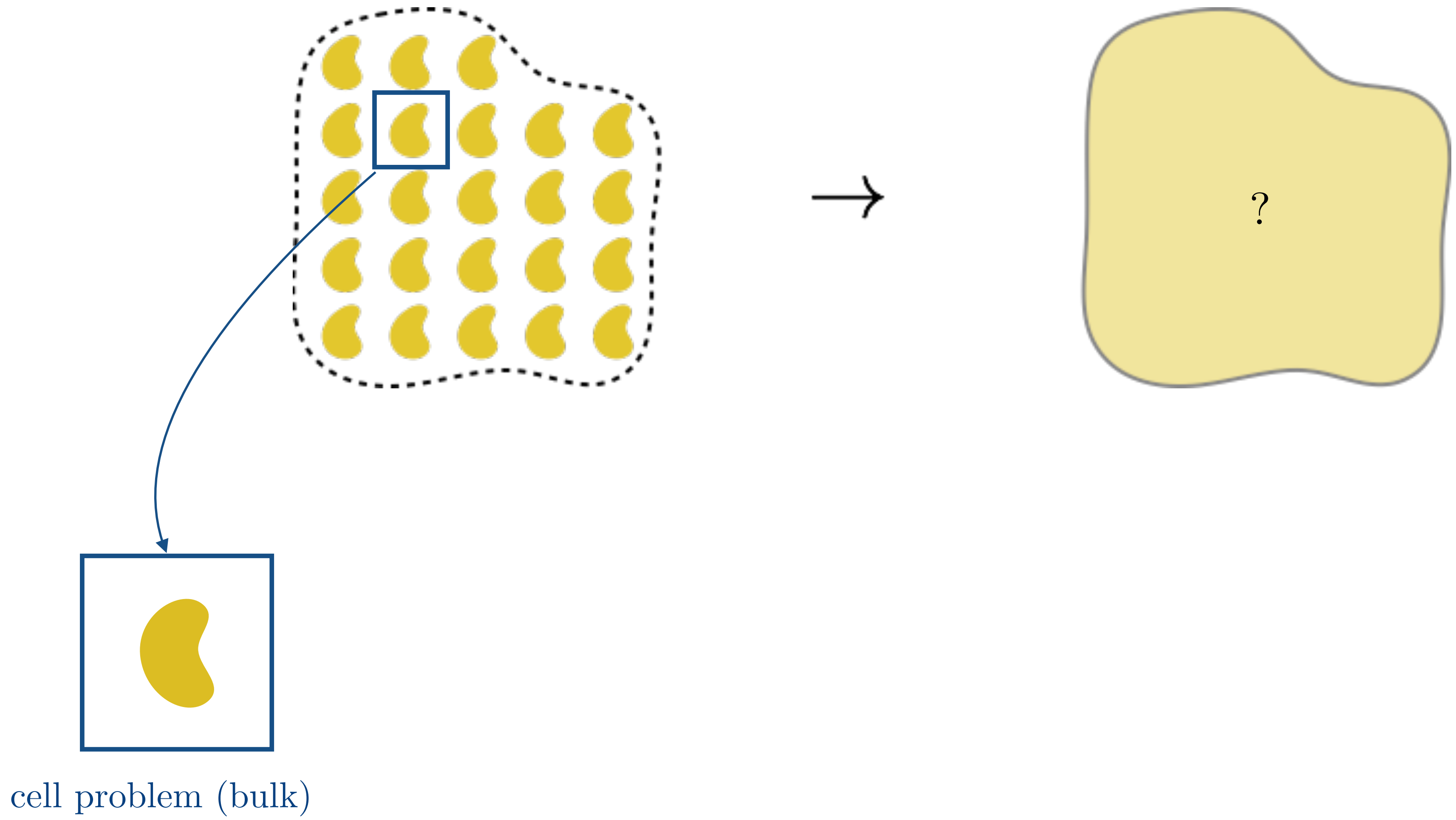

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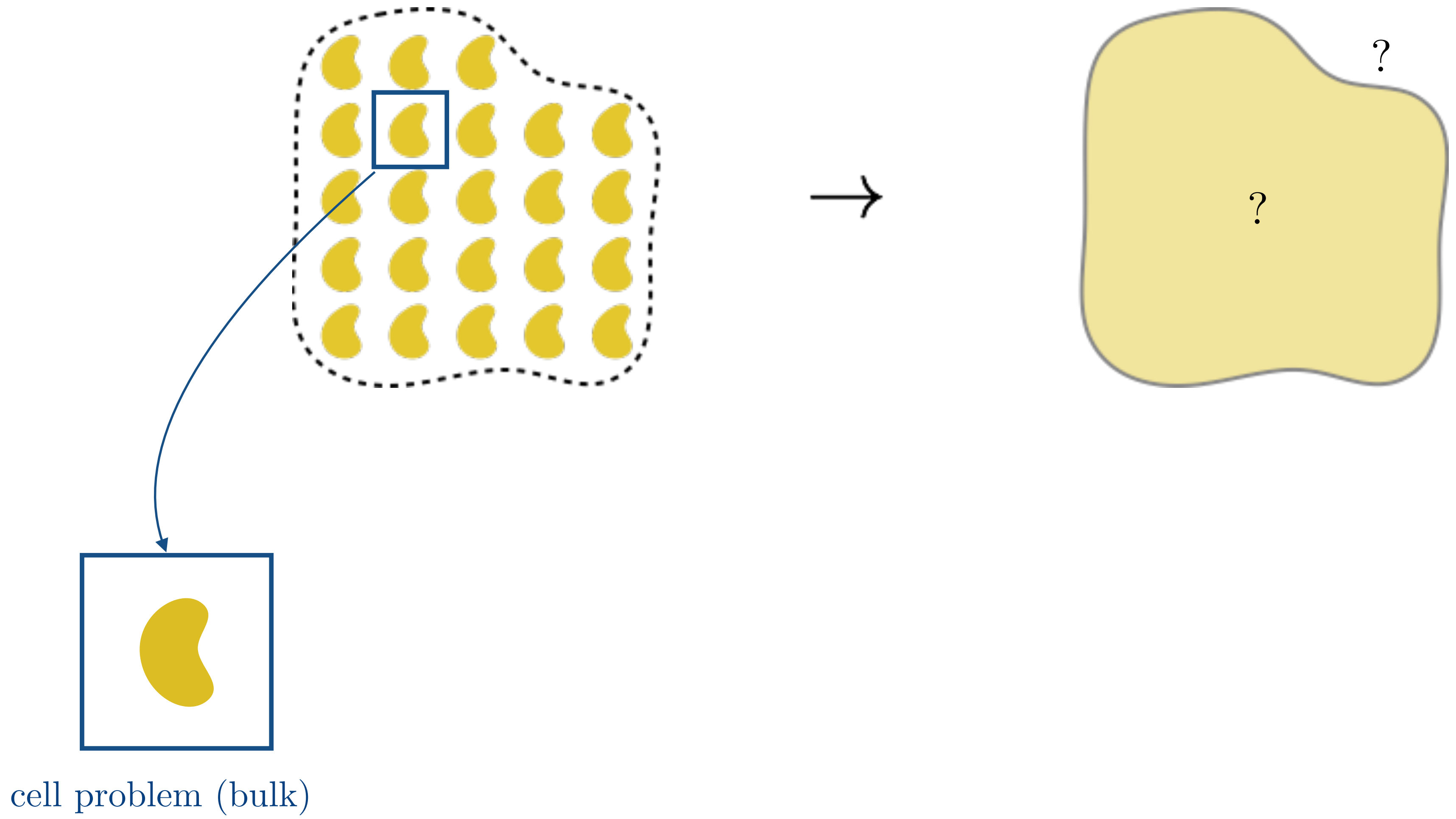

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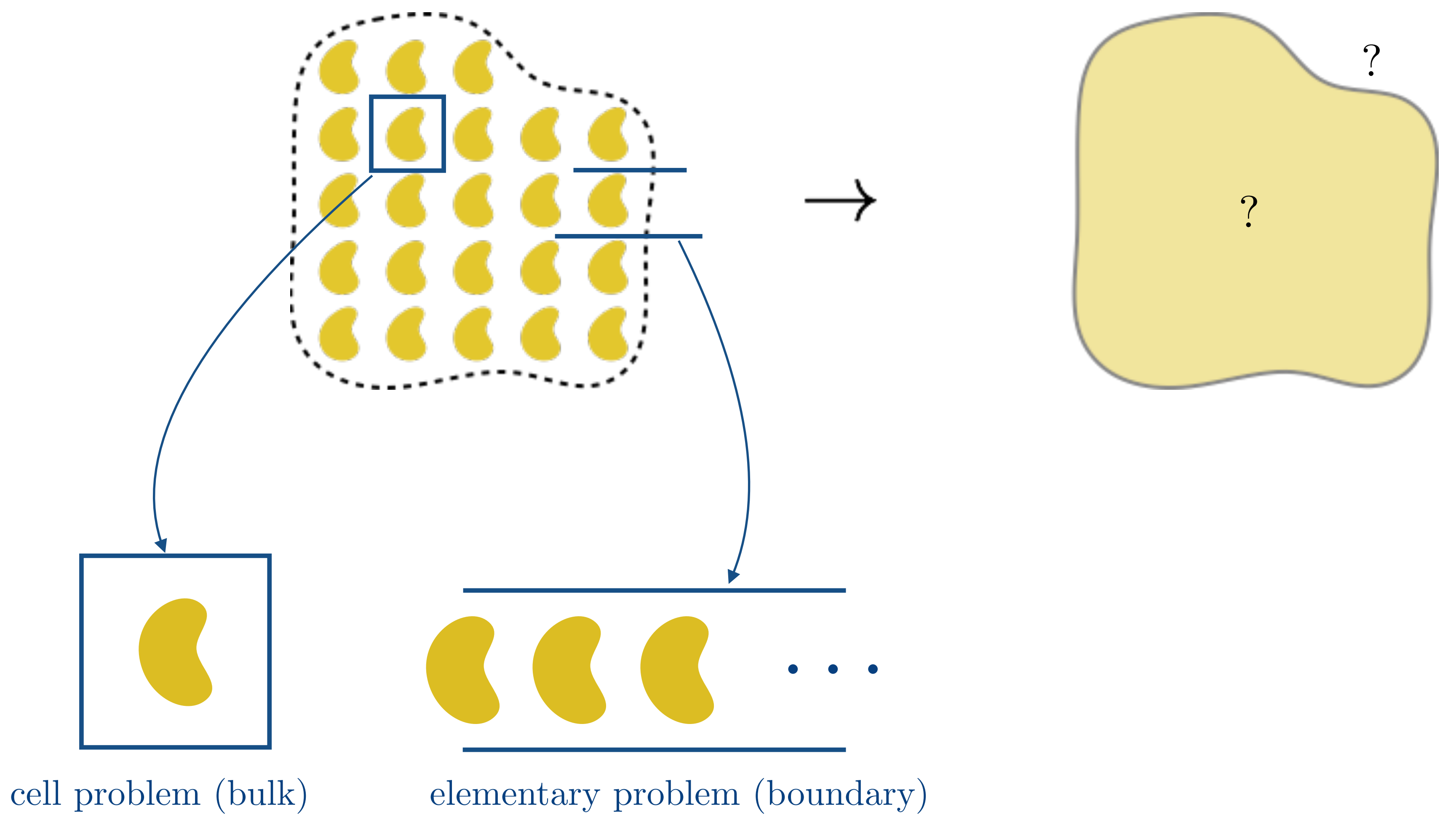

Asymptotic homogenization

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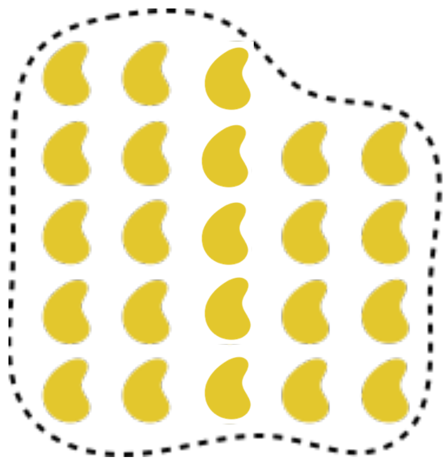
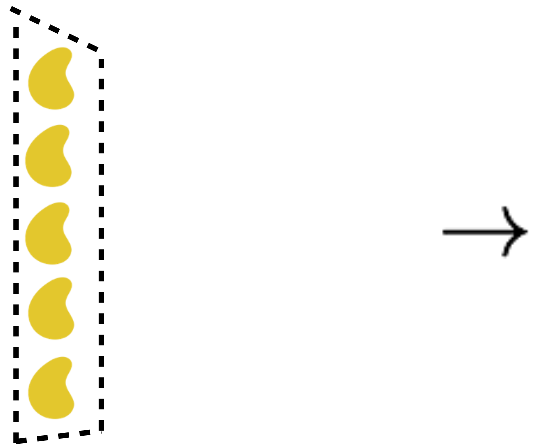


$\rightarrow$

A particular case is that of small thickness of the structure

Asymptotic homogenization

2) Propagation and boundary layer effects



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Asymptotic homogenization

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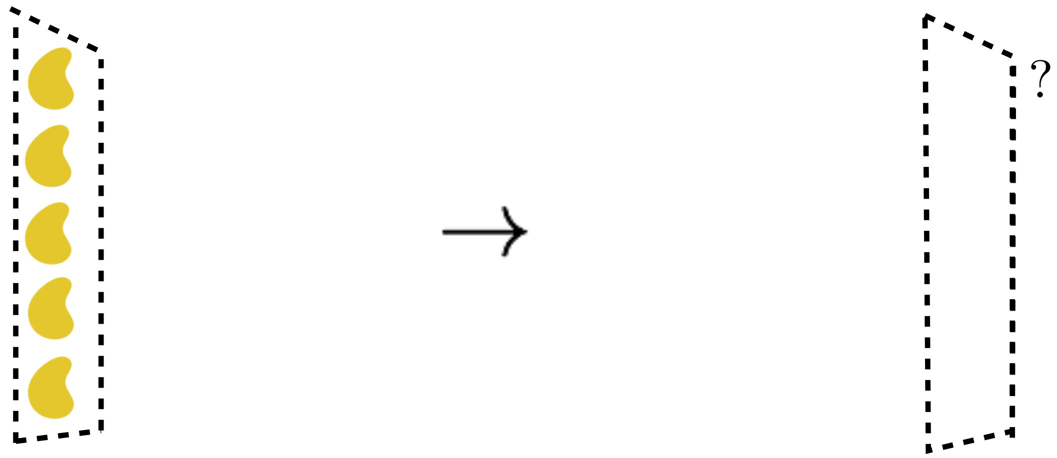

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Asymptotic homogenization

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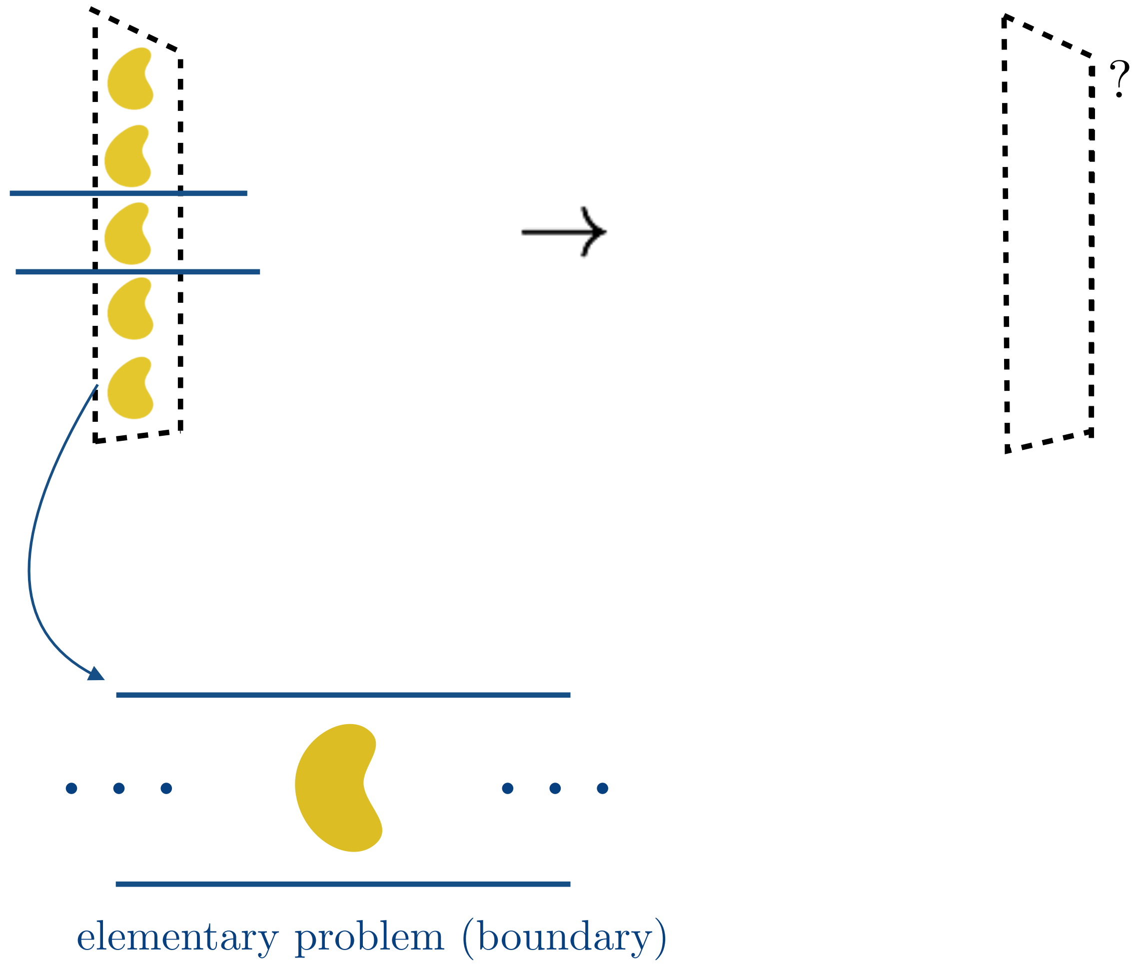

