Asymptotic Analysis of Arrays of Beams on the Top of an Elastic Half-Space

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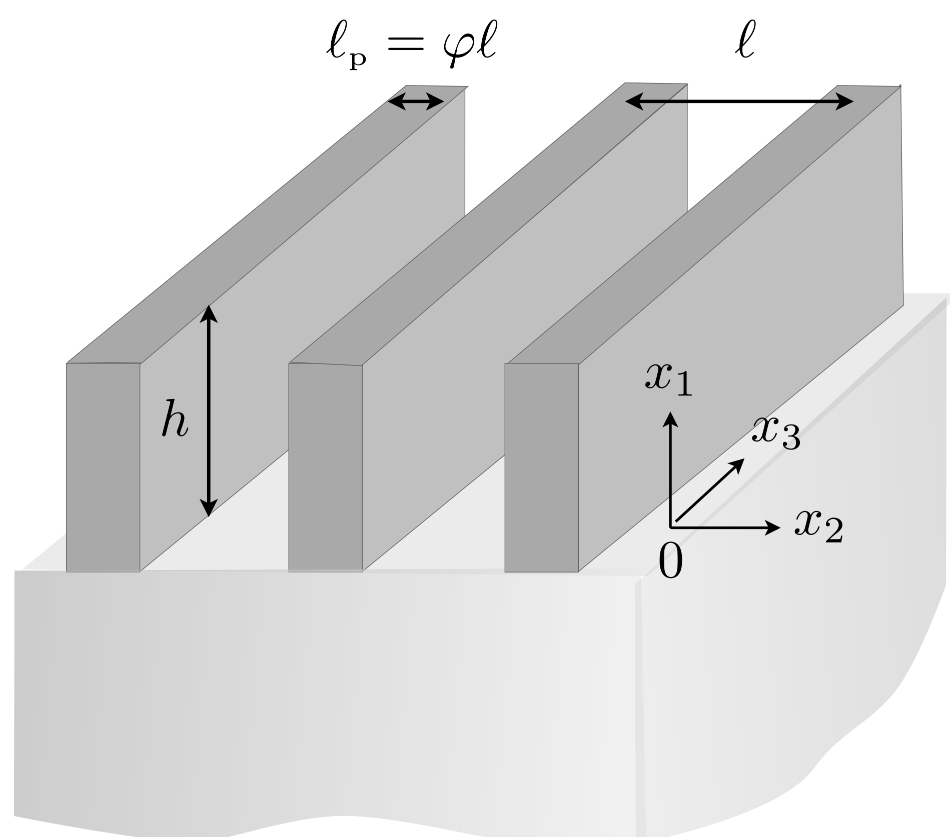

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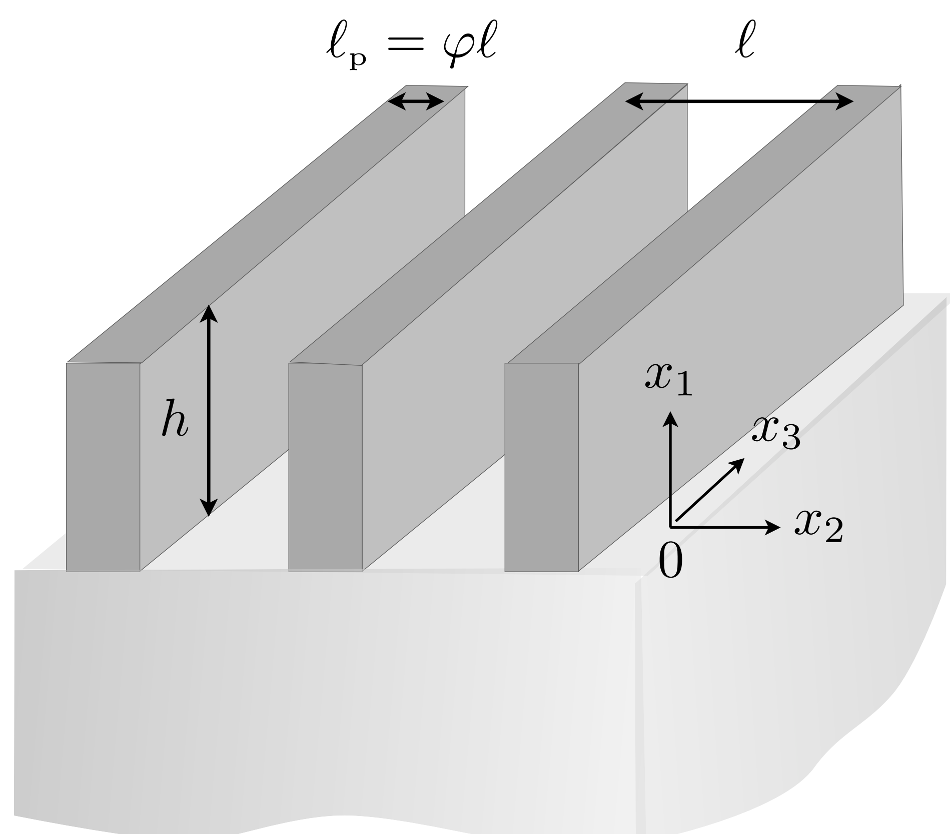

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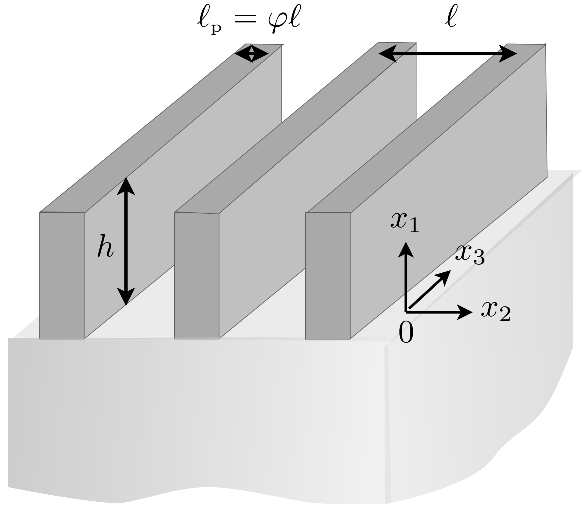

