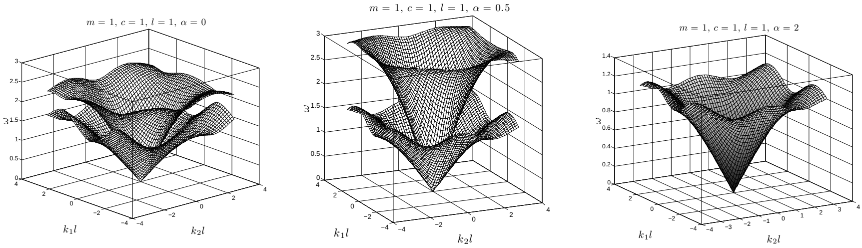
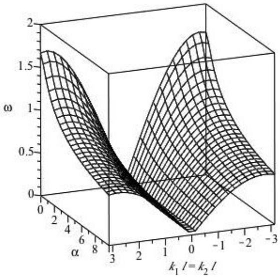
Dispersion surfaces



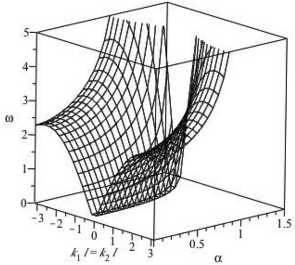
$\alpha = 0.$ $\alpha = 0.5$ $\alpha = 2.0$

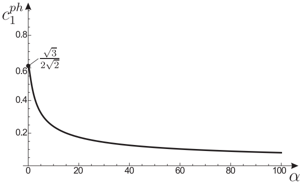
Influence of the Spinners

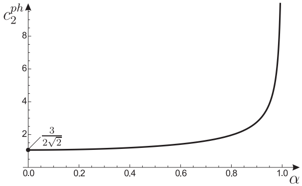
Lower dispersion surface



Upper dispersion surface



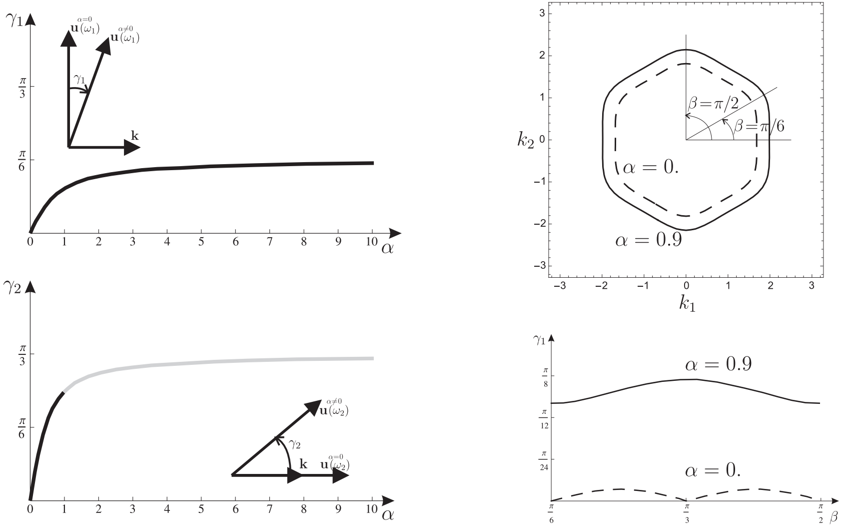




$$c_1^{ph} = \sqrt{\frac{3\left(2-\sqrt{1+3\,(\alpha/m)^2}\right)}{8(1-(\alpha/m)^2)}}$$

$$c_2^{ph} = \sqrt{\frac{3\left(2+\sqrt{1+3\,(\alpha/m)^2}\right)}{8(1-(\alpha/m)^2)}}$$

Polarisation

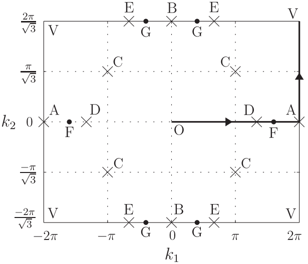


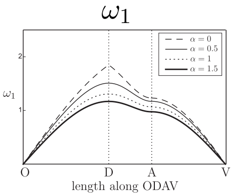
$$\gamma_1 = \frac{\pi}{2} - \arccos\left|\frac{k_1+\Psi(\omega_1)k_2}{\sqrt{1+\Psi(\omega_1)\bar{\Psi}(\omega_1)}}\right|, \qquad \gamma_2 = \arccos\left|\frac{k_1+\Psi(\omega_2)k_2}{\sqrt{1+\Psi(\omega_2)\bar{\Psi}(\omega_2)}}\right|, \qquad (\Psi = u_2/u_1)$$