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Asymptotic homogenization

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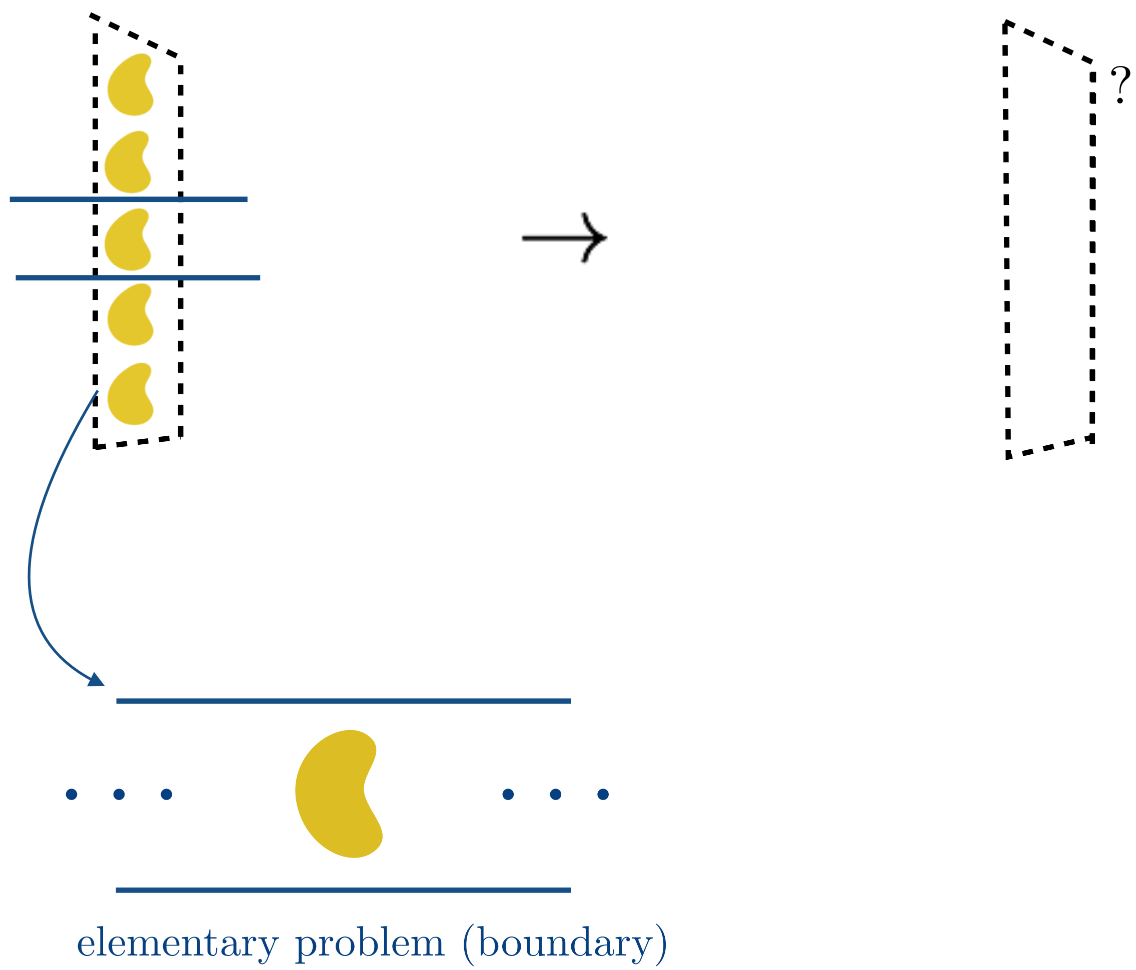


A particular case is that of small thickness of the structure

Other strategies of homogenization → Course of Bérangère Delourme

Asymptotic homogenization

2) Propagation and boundary layer effects

2 different questions that have to be addressed

Asymptotic homogenization

3) Examples (local resonance or not)

Asymptotic homogenization

3) Examples (local resonance or not)

Asymptotic analysis : $\eta = kh \to 0$

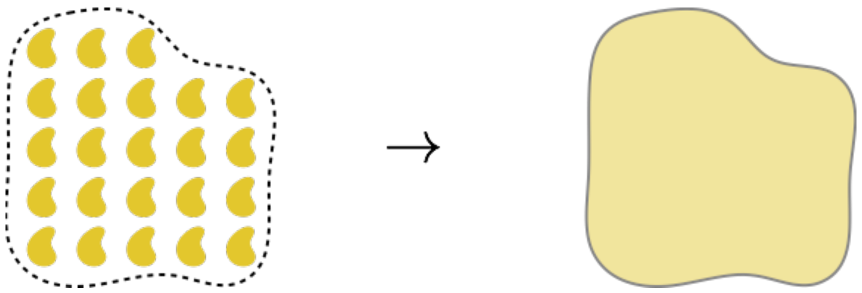
Asymptotic homogenization

3) Examples (local resonance or not)

Asymptotic analysis : $\eta = kh \to 0$

We can expect resonances of the resulting medium:

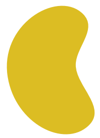
- if resonances take place in the resulting structure



Faraday cage, FP interferometer $\cdots$

$\to$ Course of Kim Pham

- if a single inclusion supports resonances



subwavelength resonance: Mie, Helmholtz, Minnaert $\cdots$

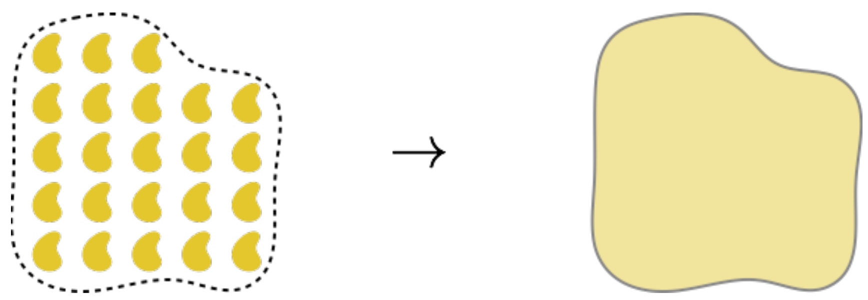
Appropriate scalings are needed to encapsulate the resonance

for instance Mie $\quad kh = \eta \ll 1$ but $k_0 h = O(1) \to c_0/c = O(\eta)$

$\to$ Course of Claude Boutin

Asymptotic homogenization

Maurel & Pham

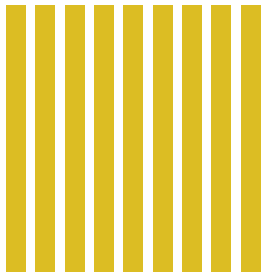


$$\mathrm{div}(a\nabla p) - b\,\frac{\partial^2 p}{\partial t^2} = 0 \qquad \mathrm{div}(\mathsf{a}_{\mathrm{eff}}\nabla p) - b_{\mathrm{eff}}\,\frac{\partial^2 p}{\partial t^2} = 0$$

classical homogenization: effective wave equation

properties of the effective parameters

Acoustic case



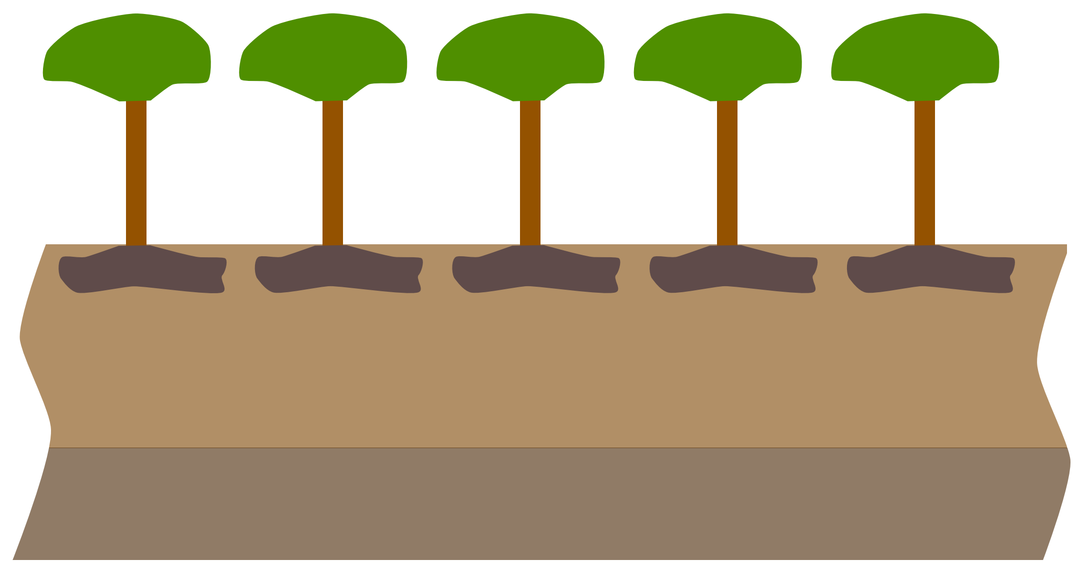
Higher order homogenization, including transmission conditions

Acoustic case

Homogenization of an array of beams on the top of an elastic half-space

2D elastic case

Conversion of surface waves in a forest of trees, a homogenization approach



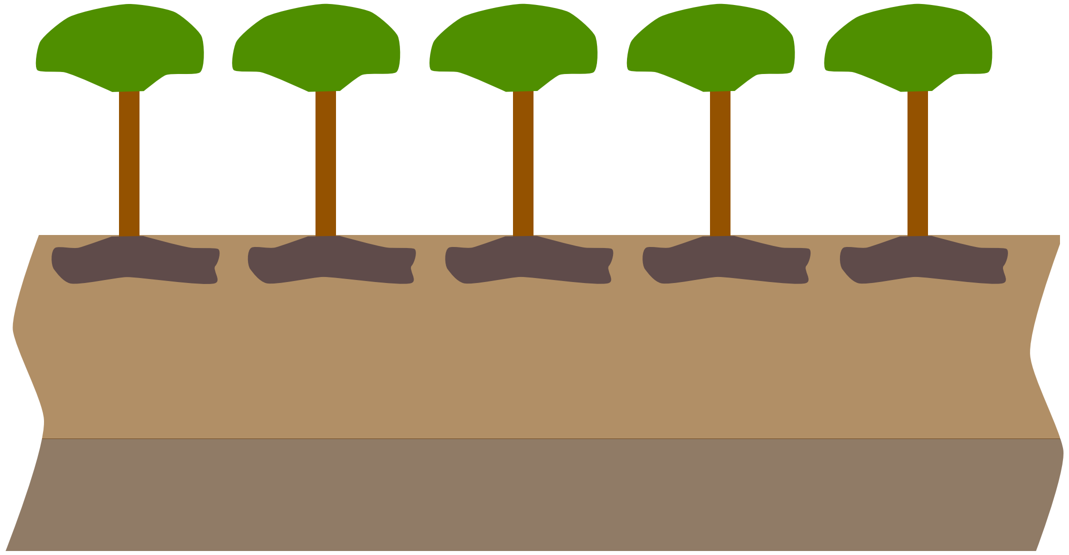
Sébastien Guenneau,
Institut Fresnel, Marseille - France

Agnès Maurel,
Institut Langevin, ESPCI, Paris - France

Jean-Jacques Marigo,
Laboratoire de Mécanique des Solides, Polytechnique, Palaiseau - France

Kim Pham,
Institut des Sciences de la Mécanique et Applications Industrielles,
ENSTA, Palaiseau - France

Conversion of surface waves in a forest of trees, a homogenization approach



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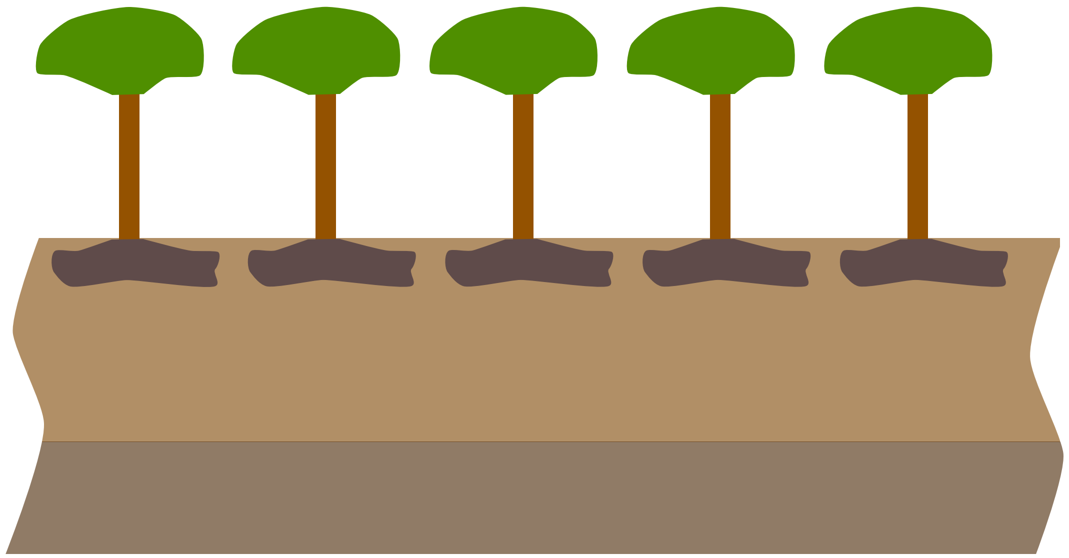
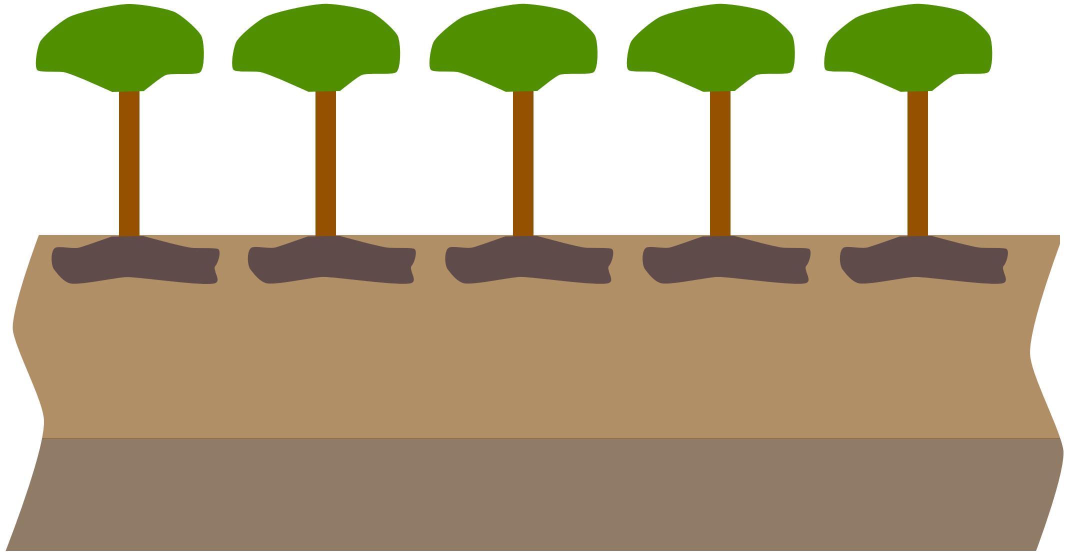


Problem setting:

Anti-plane elasticity

Conversion of surface waves in a forest of trees, a homogenization approach



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Anti-plane elasticity

- Guiding layer over a substrate

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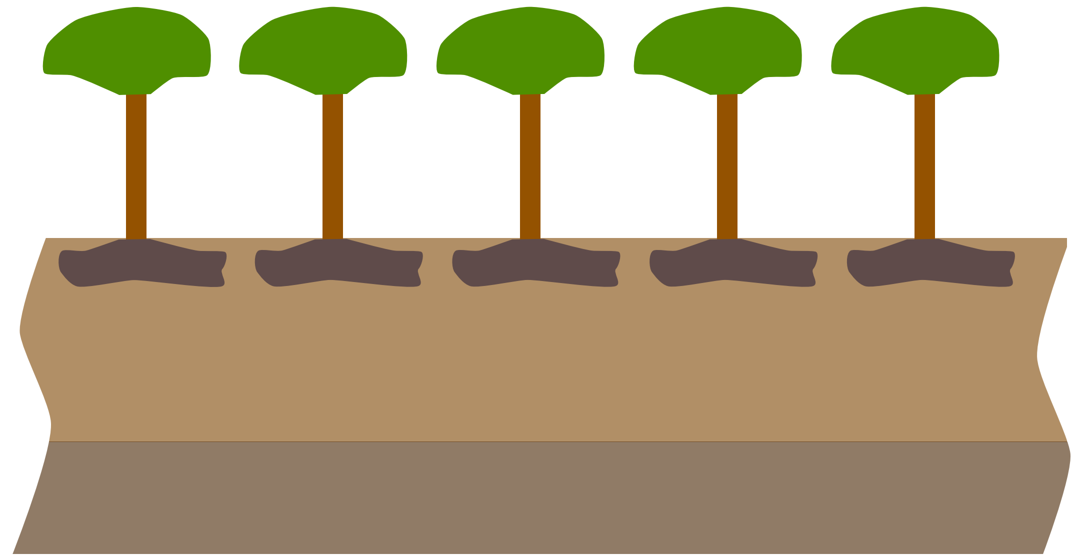

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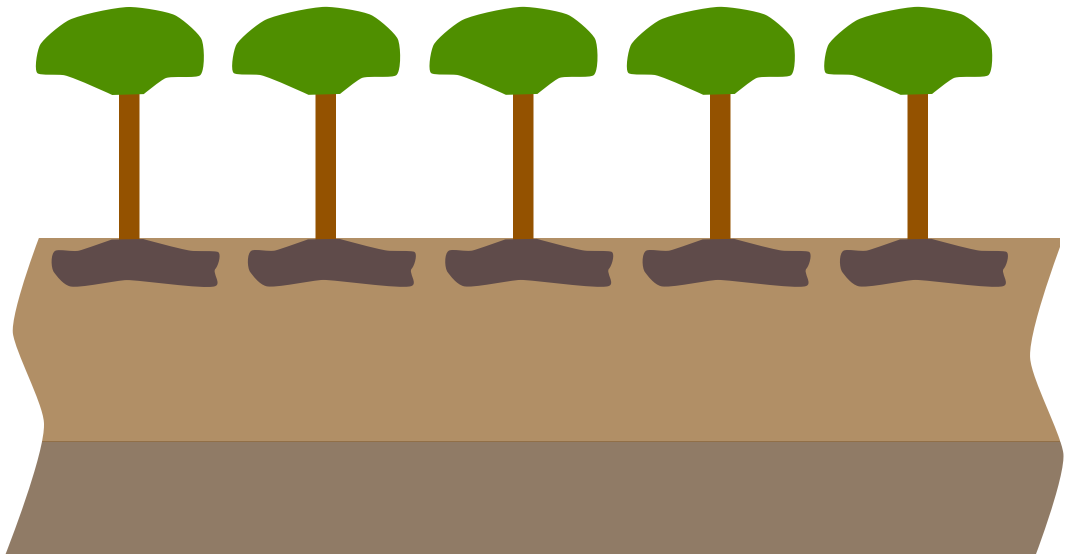

Problem setting:

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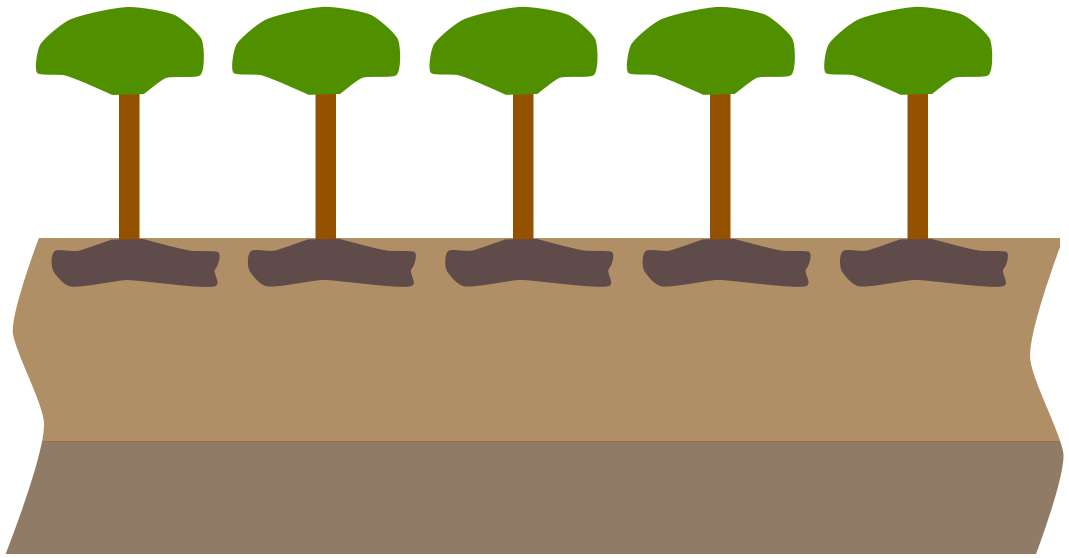
How do Love waves propagate and interact with trees ?