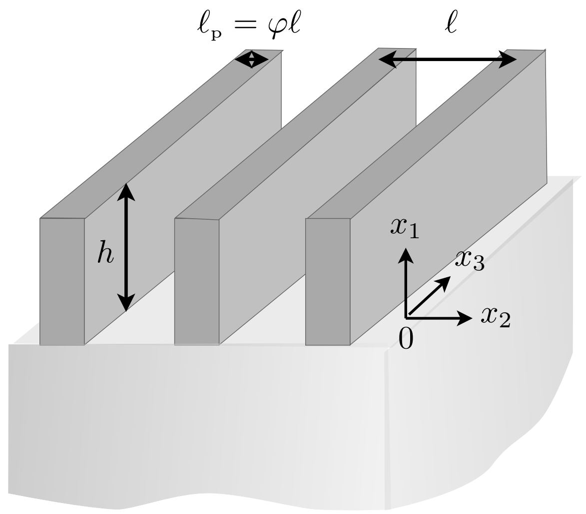
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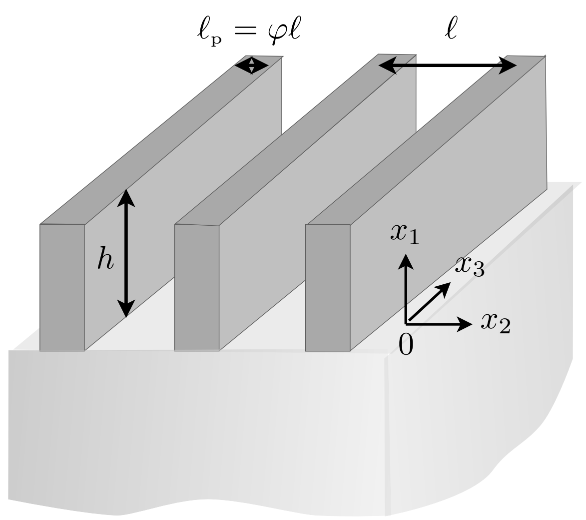

in the substrate, $x_1 \in (-\infty, 0)$:

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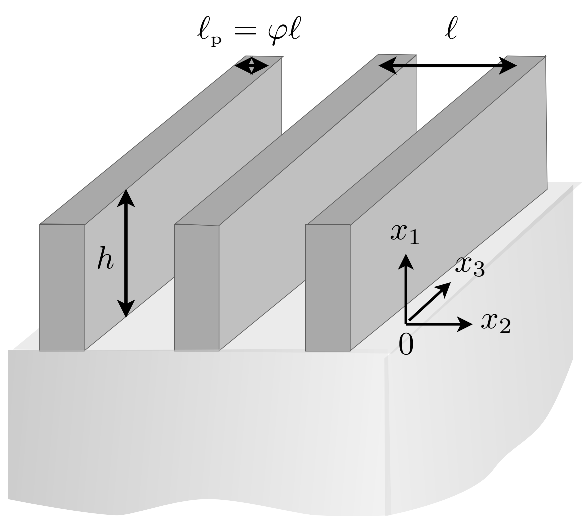

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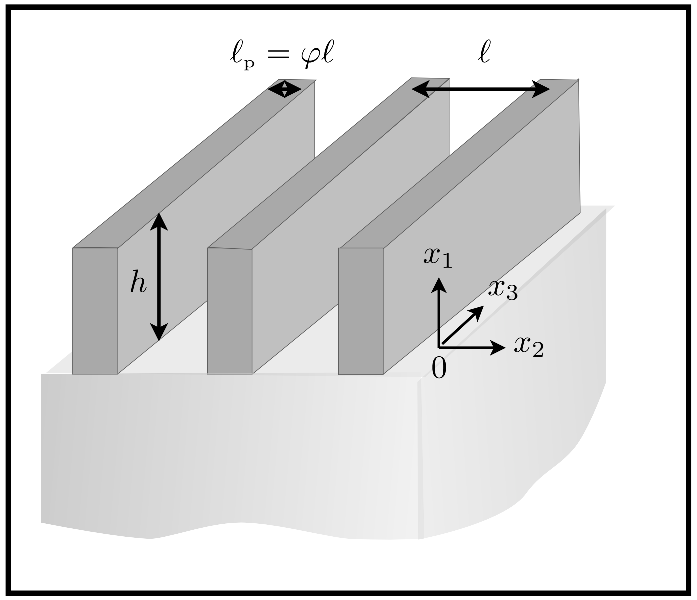

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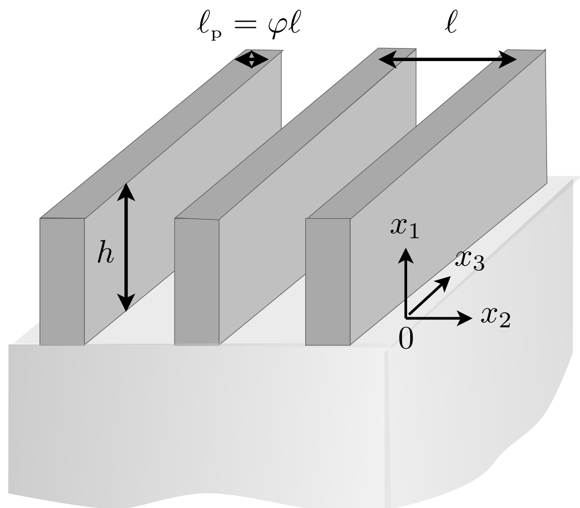