Polarisation

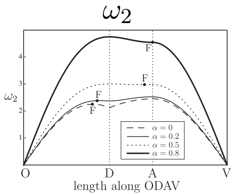
Eigenmodes at a saddle-point

Dispersion surfaces: stationary points





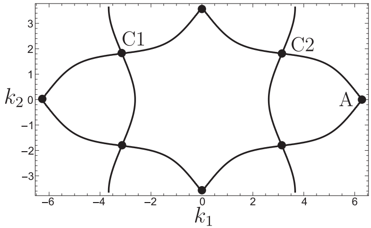
point	type
A, B, C	saddle points
D, E	maxima



point	type	
A, B, C	$\alpha < \sqrt{7/27}$ maxima	$\alpha > \sqrt{7/27}$ saddle points
D, E	$\alpha < 1/3$ minima	$\alpha > 1/3$ maxima
F, G	$\alpha < \sqrt{7/27}$ saddle points	$\alpha > \sqrt{7/27}$ $\equiv$ A, B

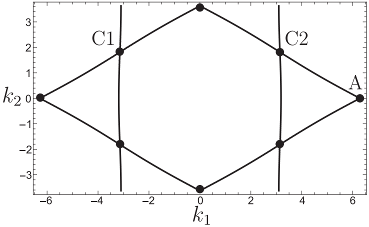
Tunable saddle-points

Lower dispersion surface $\omega_1 = \sqrt{6/(2+\sqrt{1+3\alpha^2})}$ at $\alpha = 0.9$



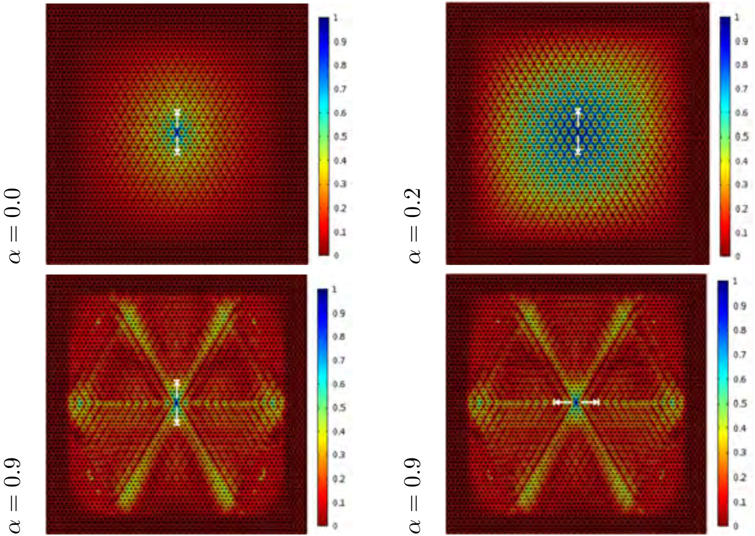
Tunable saddle-points

Upper dispersion surface $\omega_2 = \sqrt{6/(2-\sqrt{1+3\alpha^2})}$ at $\alpha = 0.9$



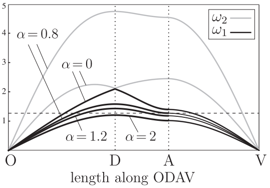
Stationary-points

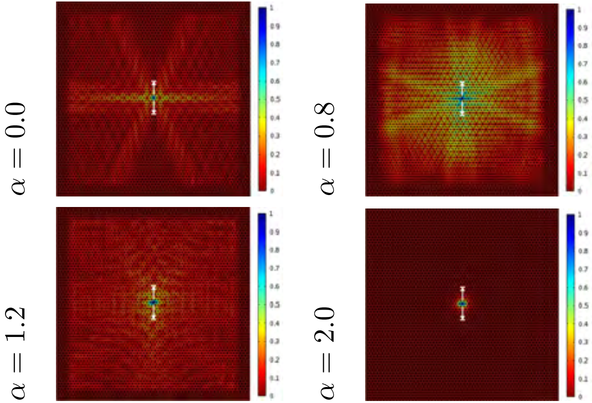
Upper dispersion surface $\omega_2 = \sqrt{6/(2-\sqrt{1+3\alpha^2})}$



Effect of the spinner constant

Propagation at $\omega = 1.27$





Homogenised continuum model

Long wave approximation for the monatomic triangular lattice.
Equations of motion for time harmonic amplitude $\mathbf{U}$