Conversion of surface waves in a forest of trees, a homogenization approach



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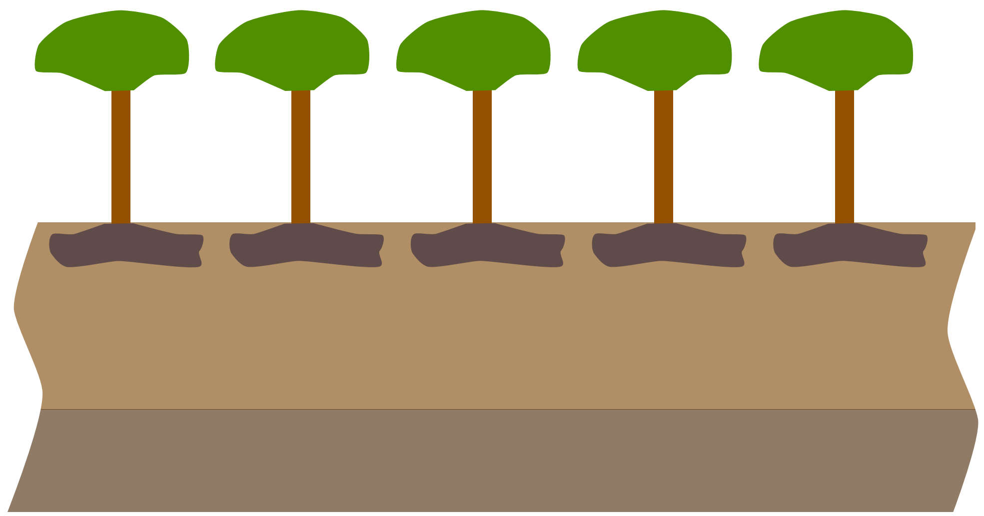
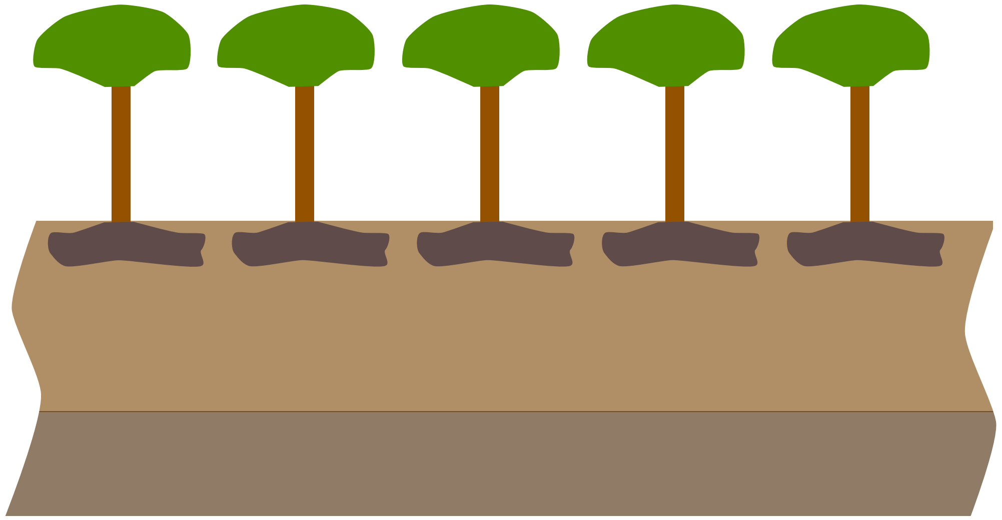


Conversion of surface waves in a forest of trees, a homogenization approach



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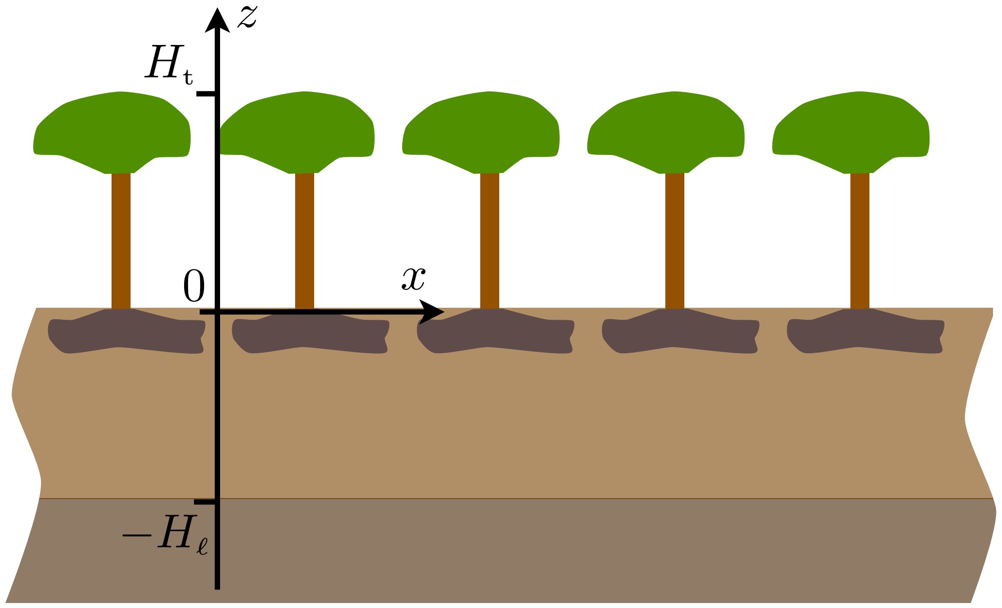

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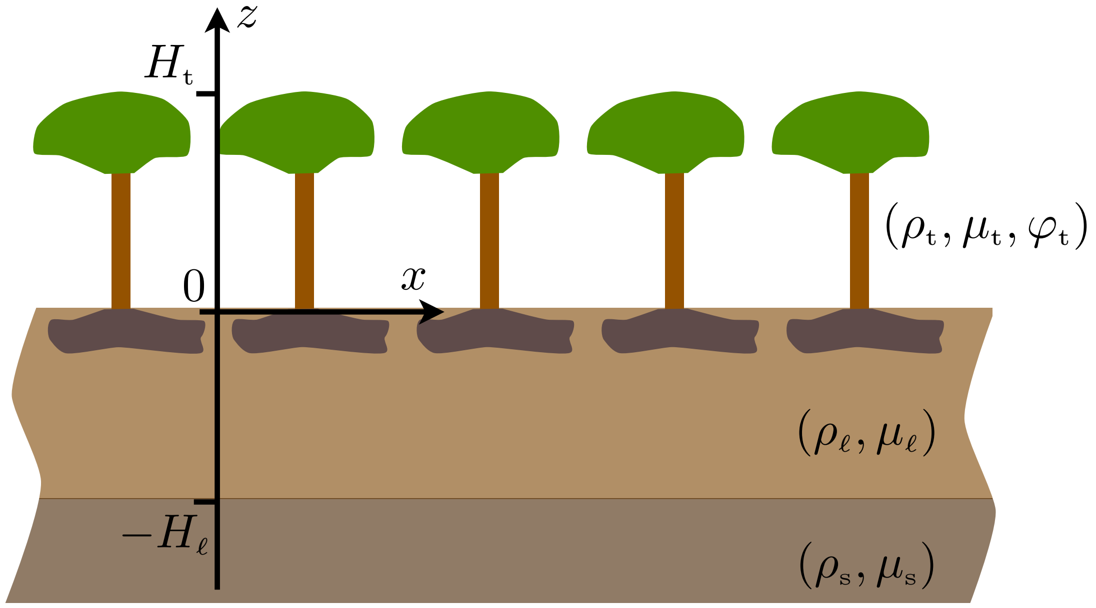

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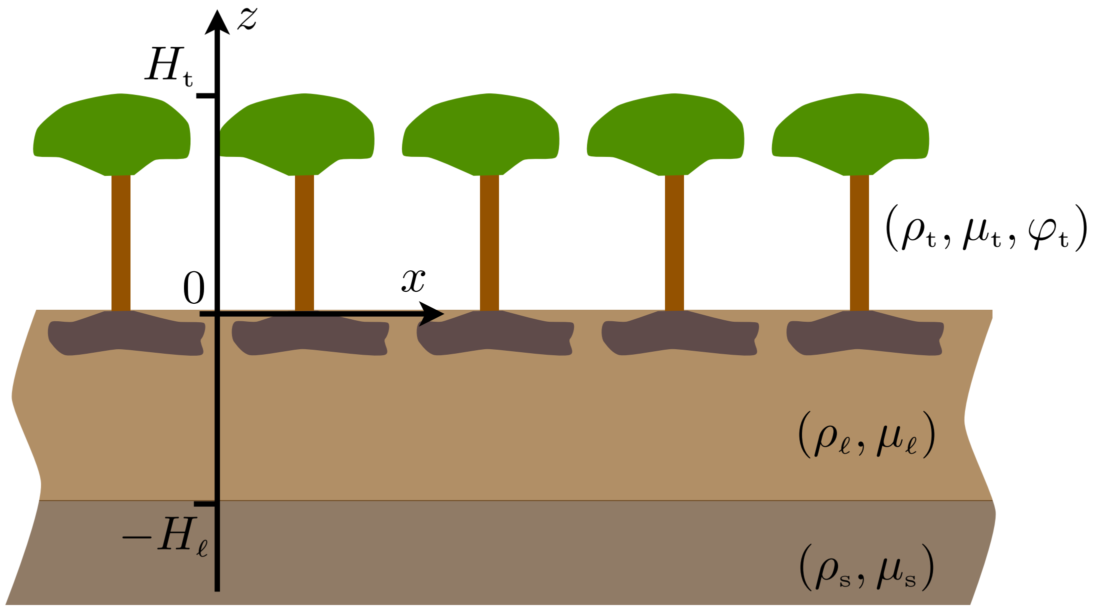
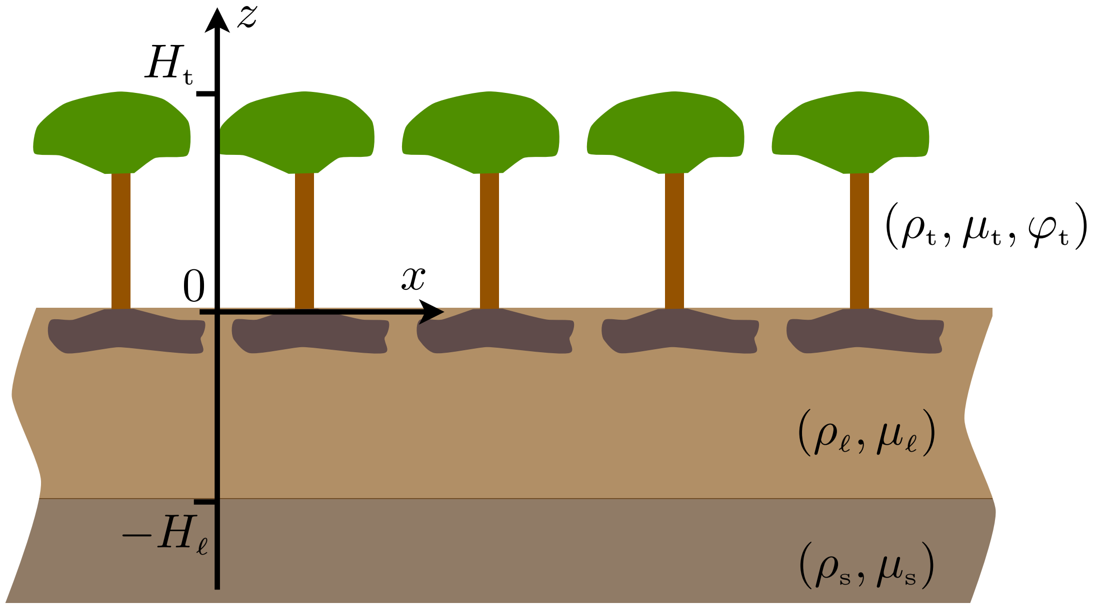


in the trees, layer, substrate

$$\boldsymbol{\sigma} = \mu \nabla u, \quad \rho \frac{\partial^2 u}{\partial t^2} = \mathrm{div} \boldsymbol{\sigma},$$

Conversion of surface waves in a forest of trees, a homogenization approach



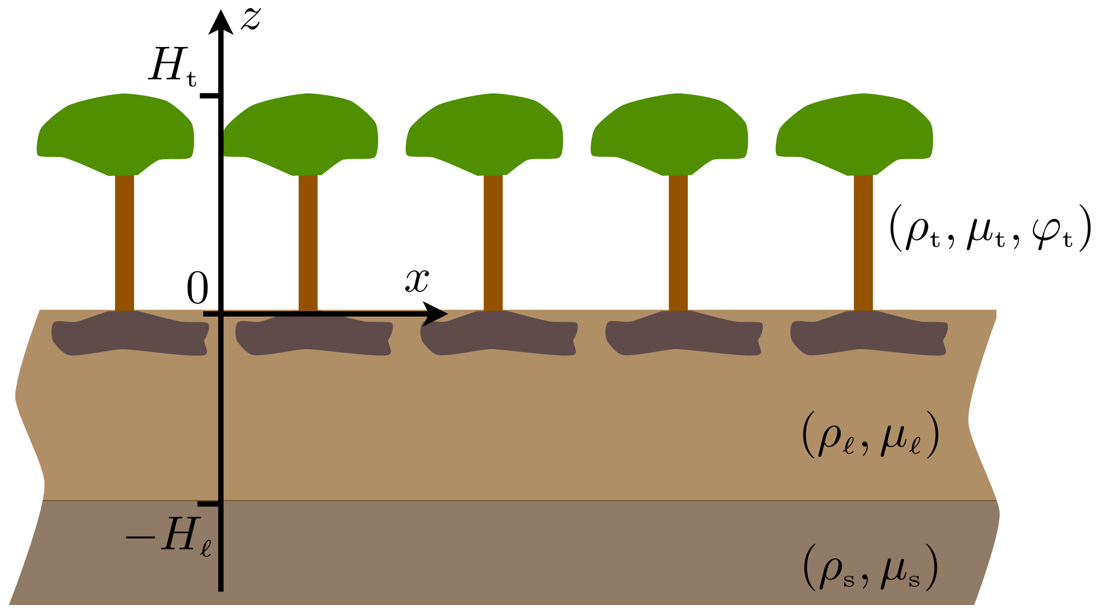
in the trees, layer, substrate

$$\boldsymbol{\sigma} = \mu \nabla u, \quad \rho \frac{\partial^2 u}{\partial t^2} = \mathrm{div} \boldsymbol{\sigma},$$

continuity of u and $\boldsymbol{\sigma} \cdot \mathbf{n}$ between two elastic media