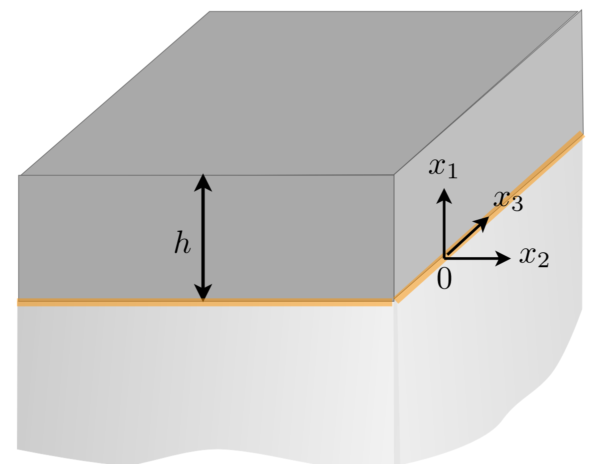

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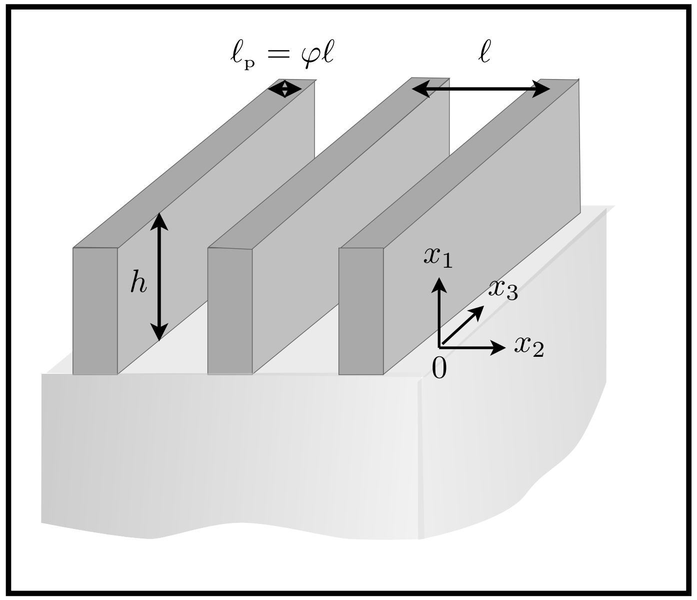

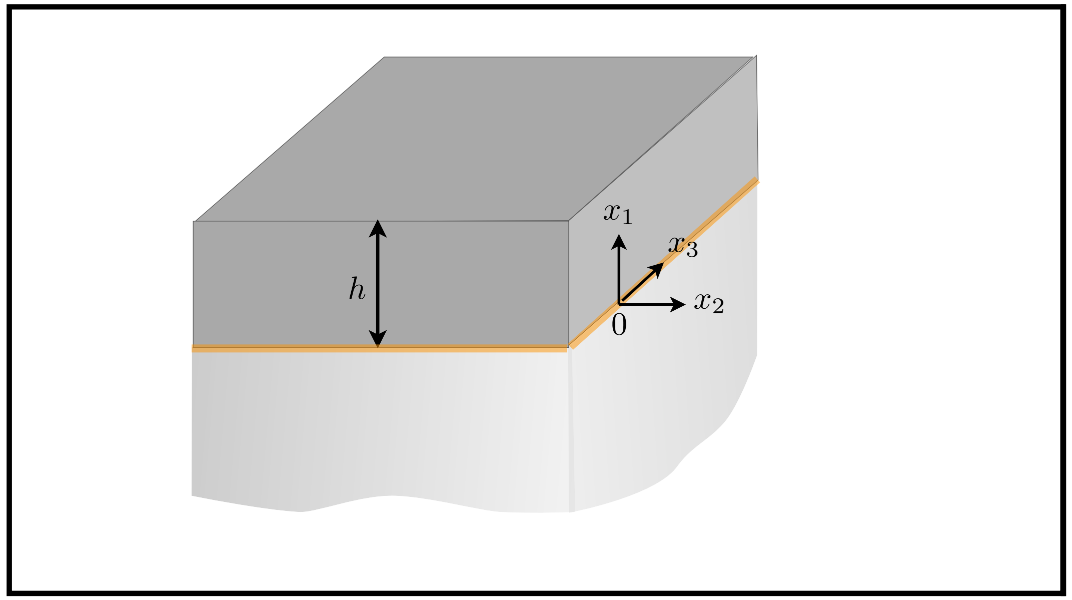

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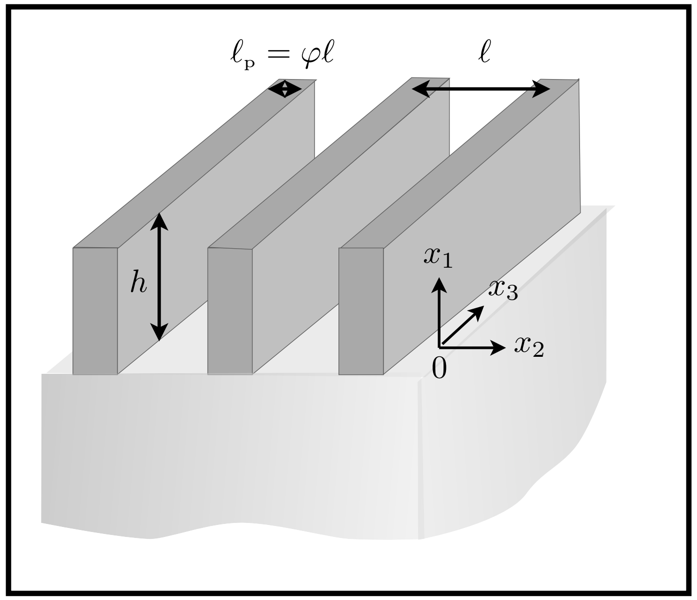