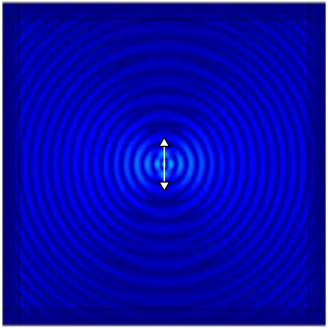
$$\mu\Big(\Delta\mathbf{U} + 2\nabla\nabla\cdot\mathbf{U}\Big) + \omega^2\boldsymbol{\Sigma}\mathbf{U} + \rho\omega^2\mathbf{U} + \mathbf{F} = 0$$

where $\lambda = \mu$, $\boldsymbol{\Sigma} = \begin{pmatrix} 0 & -i\alpha \\ i\alpha & 0 \end{pmatrix}$.

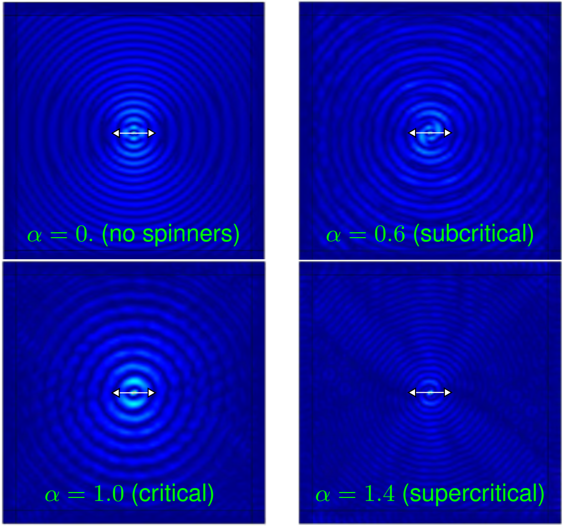
In the computation $\rho = 1.0$, $\mu = 1.0$,

- $\alpha > 1$ supercritical
- $\alpha < 1$ subcritical

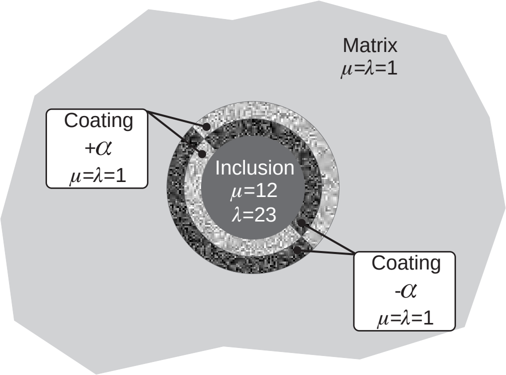
Continuum VS Discrete



Green's function

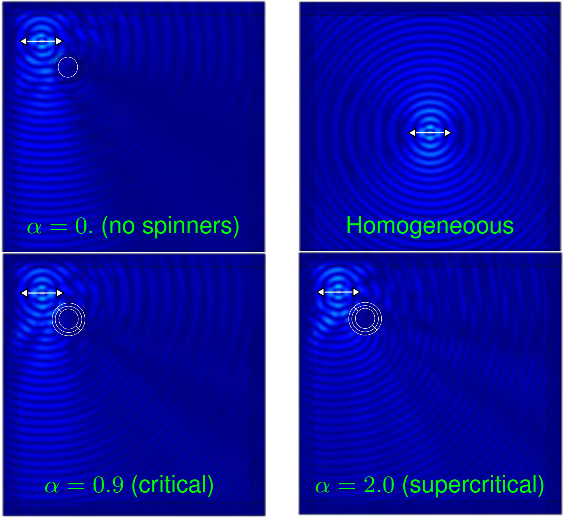


Elastic Inclusion with Chiral Coating

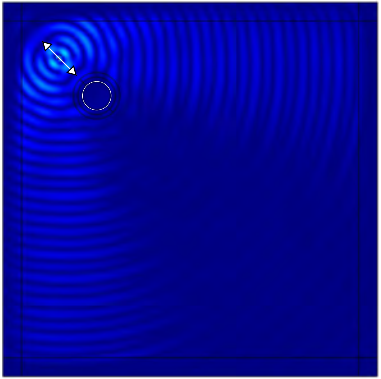


The coating has the same elastic properties as the ambient medium

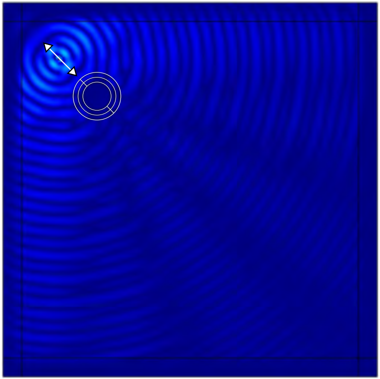
Inclusion placed in a region dominated by shear waves