Neumann b.c. $\boldsymbol{\sigma} \cdot \mathbf{n} = 0$ elastic medium/air

Conversion of surface waves in a forest of trees, a homogenization approach



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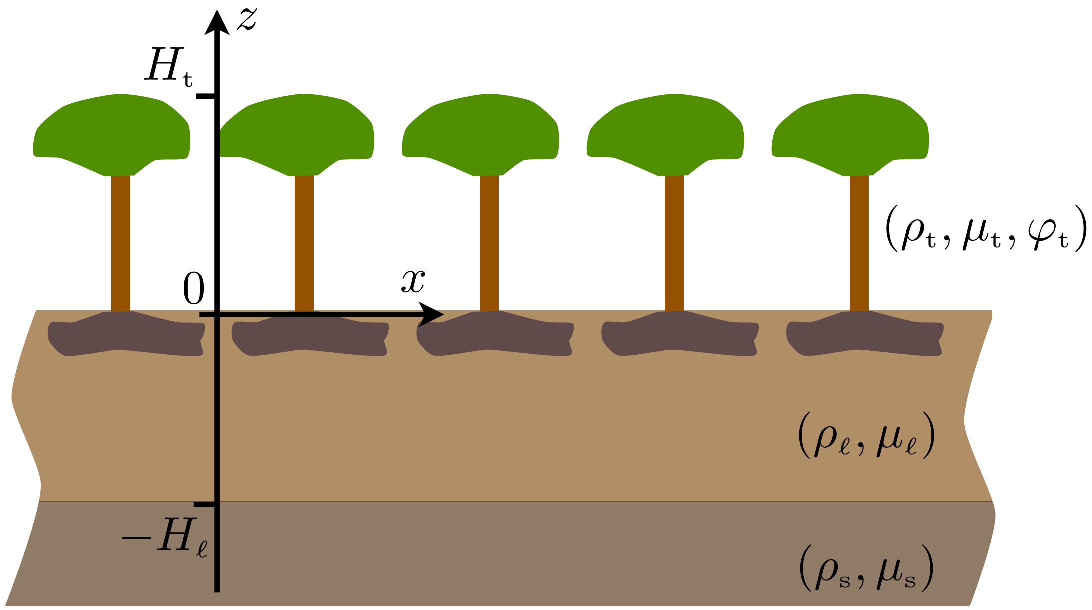

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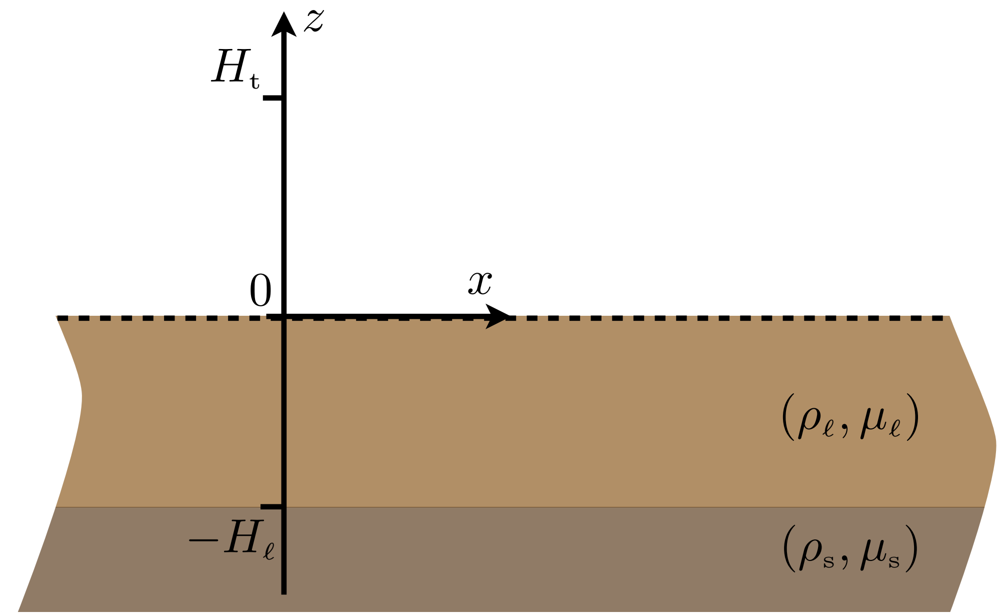
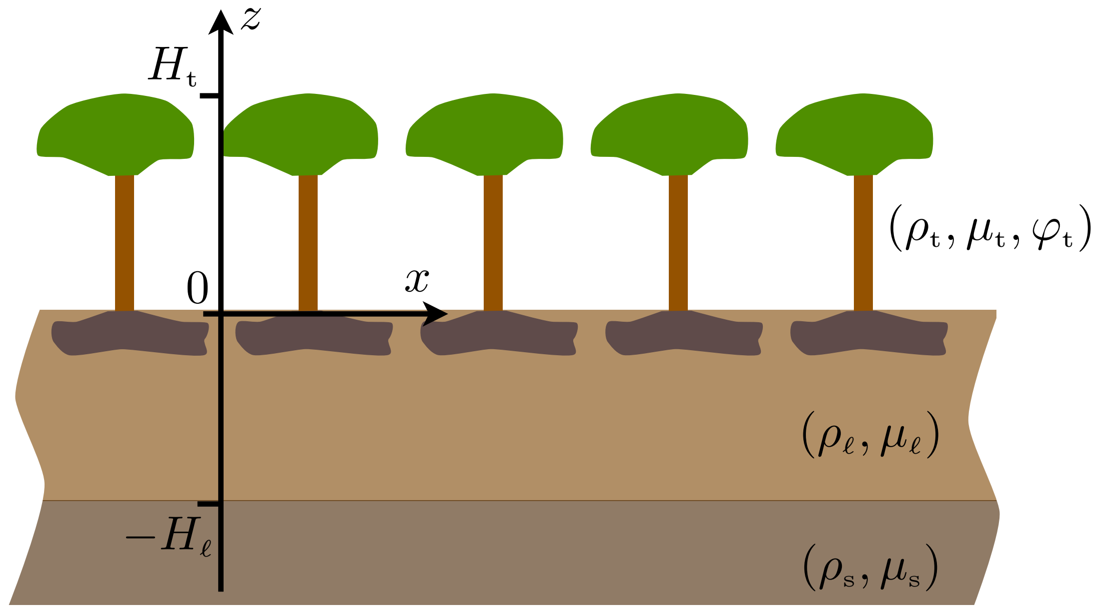
Neumann b.c. $\boldsymbol{\sigma} \cdot \mathbf{n} = 0$ elastic medium/air



in the layer, substrate

$$\boldsymbol{\sigma} = \mu \nabla u, \quad \rho \frac{\partial^2 u}{\partial t^2} = \mathrm{div}\boldsymbol{\sigma},$$

Conversion of surface waves in a forest of trees, a homogenization approach



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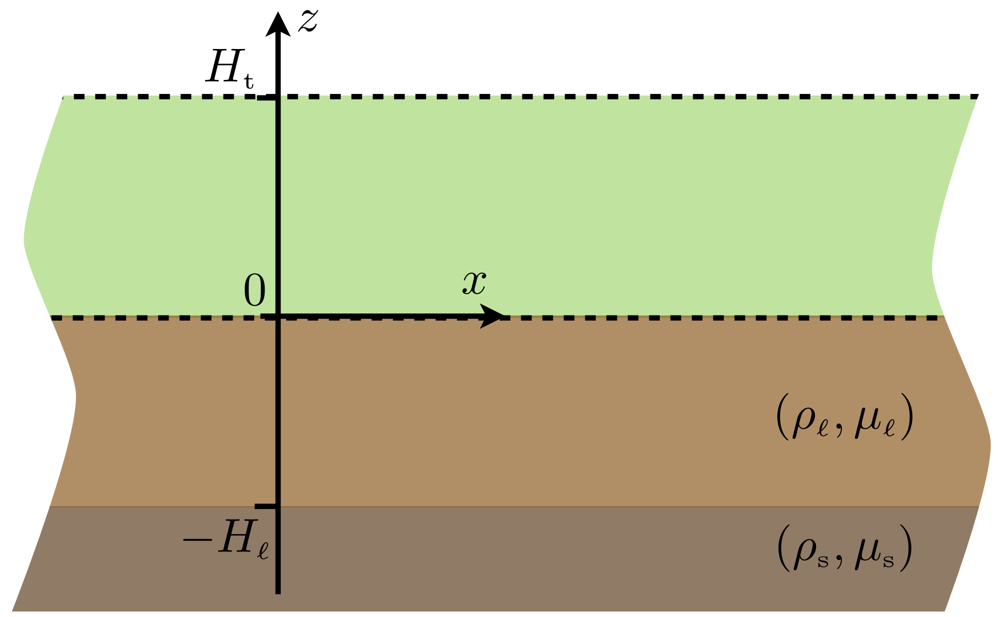

in the effective medium (trees)

$$\boldsymbol{\sigma} = \mu_t \begin{pmatrix} 0 & 0 \\ 0 & \varphi_t \end{pmatrix} \nabla u, \quad \rho_t \frac{\partial^2 u}{\partial t^2} = \mathrm{div}\boldsymbol{\sigma},$$

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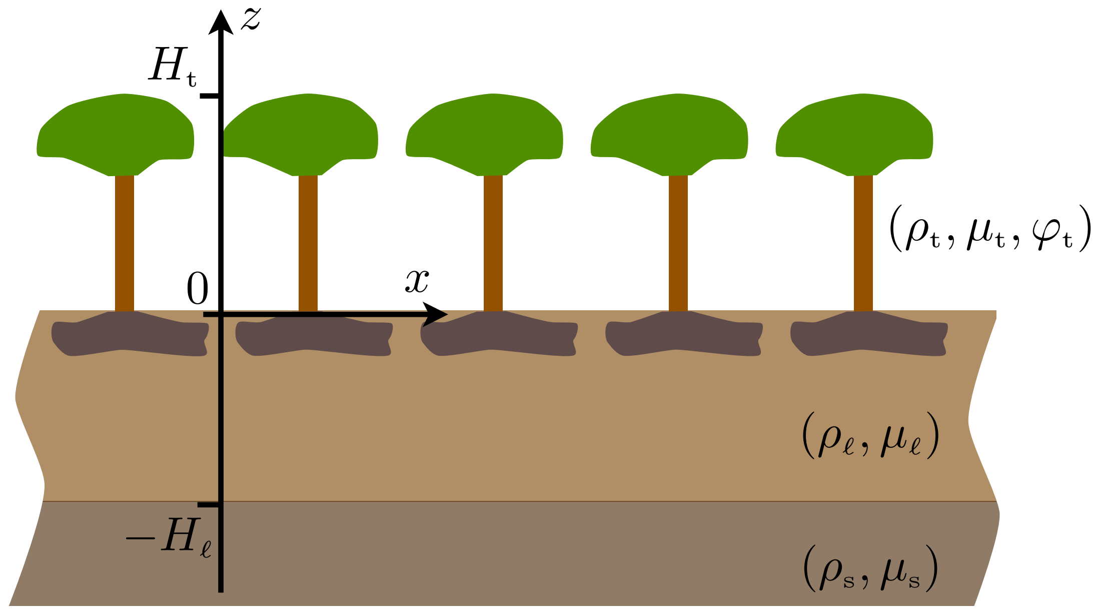

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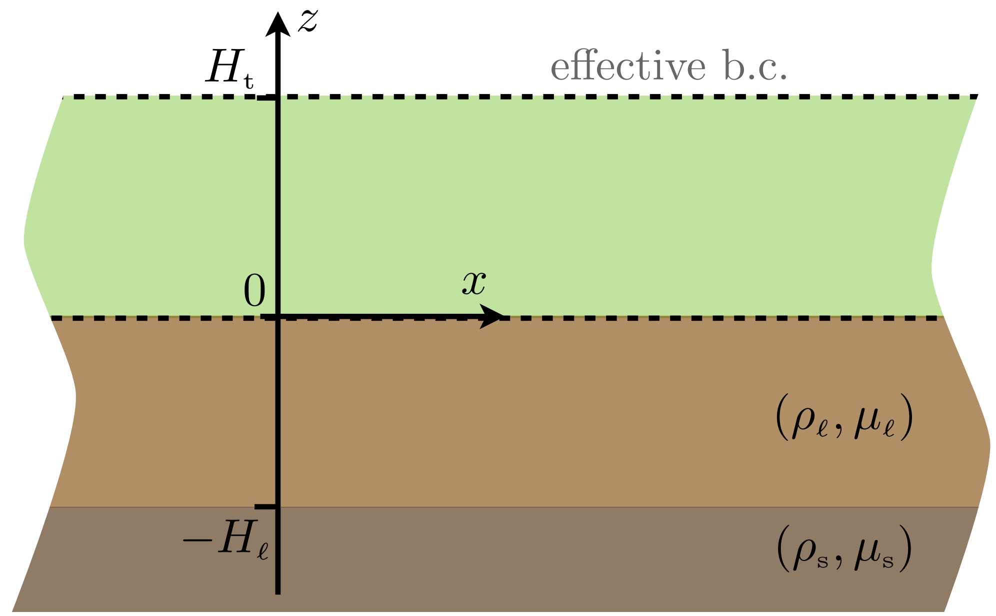

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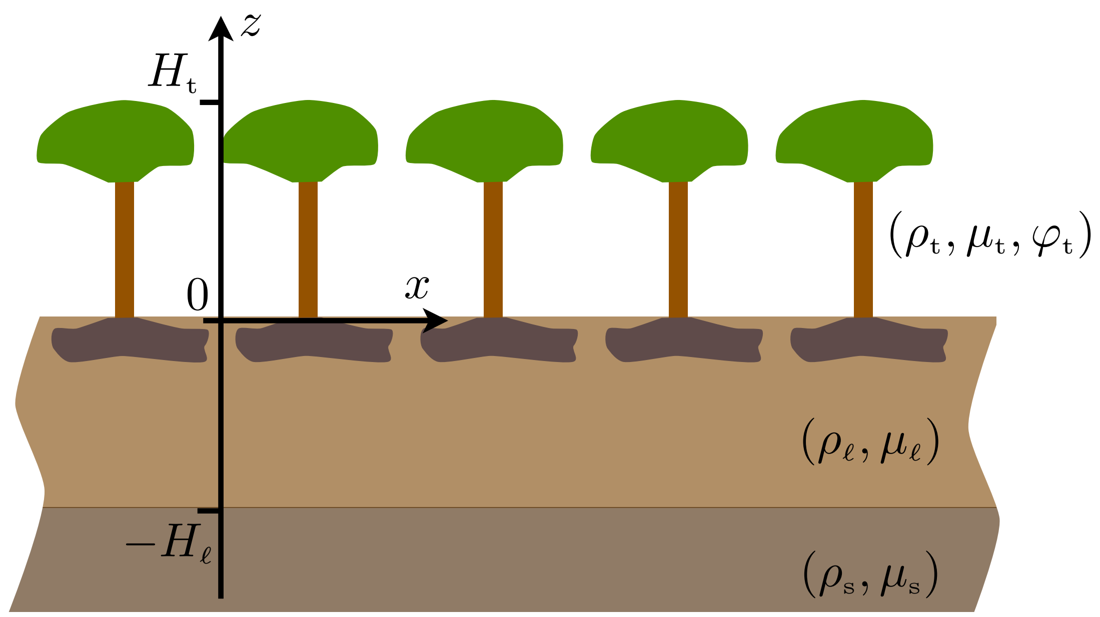
Conversion of surface waves in a forest of trees, a homogenization approach



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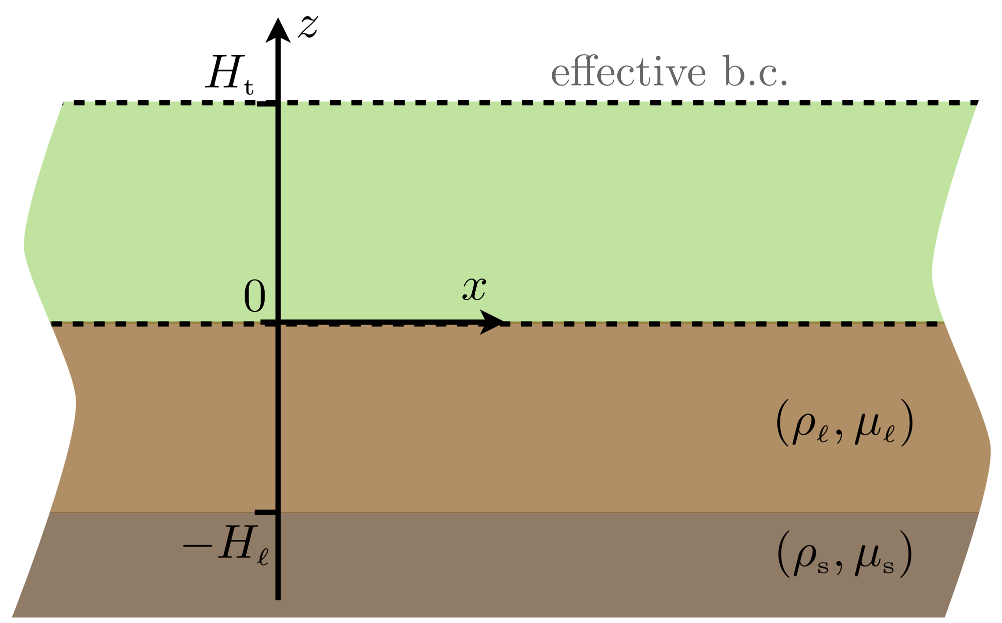

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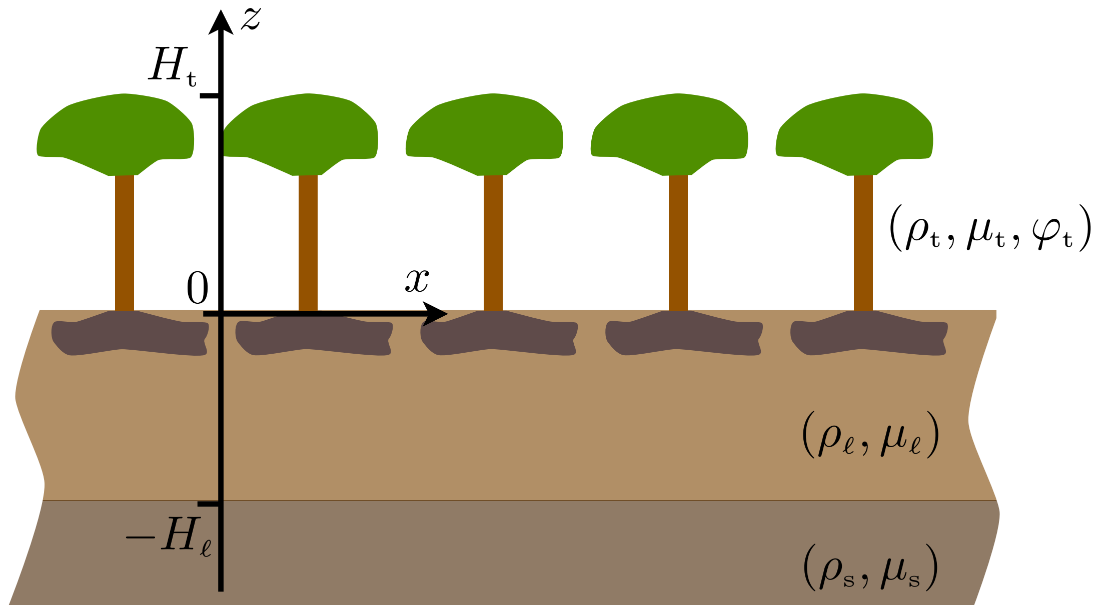
- $z = H_t, \qquad \sigma_z = -L_e \dfrac{\partial \sigma_z}{\partial z}$