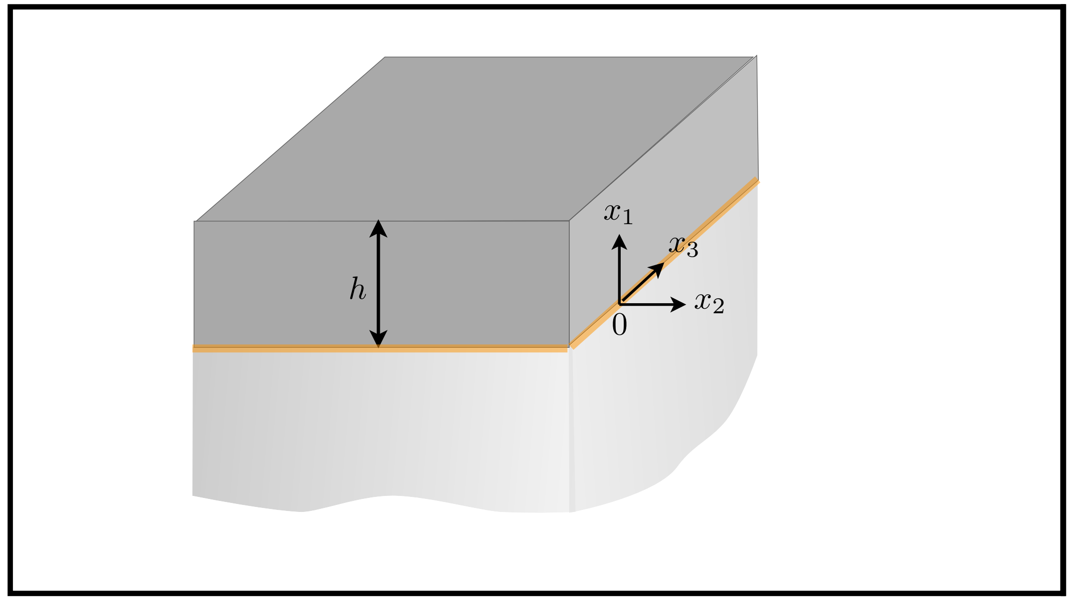

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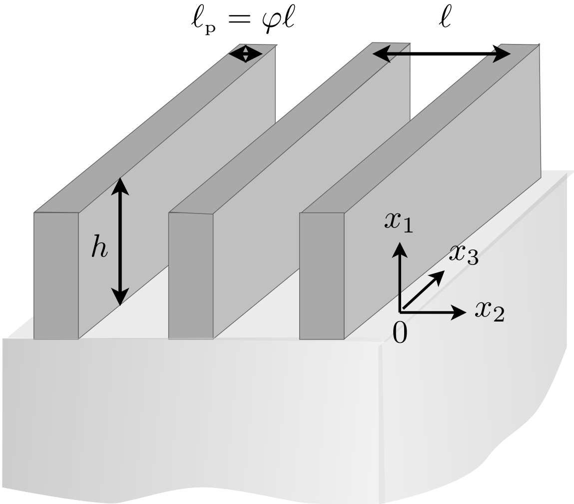

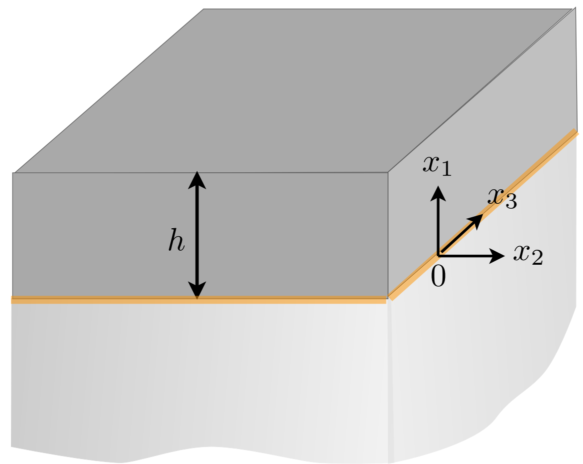

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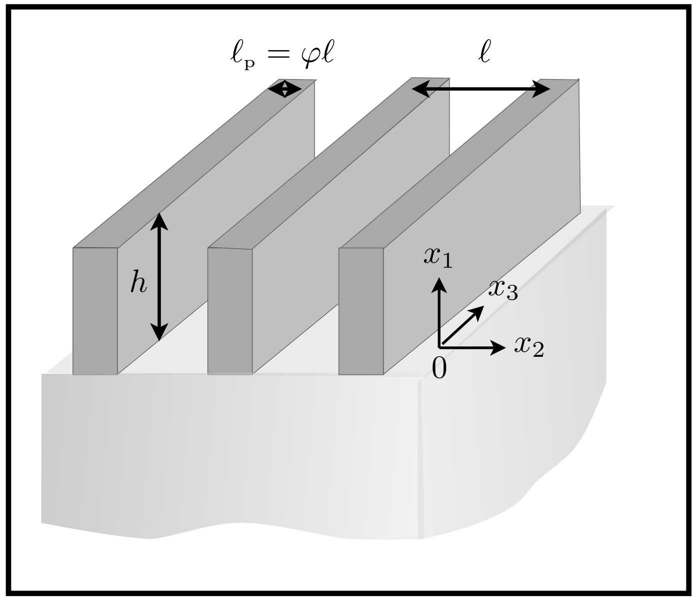