A point force in the direction of the inclusion

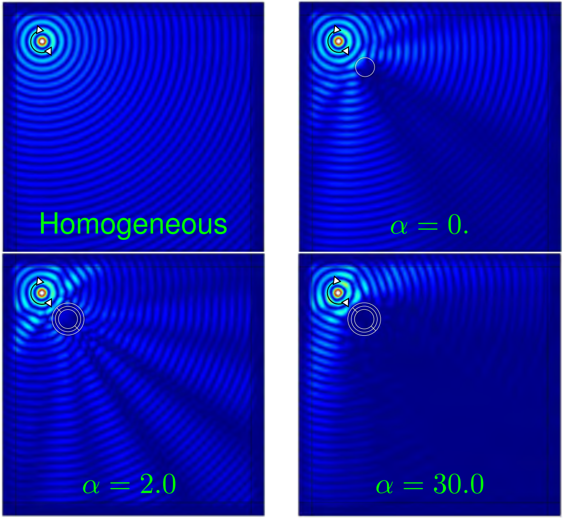


$\alpha = 0$

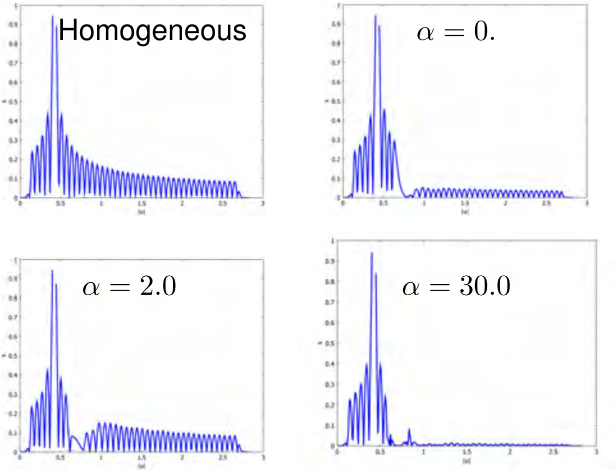


$\alpha = 1.5$

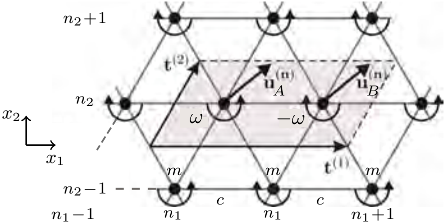
Shear waves



Shear waves



Triangular gyro-elastic lattice: Highly localised waveforms



In the lattice, the gyricity is distributed so that

$$|\Omega| = \omega$$

Governing equations:

$$\begin{aligned}\ddot{\mathbf{u}}_A^{(n_1,n_2)} = & \Big[\mathbf{a}^{(1)}\cdot\Big(\mathbf{u}_B^{(n_1,n_2)} + \mathbf{u}_B^{(n_1-1,n_2)} - 2\mathbf{u}_A^{(n_1,n_2)}\Big)\mathbf{a}^{(1)} \\ & + \mathbf{a}^{(2)}\cdot\Big(\mathbf{u}_B^{(n_1-1,n_2+1)} + \mathbf{u}_B^{(n_1,n_2-1)} - 2\mathbf{u}_A^{(n_1,n_2)}\Big)\mathbf{a}^{(2)} \\ & + \mathbf{a}^{(3)}\cdot\Big(\mathbf{u}_A^{(n_1,n_2+1)} + \mathbf{u}_A^{(n_1,n_2-1)} - 2\mathbf{u}_A^{(n_1,n_2)}\Big)\mathbf{a}^{(3)}\Big] - \alpha_1\omega\mathbf{R}\dot{\mathbf{u}}_A ,\end{aligned}$$