Conversion of surface waves in a forest of trees, a homogenization approach



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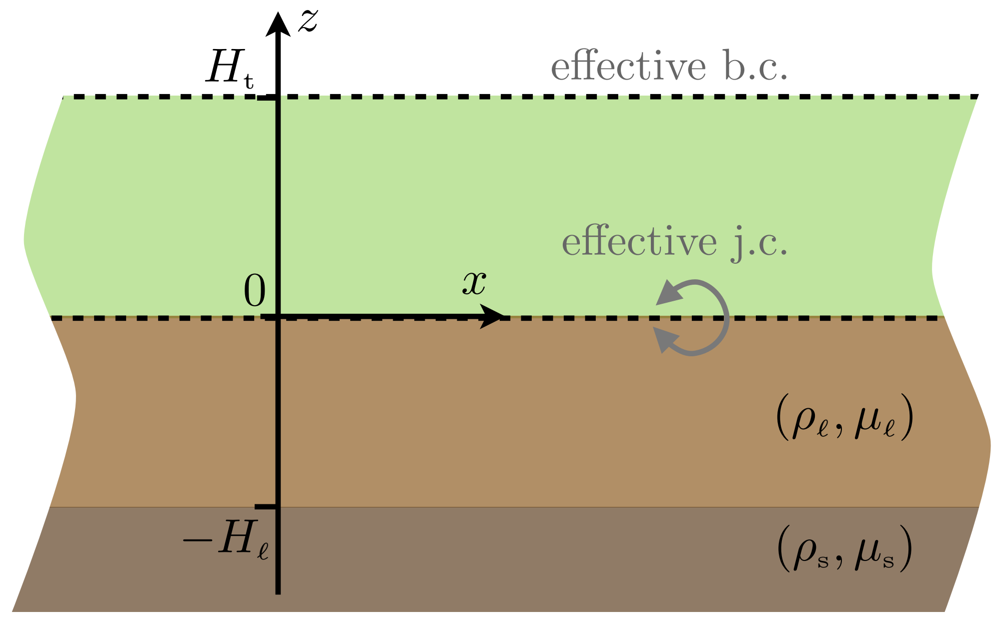

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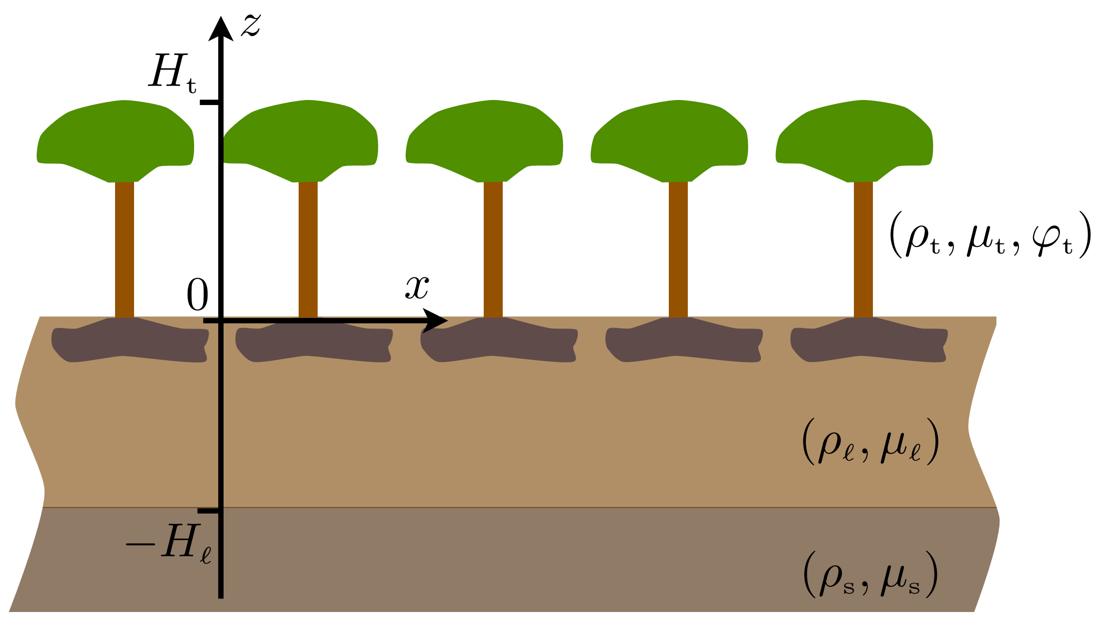
Conversion of surface waves in a forest of trees, a homogenization approach



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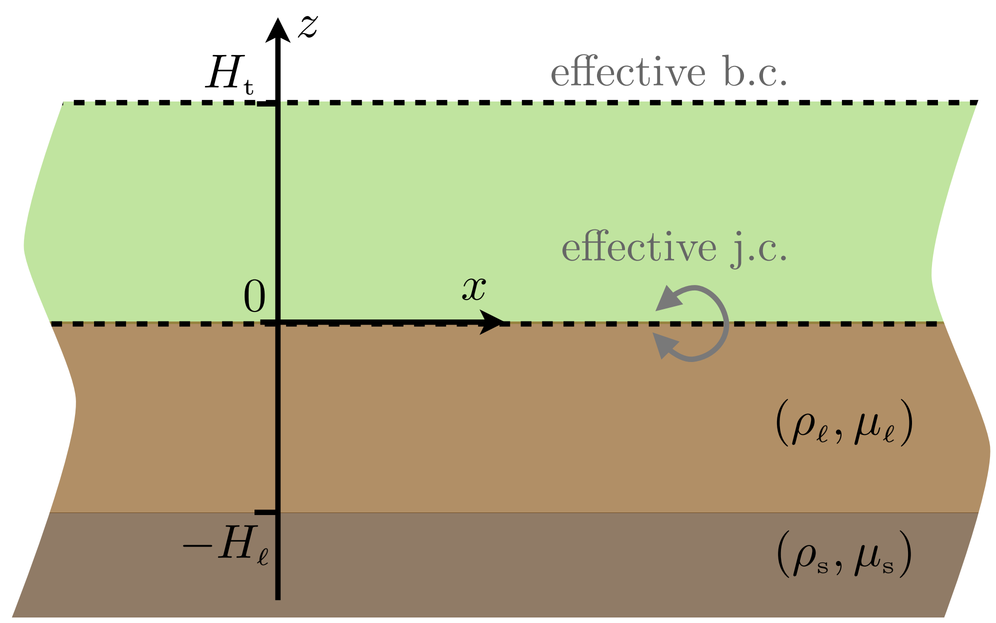

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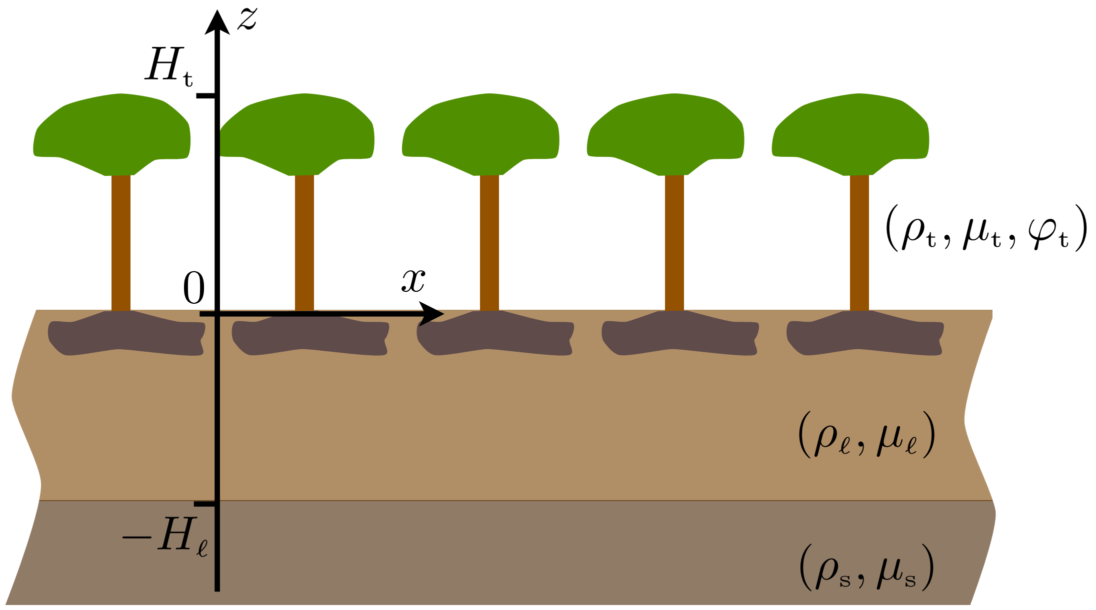
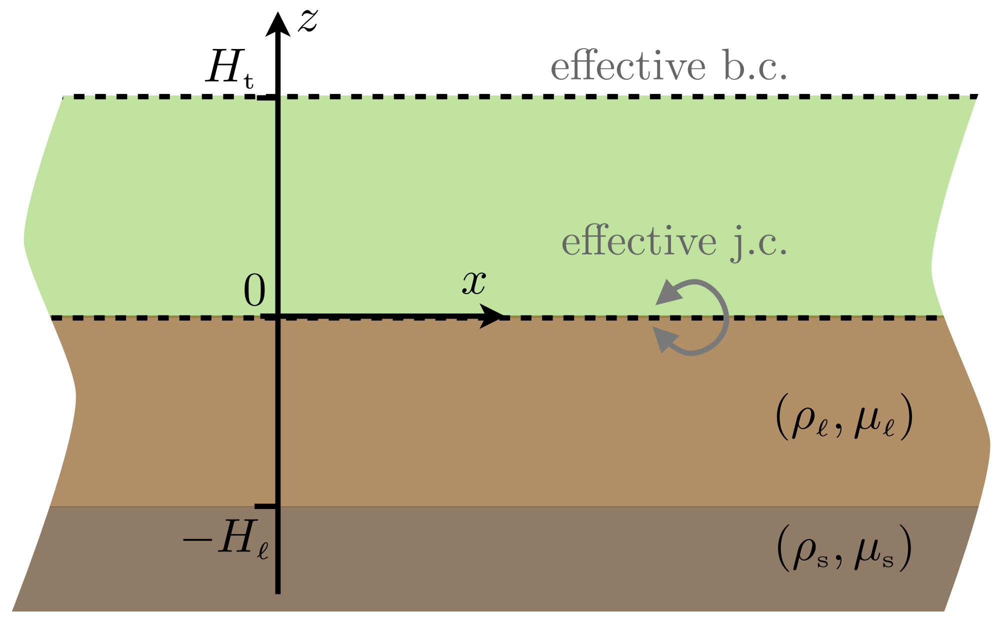
- $z = H_t,$ $\quad \sigma_z = -L_e \dfrac{\partial \sigma_z}{\partial z}$

- $z = 0,$ $\quad [\![u]\!] = \mathcal{B}_1 \sigma_z + \mathcal{B}_2 \dfrac{\partial u}{\partial x}$, $\quad [\![\sigma_z]\!] = \mathcal{B}_2 \dfrac{\partial \sigma_z}{\partial x} - \mathcal{B}_3 \dfrac{\partial^2 u}{\partial x^2} + \rho_e \dfrac{\partial^2 u}{\partial t^2}$

Conversion of surface waves in a forest of trees, a homogenization approach





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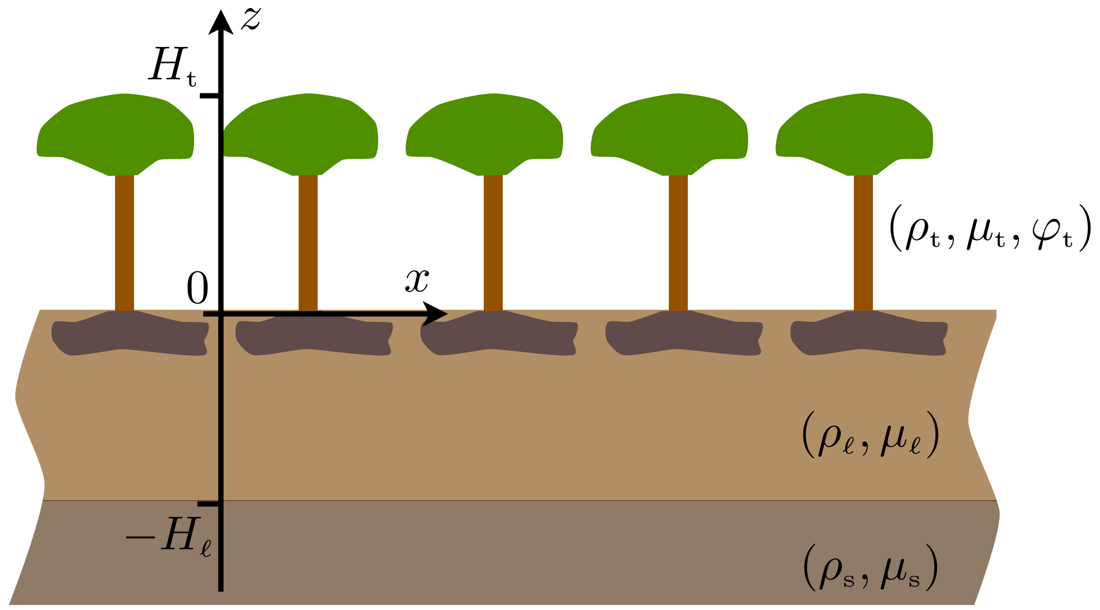
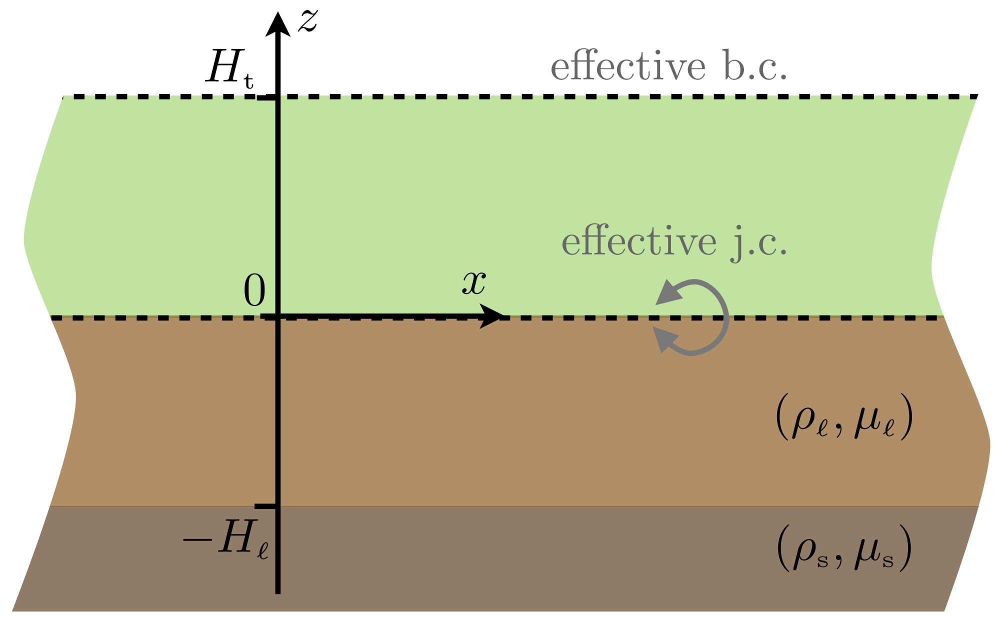
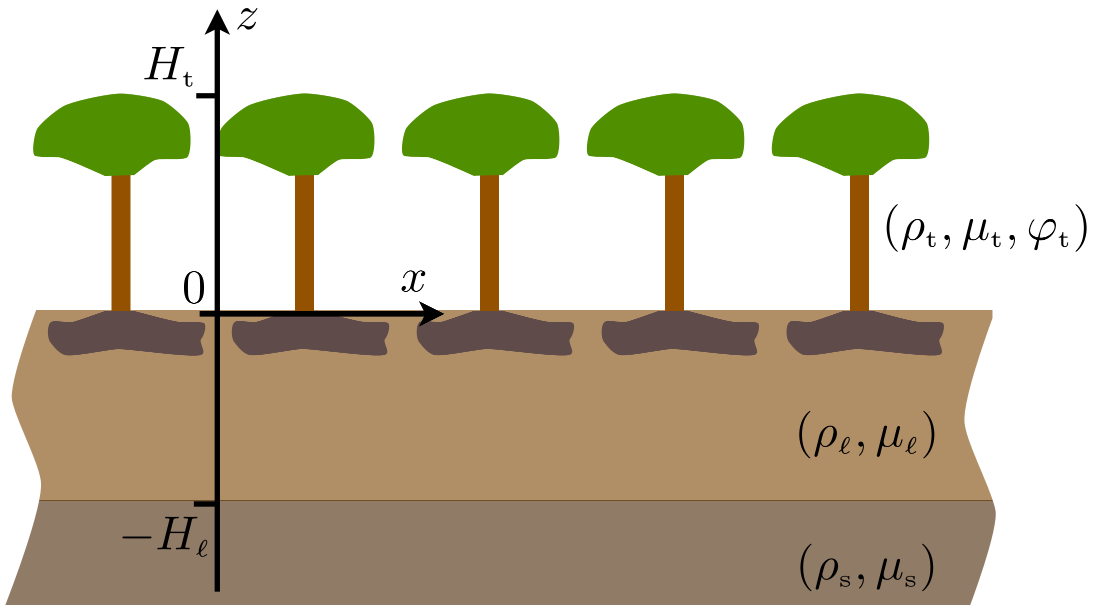
Conversion of surface waves in a forest of trees, a homogenization approach