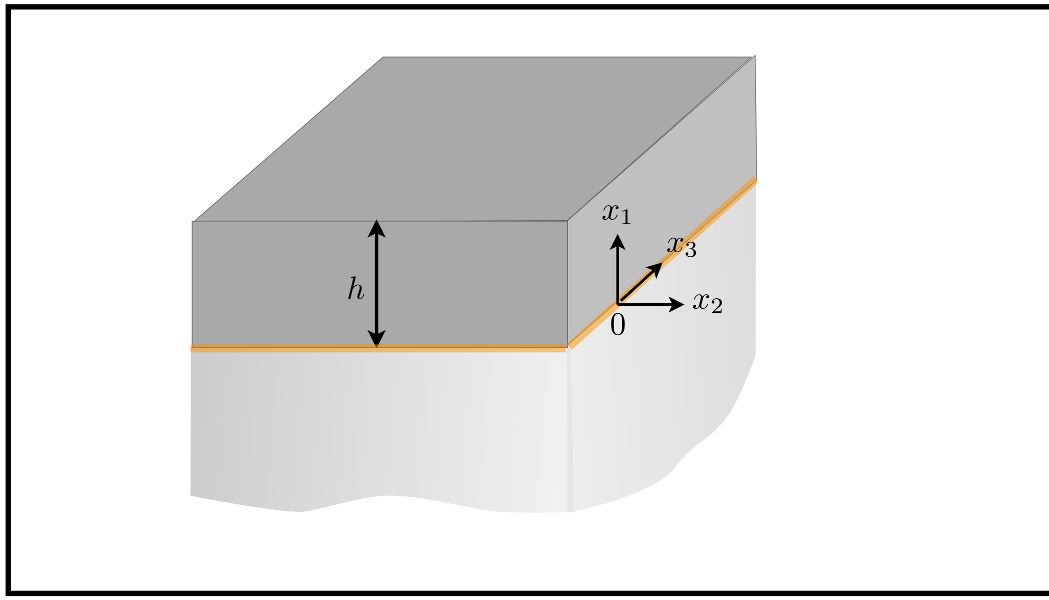

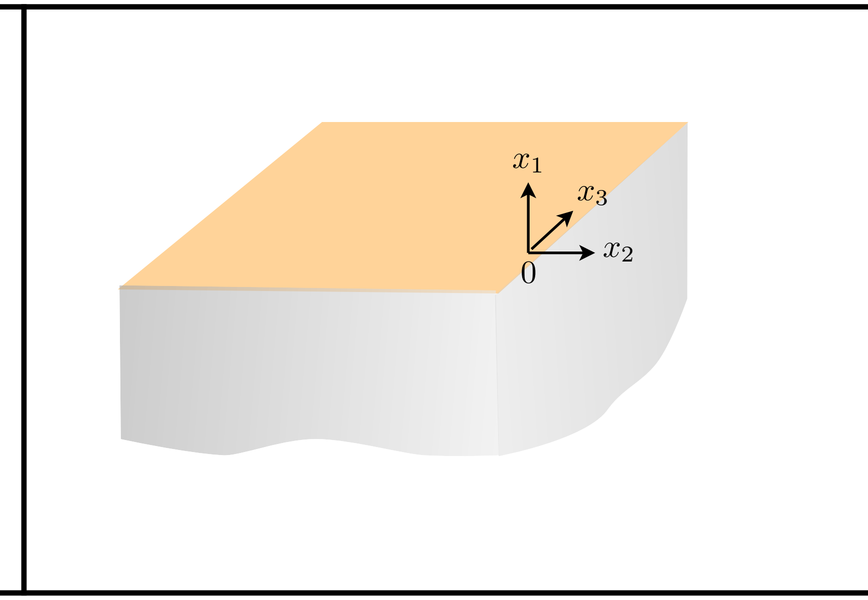

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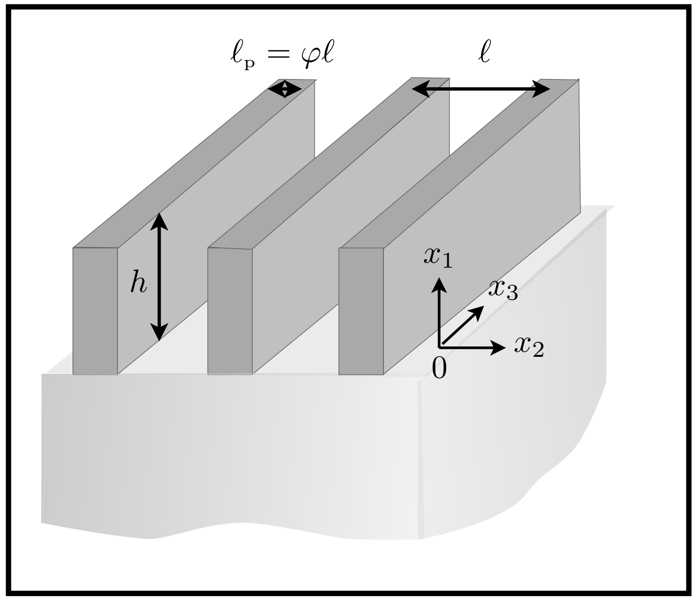