$$\begin{aligned}\ddot{\mathbf{u}}_B^{(n_1,n_2)} = & \Big[\mathbf{a}^{(1)}\cdot\Big(\mathbf{u}_A^{(n_1+1,n_2)} + \mathbf{u}_A^{(n_1,n_2)} - 2\mathbf{u}_B^{(n_1,n_2)}\Big)\mathbf{a}^{(1)} \\ & + \mathbf{a}^{(2)}\cdot\Big(\mathbf{u}_A^{(n_1,n_2+1)} + \mathbf{u}_A^{(n_1+1,n_2-1)} - 2\mathbf{u}_B^{(n_1,n_2)}\Big)\mathbf{a}^{(2)} \\ & + \mathbf{a}^{(3)}\cdot\Big(\mathbf{u}_B^{(n_1,n_2+1)} + \mathbf{u}_B^{(n_1,n_2-1)} - 2\mathbf{u}_B^{(n_1,n_2)}\Big)\mathbf{a}^{(3)}\Big] + \alpha_2\omega\mathbf{R}\dot{\mathbf{u}}_B ,\end{aligned}$$

Floquet-Bloch waves in a triangular gyro-elastic lattice

To analyse vibrations in the lattice, we introduce the time harmonic solutions:

$$\mathbf{u}_j^{(\mathbf{n})} = \mathbf{U}_j^{(\mathbf{n})} e^{\mathrm{i}\omega t} \ , \quad j = A, B \ .$$

whose amplitude satisfies the Floquet-Bloch conditions

$$\mathbf{U}_j^{(\mathbf{n}+\mathbf{a})} = \mathbf{U}_j^{(\mathbf{n})} e^{\mathrm{i}\mathbf{k}\cdot\mathbf{T}\mathbf{a}} \ , \qquad \mathbf{T} = [\mathbf{t}^{(1)}, \mathbf{t}^{(2)}] = \begin{pmatrix} 2 & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} \end{pmatrix} .$$

We obtain this characteristic equation for the eigenfrequencies:

$$\det[\mathbf{C} - \omega^2(\mathbf{I}_4 - \mathbf{S})] = 0 \ ,$$

where $\mathbf{S}$ represents the contribution of the spinners

$$\mathbf{S} = \mathrm{i}\ \mathrm{diag}\,(\alpha_1\mathbf{R}, -\alpha_2\mathbf{R}) \ .$$

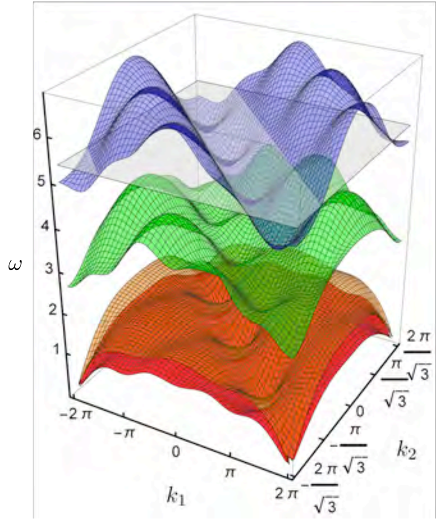
and $\mathbf{C}$ represents the influence of the elastic connections

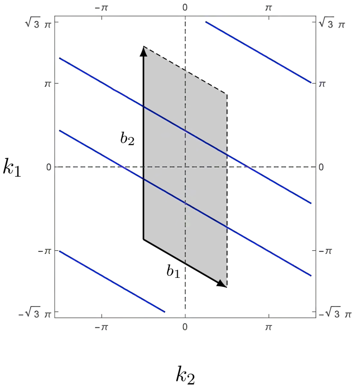
$$\mathbf{C} = \begin{pmatrix} \mathbf{C}^{(1)} & \mathbf{C}^{(2)} \\ \overline{\mathbf{C}^{(2)}} & \mathbf{C}^{(1)} \end{pmatrix} , \qquad \mathbf{C}^{(1)} = \begin{pmatrix} 3 - \frac{1}{2}\cos(\zeta) & -\frac{\sqrt{3}}{2}\cos(\zeta) \\ -\frac{\sqrt{3}}{2}\cos(\zeta) & 3\left(1 - \frac{1}{2}\cos(\zeta)\right) \end{pmatrix}$$

$$\mathbf{C}^{(2)} = \begin{pmatrix} -e^{-\mathrm{i}(\zeta+\xi)}\left(2\cos(\zeta+\xi) + \frac{1}{2}\cos(\xi)\right) & \frac{\sqrt{3}}{2}e^{-\mathrm{i}(\zeta+\xi)}\cos(\xi) \\ \frac{\sqrt{3}}{2}e^{-\mathrm{i}(\zeta+\xi)}\cos(\xi) & -\frac{3}{2}e^{-\mathrm{i}(\zeta+\xi)}\cos(\xi) \end{pmatrix} ,$$

with $\xi = (k_1 - \sqrt{3}k_2)/2$ and $\zeta = (k_1 + \sqrt{3}k_2)/2$.