Conversion of surface waves in a forest of trees, a homogenization approach



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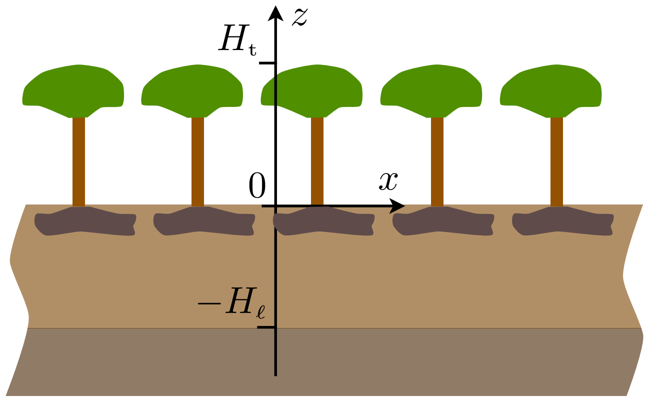
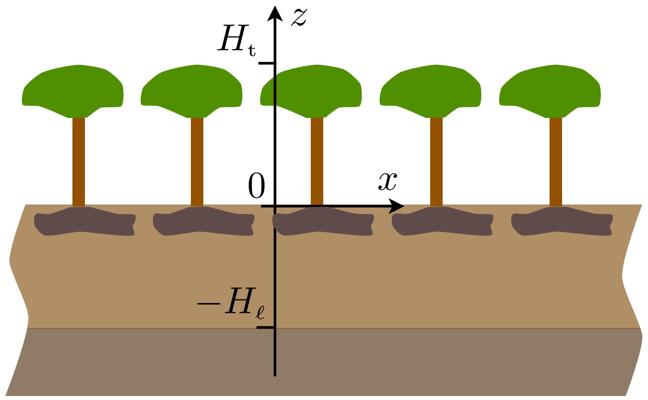
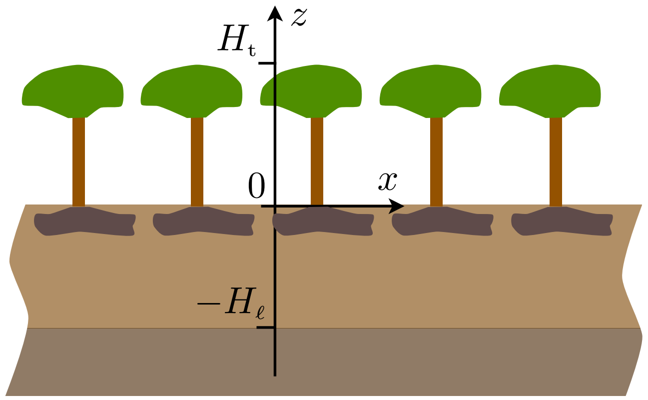


Conversion of surface waves in a forest of trees, a homogenization approach



Conversion of surface waves in a forest of trees, a homogenization approach

Results in the harmonic regime



Conversion of surface waves in a forest of trees, a homogenization approach

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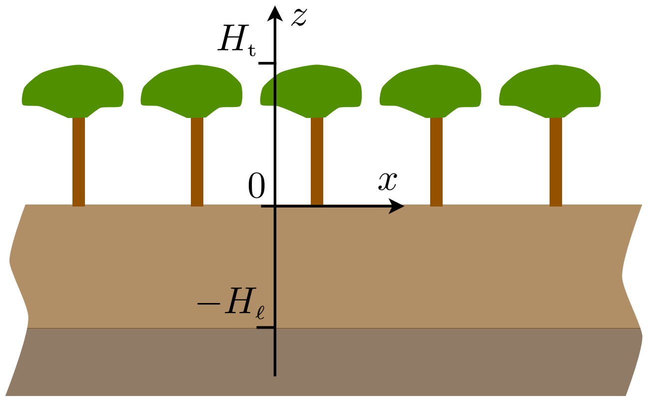

Conversion of surface waves in a forest of trees, a homogenization approach

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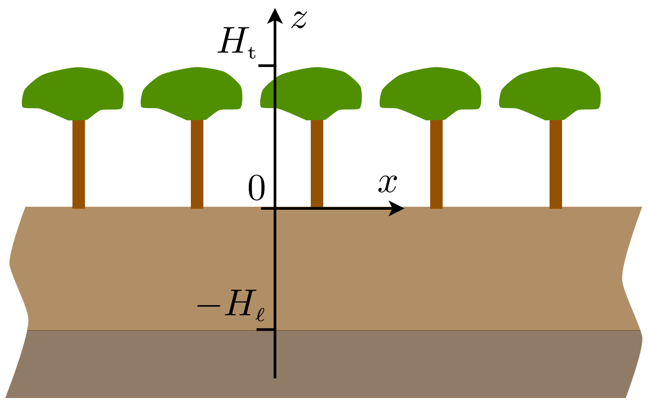
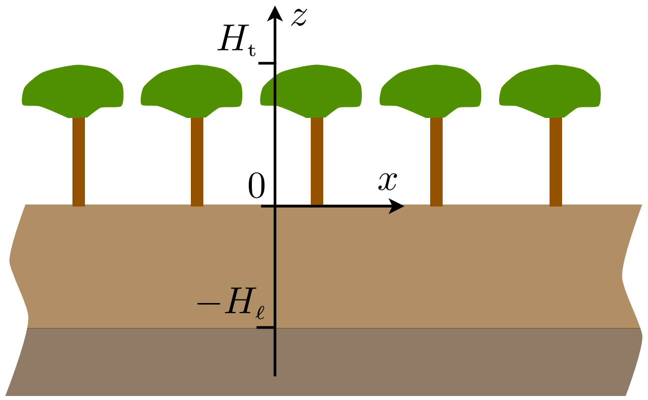


Guided waves for the homogenized problem

Conversion of surface waves in a forest of trees, a homogenization approach

Results in the harmonic regime

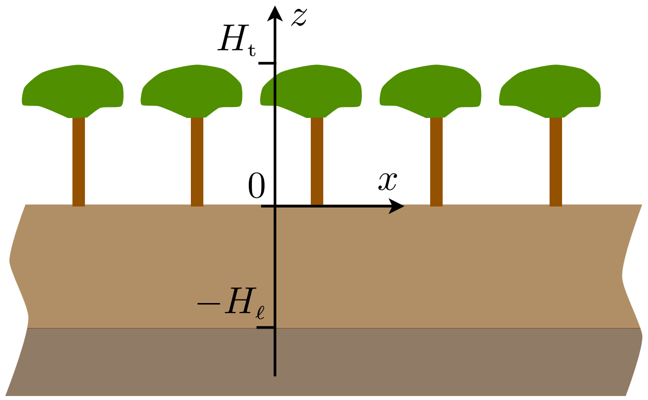


Guided waves for the homogenized problem

$$u(x,z) = \begin{cases} e^{\alpha_s z} e^{i\beta x}, & z < -H_\ell, \\ (A \cos k_\ell z + B \sin k_\ell z) e^{i\beta x}, & -H_\ell < z < 0 \\ (C \cos k_t z + D \sin k_t z) e^{i\beta x}, & 0 < z < H_t \end{cases}$$

Conversion of surface waves in a forest of trees, a homogenization approach

Results in the harmonic regime



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$$\alpha_s = \sqrt{\beta^2 - \frac{\omega^2}{c_s^2}}$$

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Conversion of surface waves in a forest of trees, a homogenization approach

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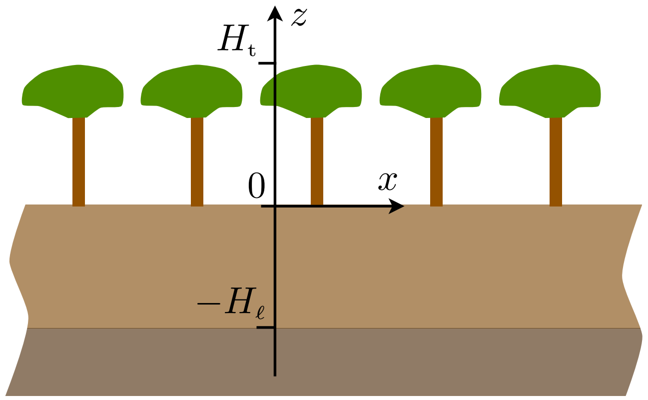

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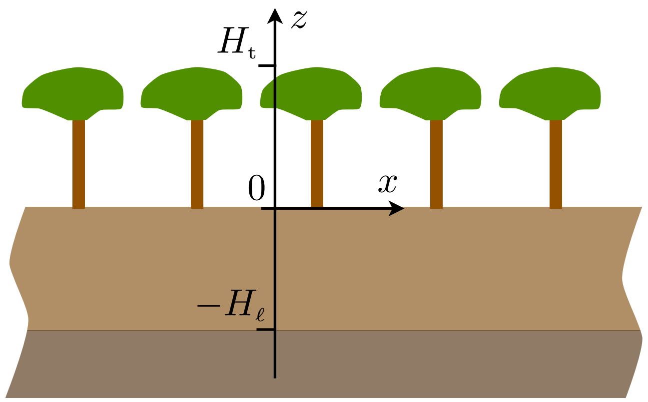
Dispersion relation:

$$1 - \frac{\mu_\ell k_\ell}{\mu_s \alpha_s} \tan k_\ell H_\ell - \frac{\mu_t k_t}{\mu_\ell k_\ell} \varphi_t \tan k_t H_t \left(\tan k_\ell H_\ell + \frac{\mu_\ell k_\ell}{\mu_s \alpha_s} \right) = 0.$$

(without foliage)

Conversion of surface waves in a forest of trees, a homogenization approach

Results in the harmonic regime



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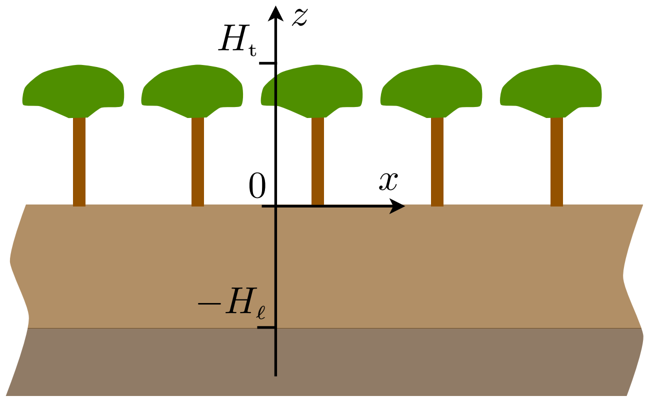
Conversion of surface waves in a forest of trees, a homogenization approach

Results in the harmonic regime



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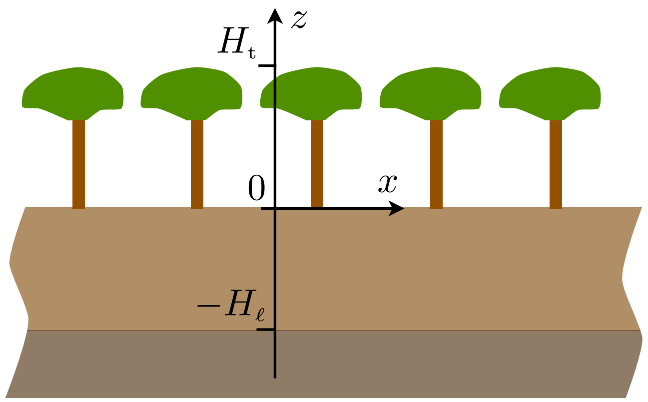
Conversion of surface waves in a forest of trees, a homogenization approach

Results in the harmonic regime



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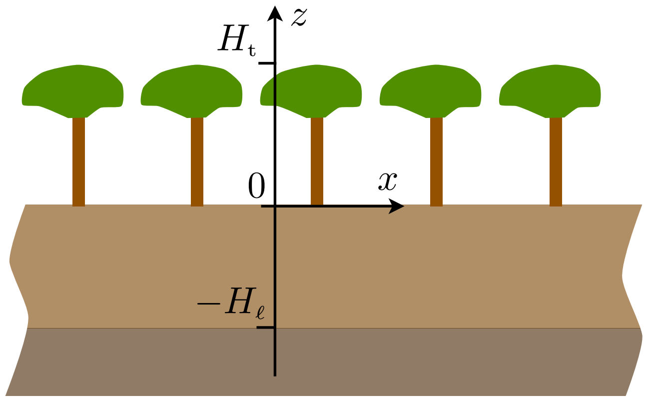
Conversion of surface waves in a forest of trees, a homogenization approach

Results in the harmonic regime



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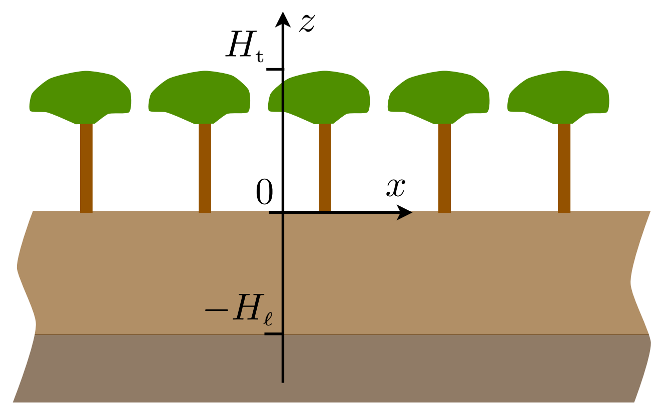
Conversion of surface waves in a forest of trees, a homogenization approach

Results in the harmonic regime



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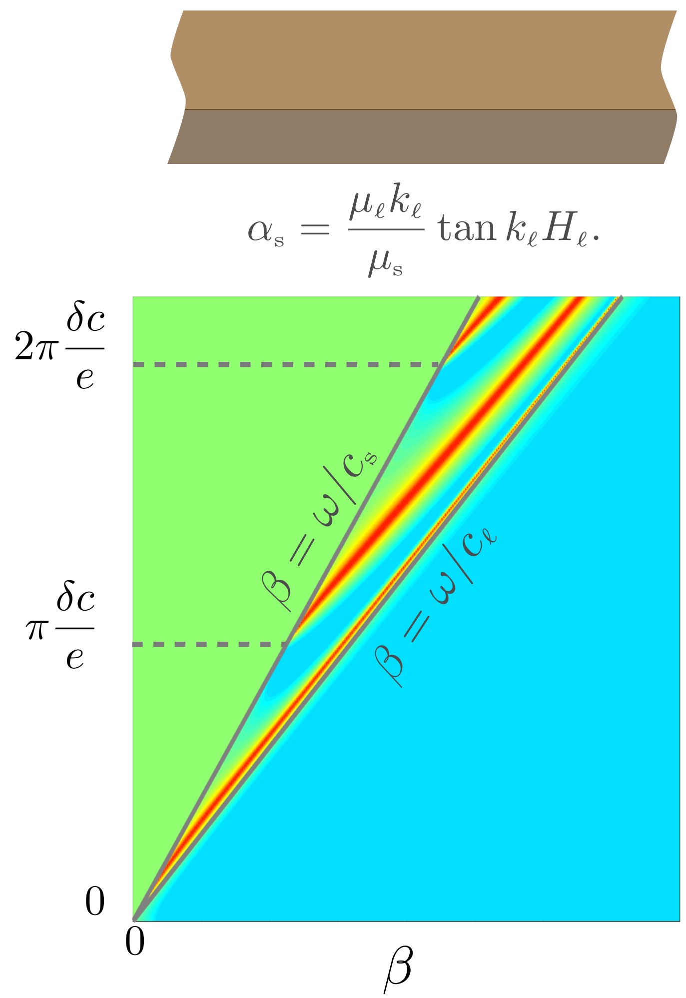
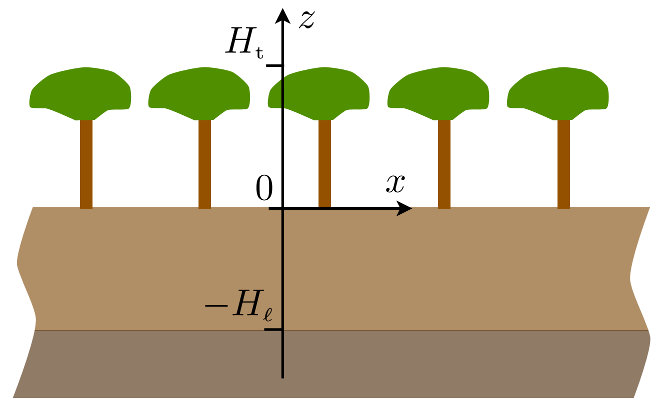
$H_t = 0$