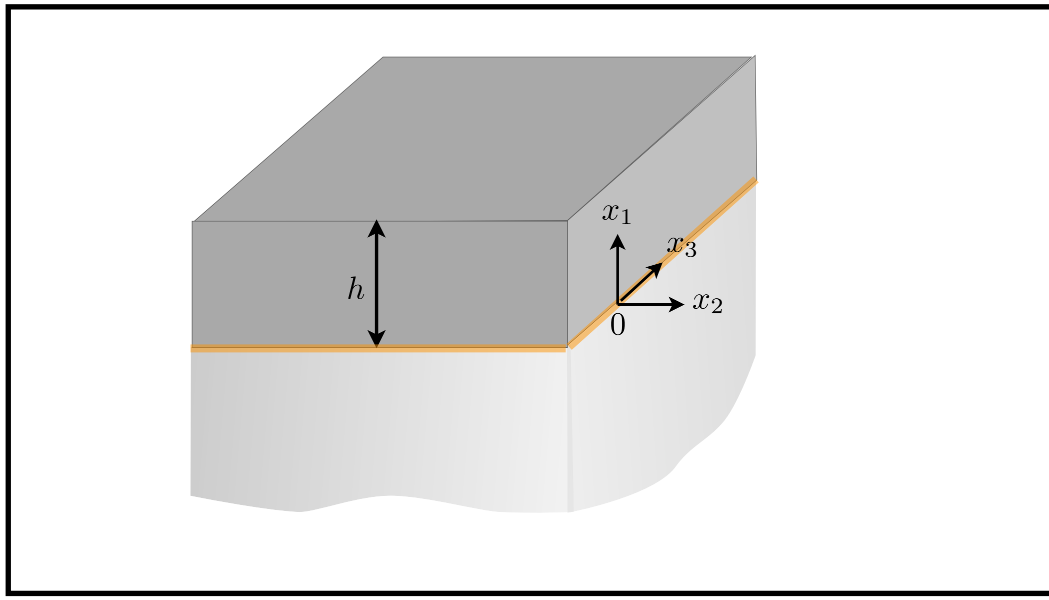

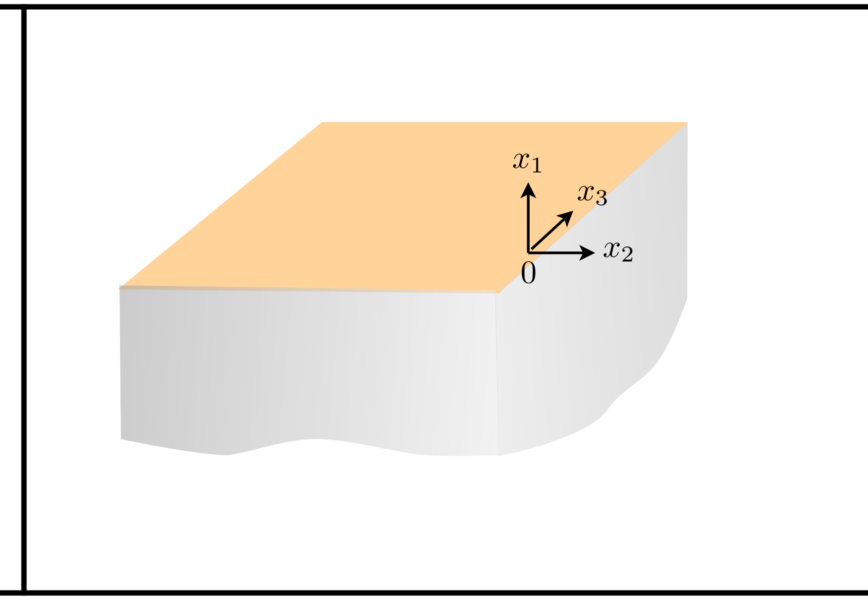

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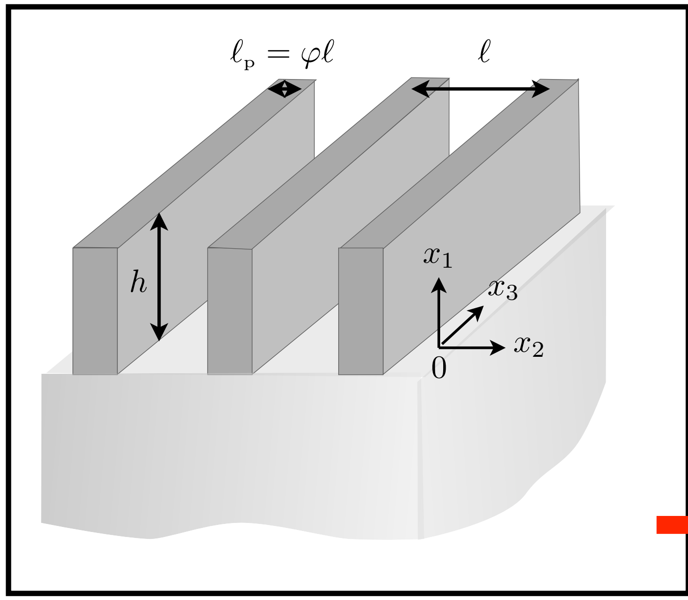