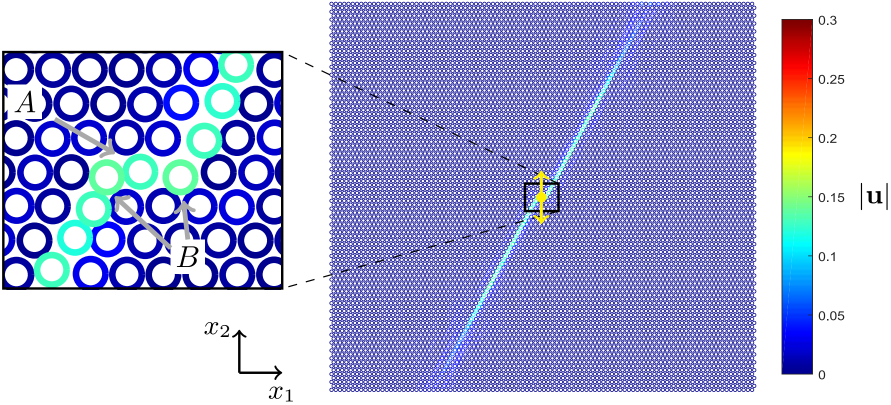
Dispersion surfaces for $\alpha_A = 0.8$ and $\alpha_B = 0.9$



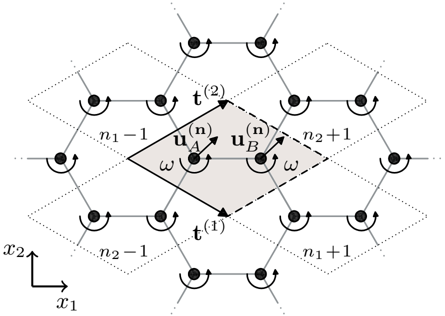


Highly localised waveform for $\alpha_A = 0.8$ and $\alpha_B = 0.9$

Example: Highly-localised waveform for $\alpha_A = 0.8$ and $\alpha_B = 0.9$



Hexagonal gyro-elastic lattice: Interfacial waveforms



Governing equations:

$$\ddot{\mathbf{u}}_A^{(\mathbf{n})} = \sum_{j=1}^{3} \mathbf{a}^{(j)} \cdot \{\mathbf{u}_B^{(\mathbf{n}-\mathbf{p}_j)} - \mathbf{u}_A^{(\mathbf{n})}\}\mathbf{a}^{(j)} - \alpha\omega\mathbf{R}\dot{\mathbf{u}}_A^{(\mathbf{n})}$$

$$\ddot{\mathbf{u}}_B^{(\mathbf{n})} = \sum_{j=1}^{3} \mathbf{a}^{(j)} \cdot \{\mathbf{u}_A^{(\mathbf{n}+\mathbf{p}_j)} - \mathbf{u}_B^{(\mathbf{n})}\}\mathbf{a}^{(j)} - \alpha\omega\mathbf{R}\dot{\mathbf{u}}_B^{(\mathbf{n})}$$

Characteristic equation: $\det[\mathbf{C} - \omega^2(\mathbf{I}_4 - \mathbf{S})] = 0$,

Stiffness matrix: $\mathbf{C} = \begin{pmatrix} \frac{3}{2}\mathbf{I}_2 & C_H \\ \overline{C_H^T} & \frac{3}{2}\mathbf{I}_2 \end{pmatrix}$ $\quad C_H = \begin{pmatrix} -1 - \dfrac{e^{-\mathrm{i}\eta} + e^{-\mathrm{i}\nu}}{4} & \dfrac{\sqrt{3}(e^{-\mathrm{i}\eta} - e^{-\mathrm{i}\nu})}{4} \\ \dfrac{\sqrt{3}(e^{-\mathrm{i}\eta} - e^{-\mathrm{i}\nu})}{4} & -\dfrac{3(e^{-\mathrm{i}\eta} + e^{-\mathrm{i}\nu})}{4} \end{pmatrix}$

with $\eta = (3k_1 - \sqrt{3}k_2)/2$ and $\nu = (3k_1 + \sqrt{3}k_2)/2$, and the gyroscopic matrix

$$\mathbf{S} = \mathrm{i}\alpha \; \mathrm{diag}(\mathbf{R}, \mathbf{R}) \; .$$