Love waves



Conversion of surface waves in a forest of trees, a homogenization approach

Results in the harmonic regime



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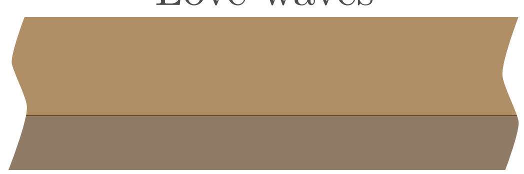
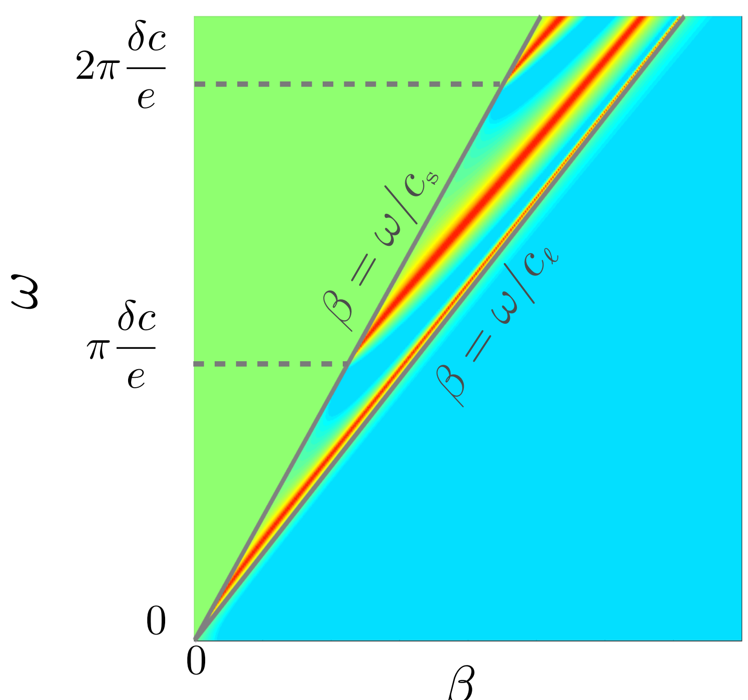
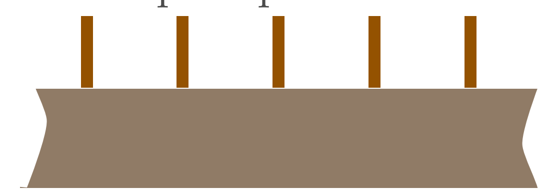


$$\alpha_s = \frac{\mu_\ell k_\ell}{\mu_s} \tan k_\ell H_\ell.$$

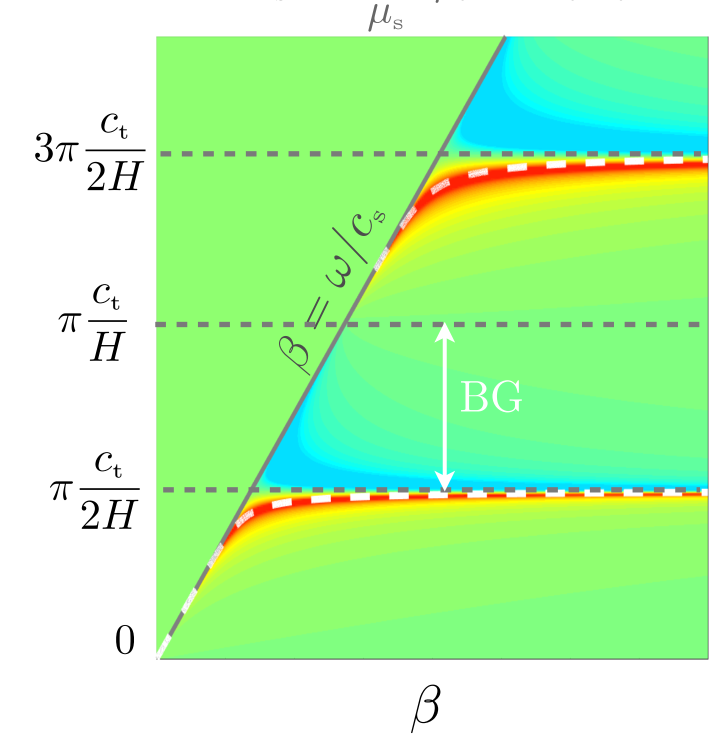


$H_\ell = 0$

Spoof plasmons

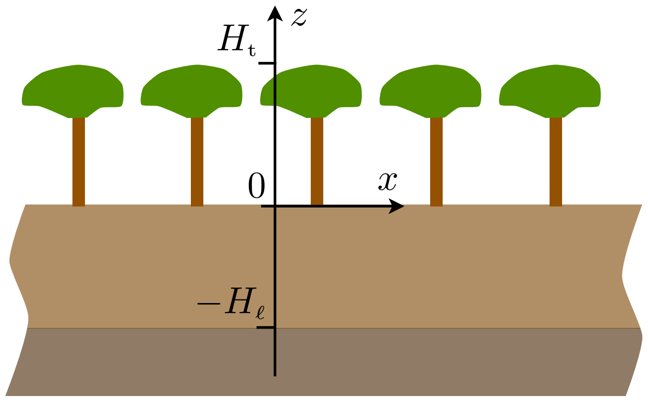


$$\alpha_s = \frac{\mu_t k_t}{\mu_s} \varphi_t \tan k_t H_t.$$



Conversion of surface waves in a forest of trees, a homogenization approach

Results in the harmonic regime



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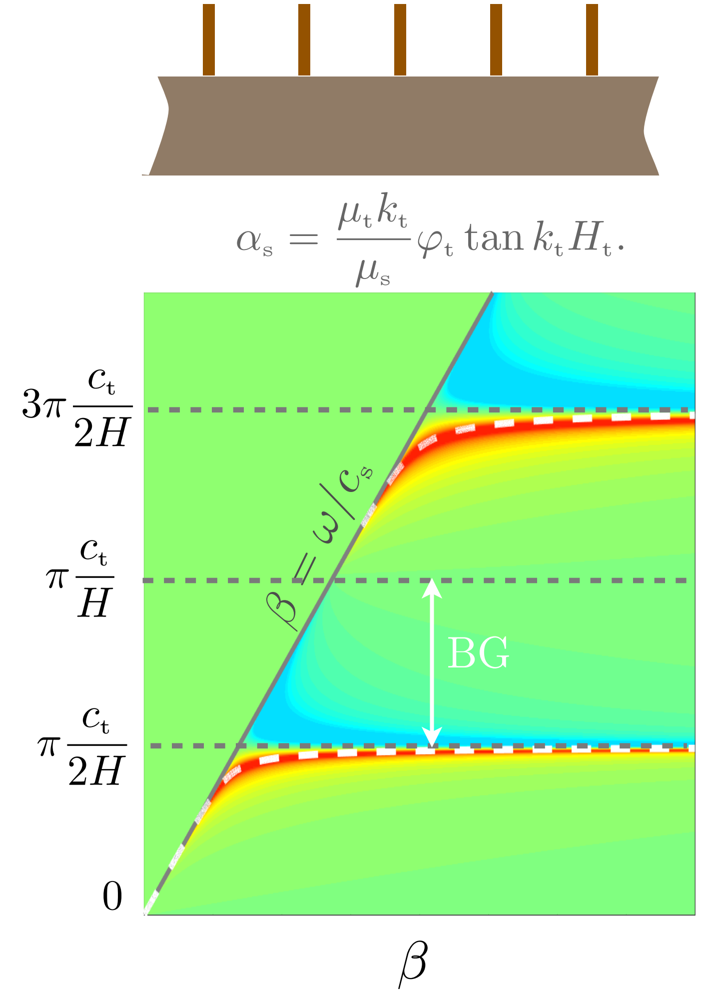
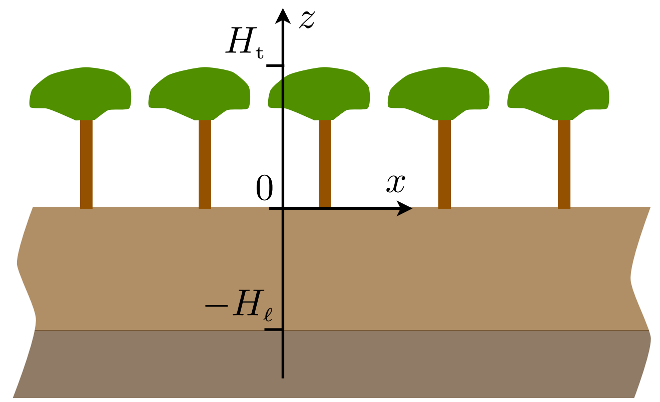


Conversion of surface waves in a forest of trees, a homogenization approach

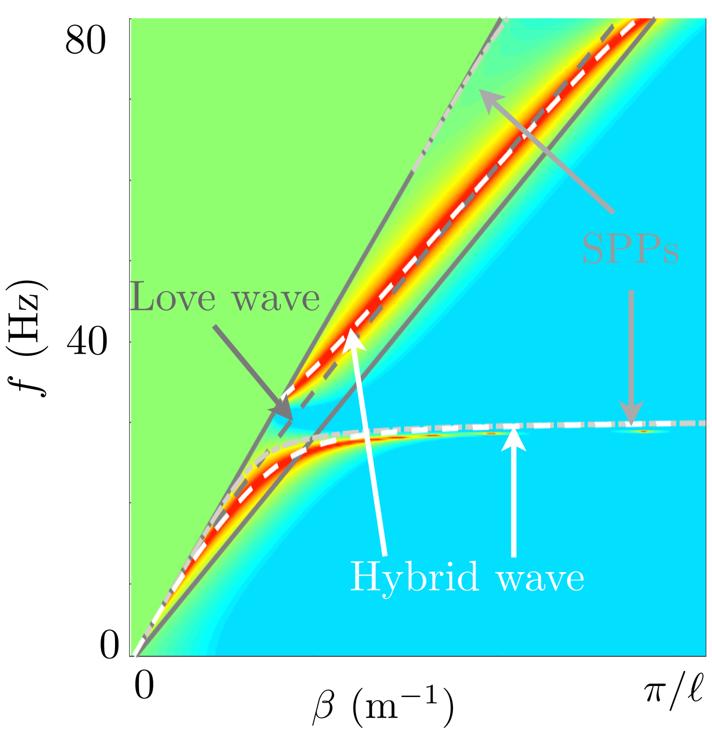
Results in the harmonic regime



$$\alpha_s = \sqrt{\beta^2 - \frac{\omega^2}{c_s^2}}$$
$$k_\ell = \sqrt{\frac{\omega^2}{c_\ell^2} - \beta^2}$$
$$k_t = \frac{\omega}{c_t}$$

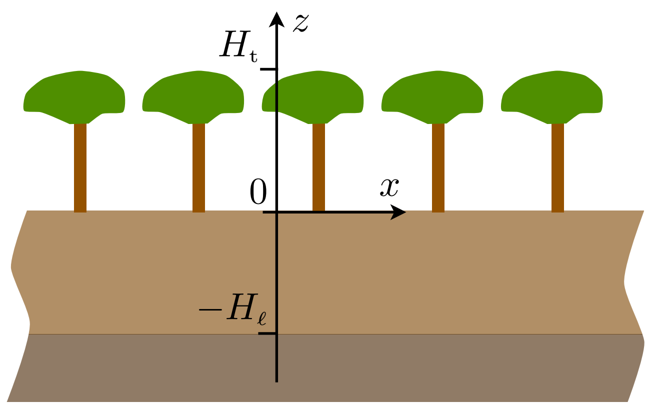
Dispersion relation: $$1 - \frac{\mu_\ell k_\ell}{\mu_s \alpha_s} \tan k_\ell H_\ell - \frac{\mu_t k_t}{\mu_\ell k_\ell} \varphi_t \tan k_t H_t \left(\tan k_\ell H_\ell + \frac{\mu_\ell k_\ell}{\mu_s \alpha_s} \right) = 0.$$

without foliage



Conversion of surface waves in a forest of trees, a homogenization approach

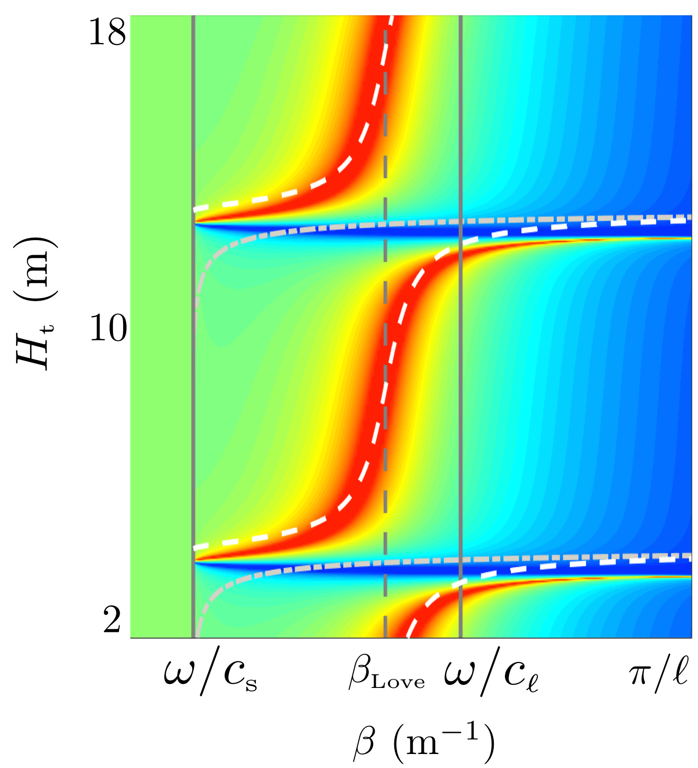
Results in the harmonic regime



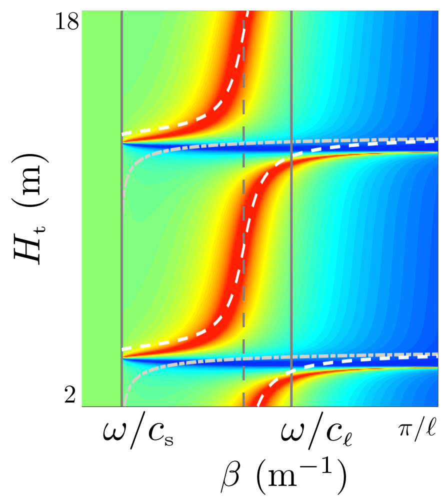
$$\alpha_s = \sqrt{\beta^2 - \frac{\omega^2}{c_s^2}}$$
$$k_\ell = \sqrt{\frac{\omega^2}{c_\ell^2} - \beta^2}$$
$$k_t = \frac{\omega}{c_t}$$

Dispersion relation: $$1 - \frac{\mu_\ell k_\ell}{\mu_s \alpha_s} \tan k_\ell H_\ell - \frac{\mu_t k_t}{\mu_\ell k_\ell} \varphi_t \tan k_t H_t \left(\tan k_\ell H_\ell + \frac{\mu_\ell k_\ell}{\mu_s \alpha_s} \right) = 0.$$

without foliage

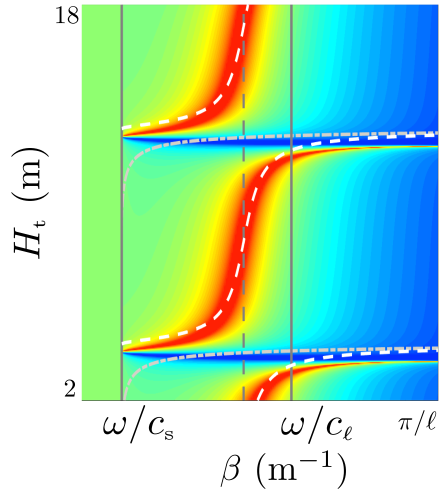


Conversion of surface waves in a forest of trees, a homogenization approach

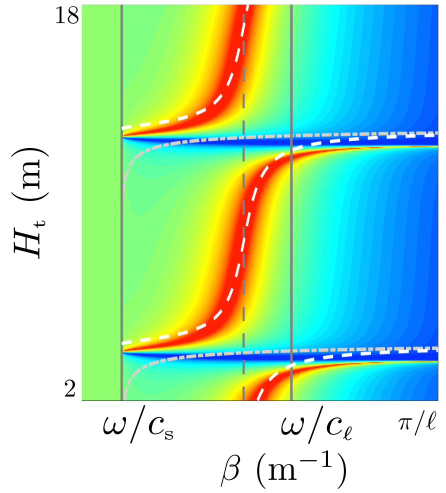


Conversion of surface waves in a forest of trees, a homogenization approach

Forest of trees with a gradient of heights

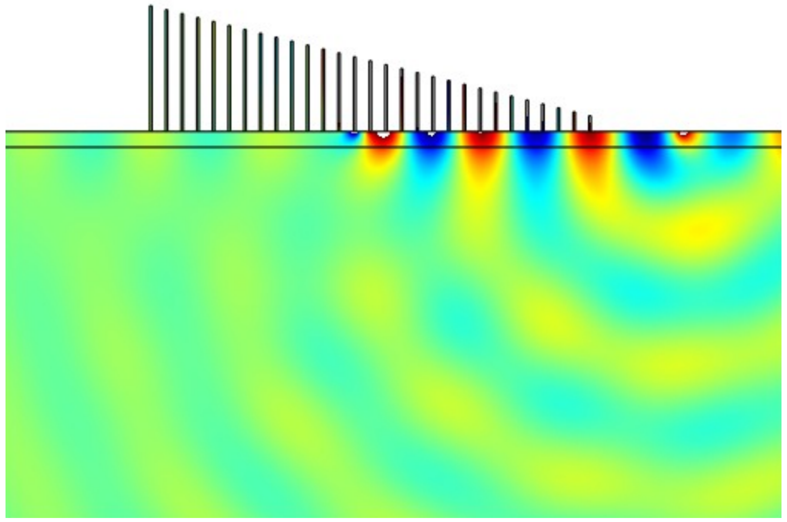


Conversion of surface waves in a forest of trees, a homogenization approach



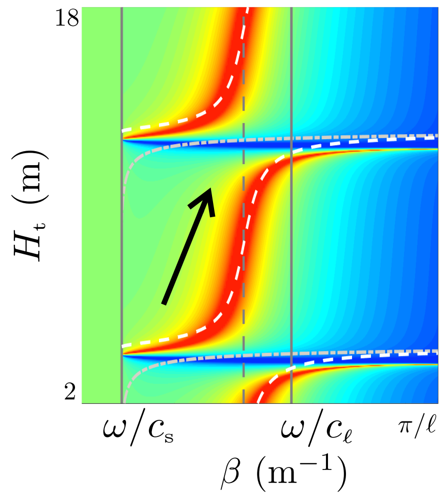
Forest of trees with a gradient of heights

without foliage



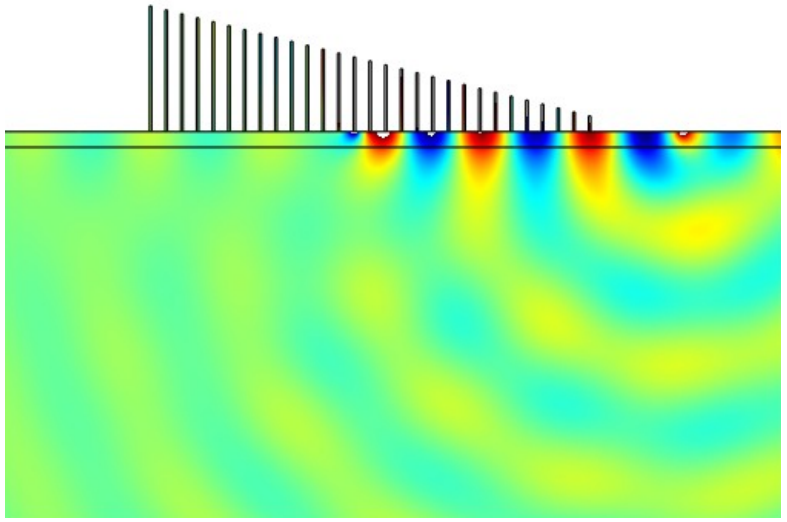
Comsol multiphysics (S. Guenneau)