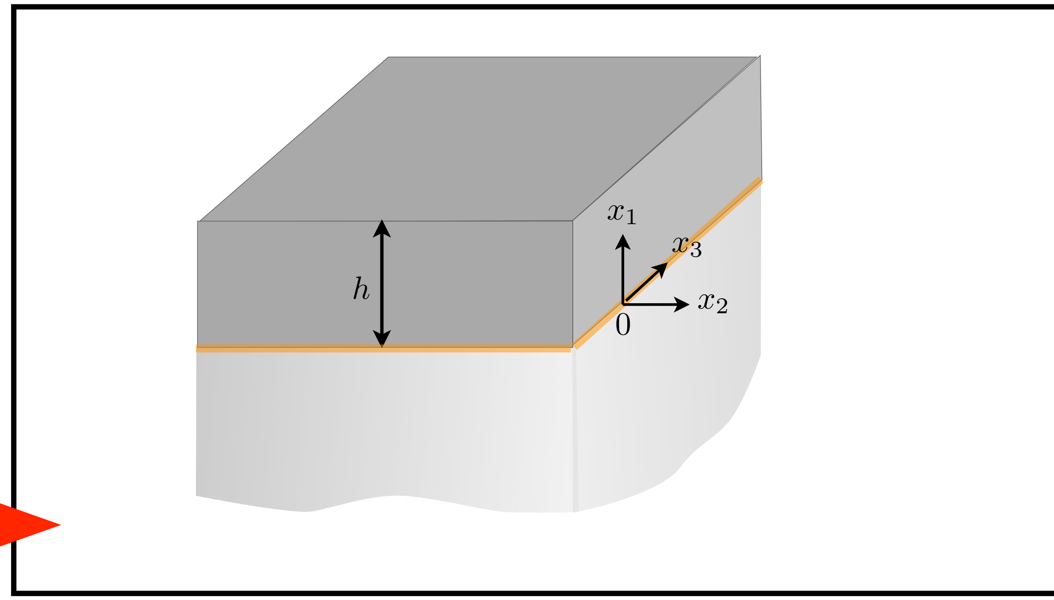

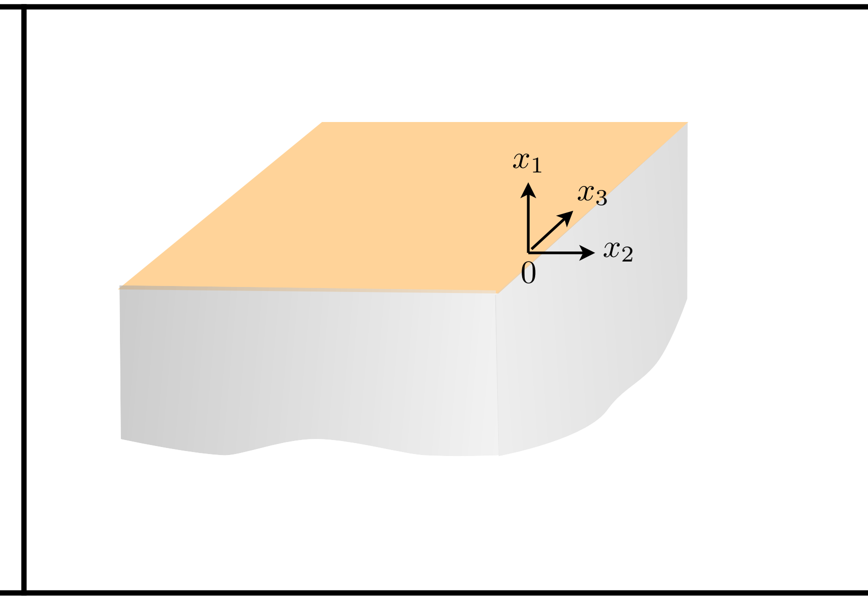

Homogenization

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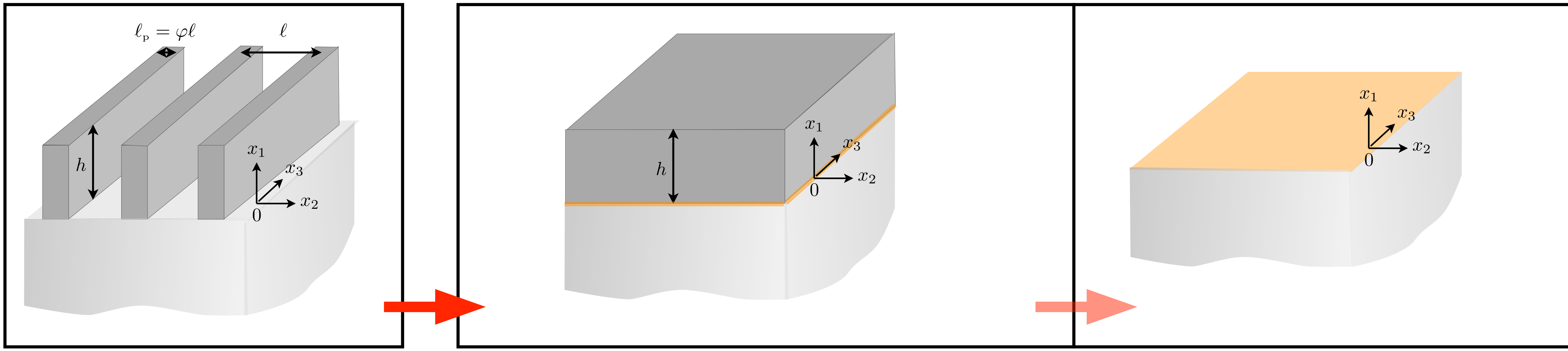

Homogenization

more incidental

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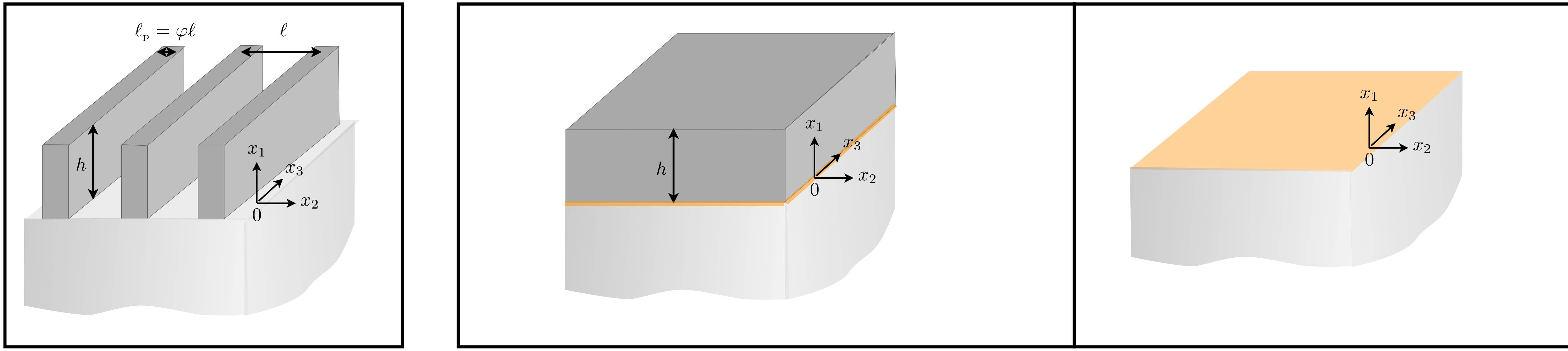

in the substrate, $x_1 \in (-\infty, 0)$:

$$\mathrm{div}\boldsymbol{\sigma} + \rho_s\omega^2\mathbf{u} = 0, \quad \boldsymbol{\sigma} = 2\mu_s\mathbf{e} + \lambda_s\mathrm{tr}(\mathbf{e})\mathrm{I}, \quad \mathbf{e} = \frac{1}{2}(\nabla\mathbf{u} + {}^t\nabla\mathbf{u}),$$