Conversion of surface waves in a forest of trees, a homogenization approach



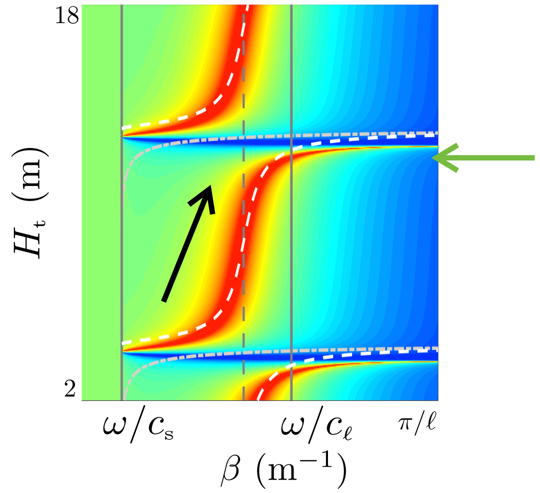
Forest of trees with a gradient of heights

without foliage



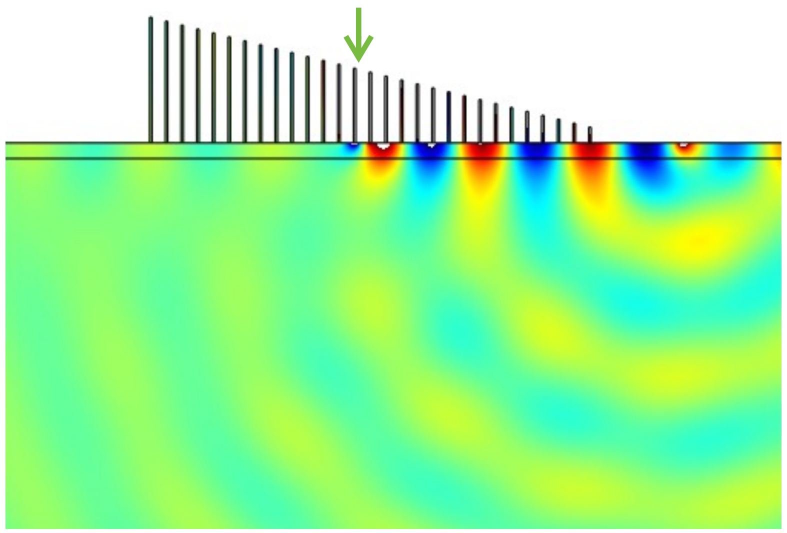
Comsol multiphysics (S. Guenneau)

Conversion of surface waves in a forest of trees, a homogenization approach



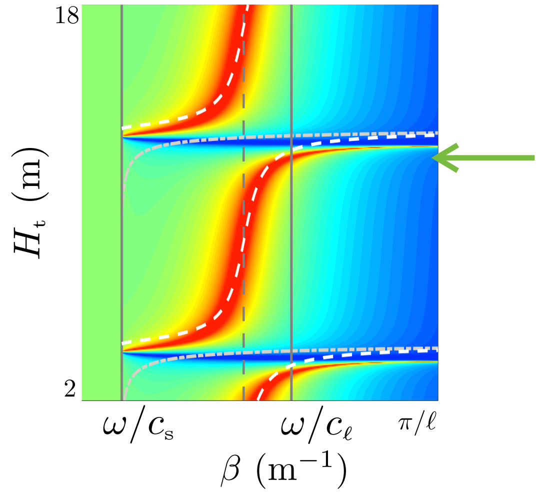
Forest of trees with a gradient of heights

without foliage



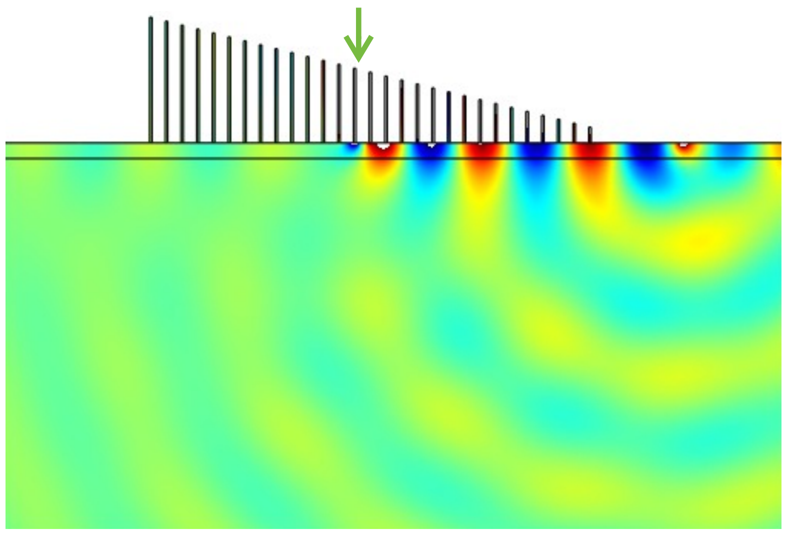
Comsol multiphysics (S. Guenneau)

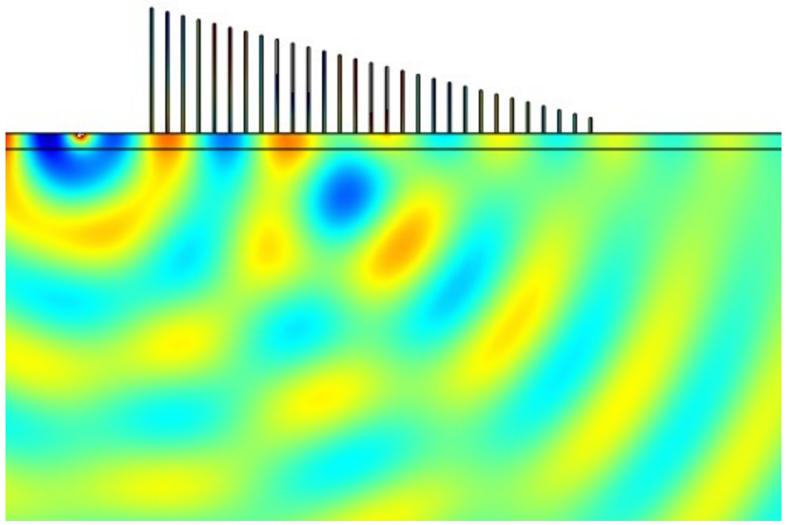
Conversion of surface waves in a forest of trees, a homogenization approach



Forest of trees with a gradient of heights

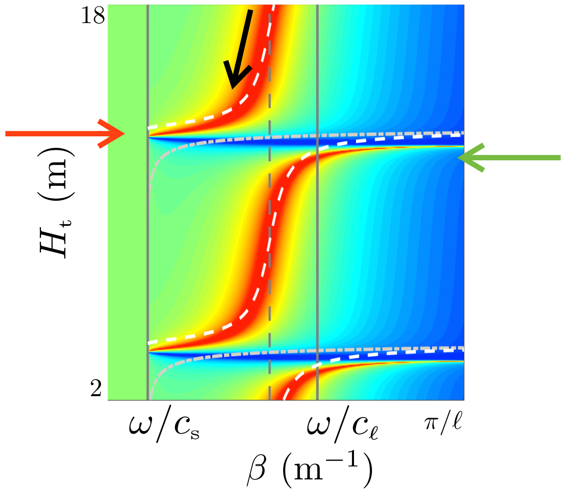
without foliage





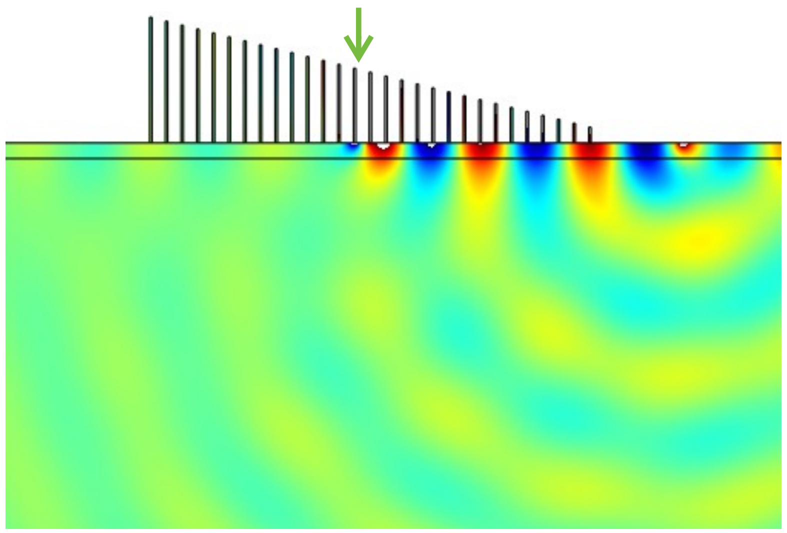
Comsol multiphysics (S. Guenneau)

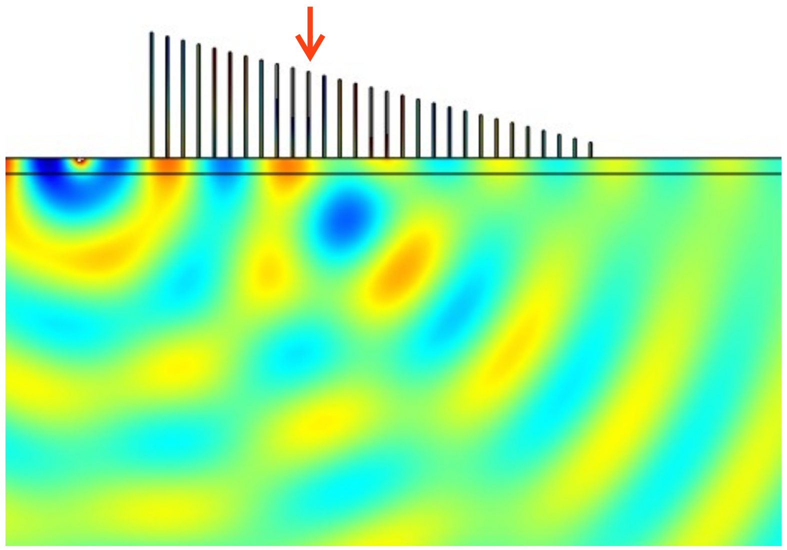
Conversion of surface waves in a forest of trees, a homogenization approach



Forest of trees with a gradient of heights

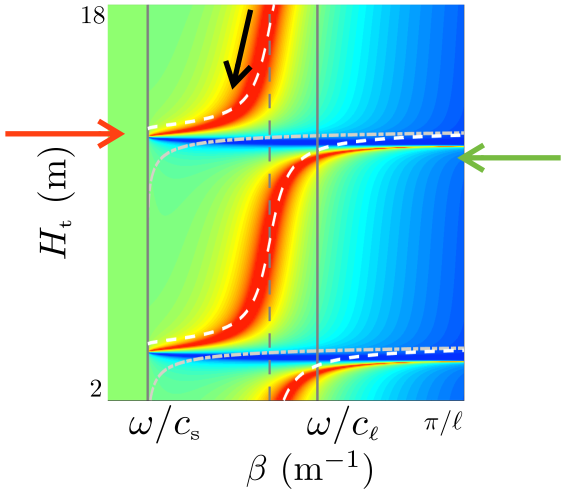
without foliage





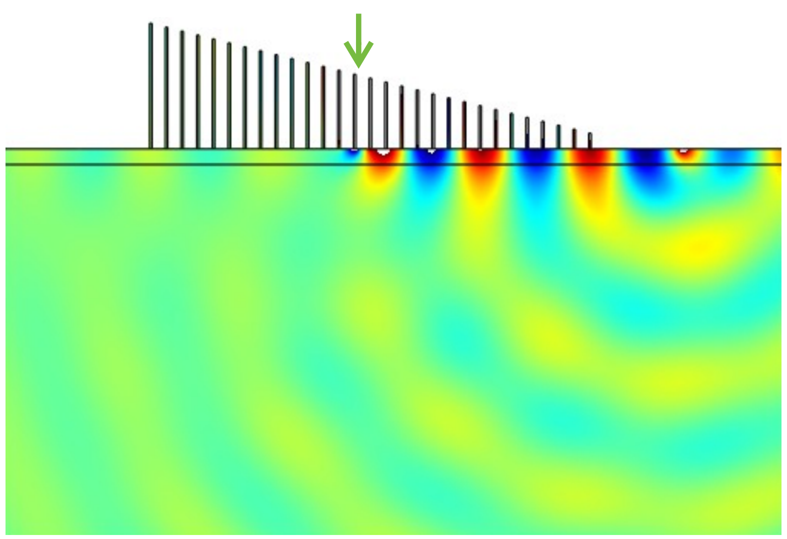
Comsol multiphysics (S. Guenneau)

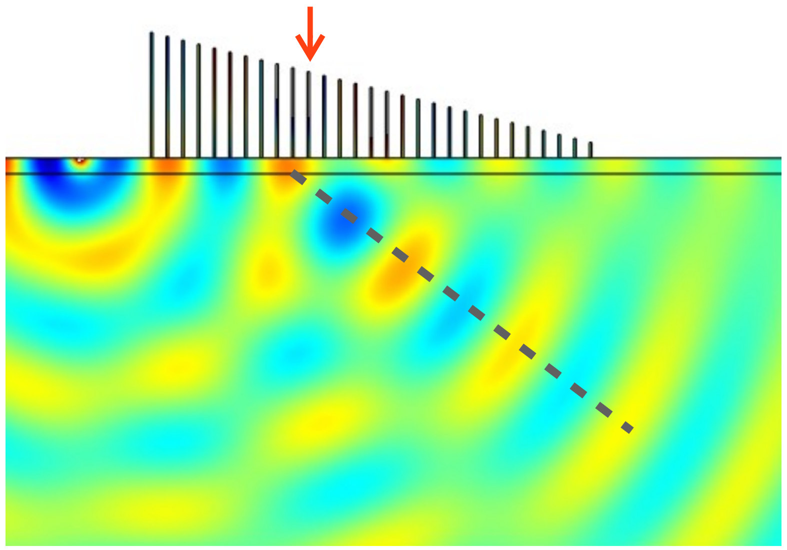
Conversion of surface waves in a forest of trees, a homogenization approach



Forest of trees with a gradient of heights

without foliage





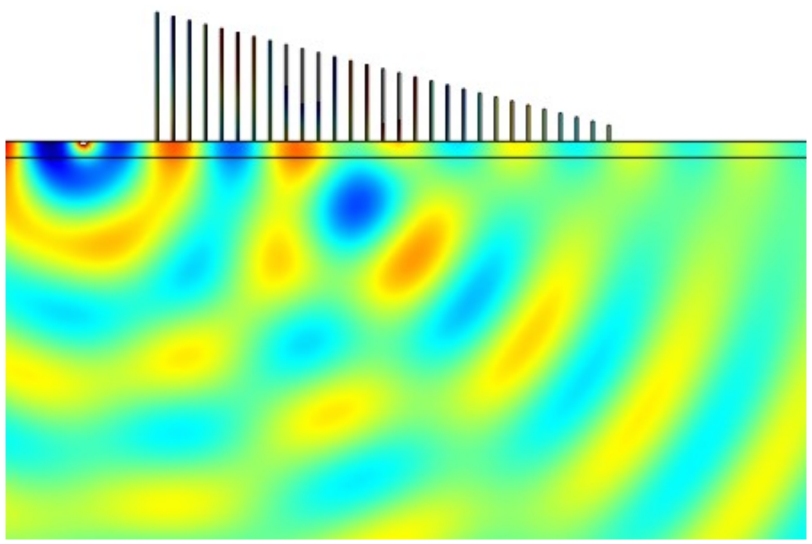
Comsol multiphysics (S. Guenneau)

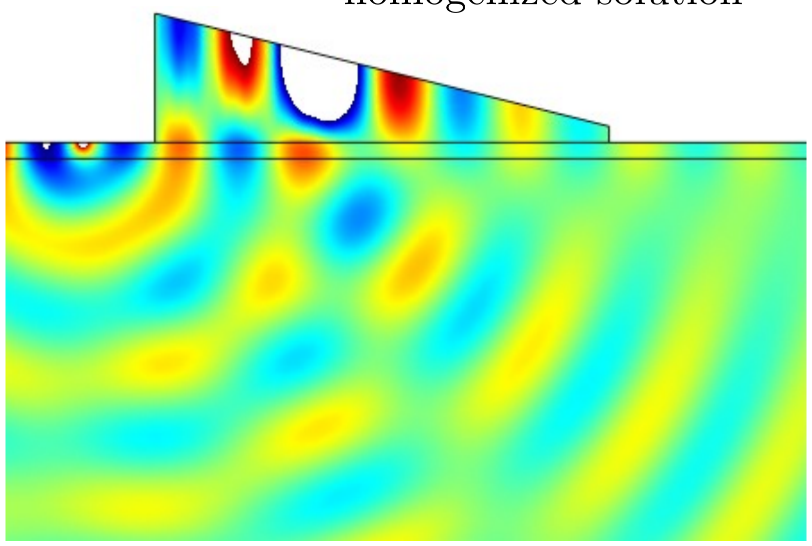
Conversion of surface waves in a forest of trees, a homogenization approach

Forest of trees with a gradient of heights

Conversion of surface waves in a forest of trees, a homogenization approach

Forest of trees with a gradient of heights

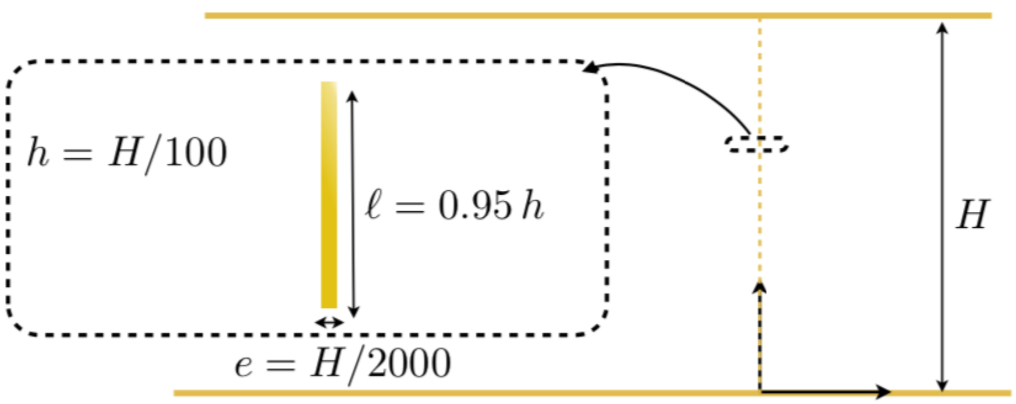




Comsol multiphysics (S. Guenneau)

Asymptotic homogenization

1) For which kind of structures ?



$kH = 100, kh = 1, ke = 0.05$

$H/e = 2000$