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+ appropriate boundary and transmission conditions

In the region of the plates, $x_1 \in (0, \ell)$

$$\frac{\partial^4 u_2}{\partial x_1^4} - \kappa^4 u_2 = 0, \quad \kappa = \left(\frac{\rho_p\omega^2\ell_p}{D_p}\right)^{1/4},$$

transmission conditions at $x_1 = 0$

$$\begin{cases} \sigma_{11}(0^-, \mathbf{x}') = \rho_p\omega^2\varphi h\, u_1(0, \mathbf{x}'), \quad \sigma_{12}(0^-, \mathbf{x}') = -\dfrac{D_p}{\ell}\dfrac{\partial^3 u_2}{\partial x_1^3}(0^+, \mathbf{x}'), \\ \sigma_{13}(0^-, \mathbf{x}') = \varphi h\left(E_p\dfrac{\partial^2 u_3}{\partial x_3^2}(0^-, \mathbf{x}') + \rho_p\omega^2 u_3(0^-, \mathbf{x}')\right), \\ u_2(0^+, \mathbf{x}') = u_2(0^-, \mathbf{x}'), \quad \dfrac{\partial u_2}{\partial x_1}(0^+, \mathbf{x}') = 0, \\ \dfrac{\partial^2 u_2}{\partial x_1^2}(h, \mathbf{x}') = \dfrac{\partial^3 u_2}{\partial x_1^3}(h, \mathbf{x}') = 0. \end{cases}$$

Instead of the plates,
Boundary conditions at $x_1 = 0$

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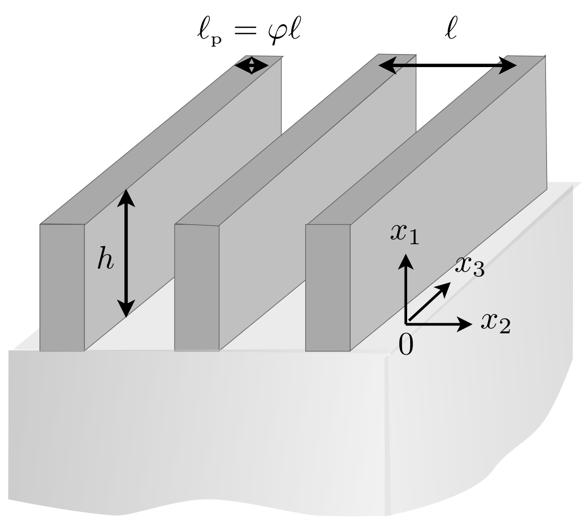

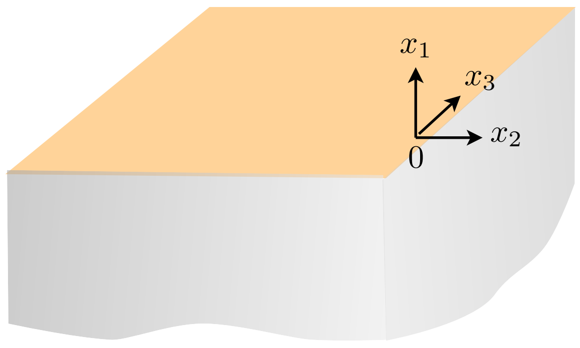

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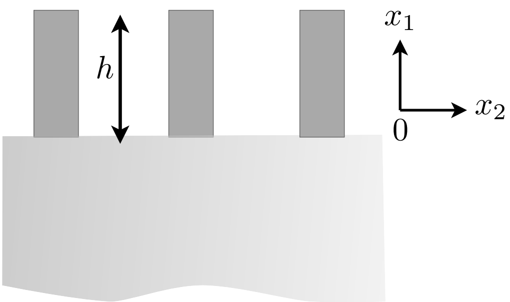

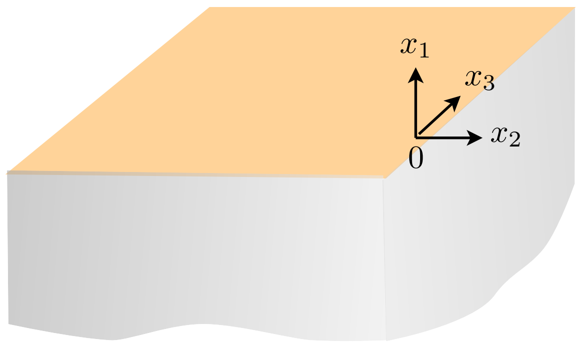

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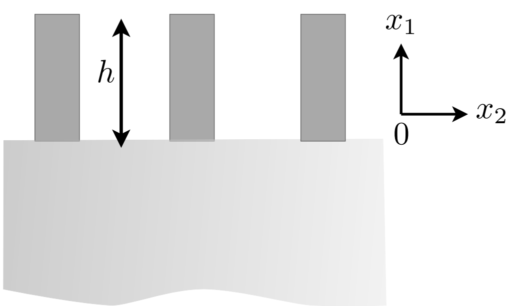

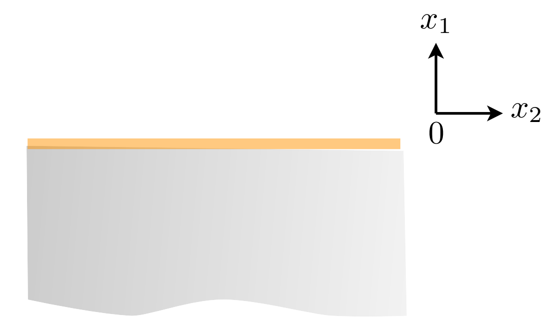

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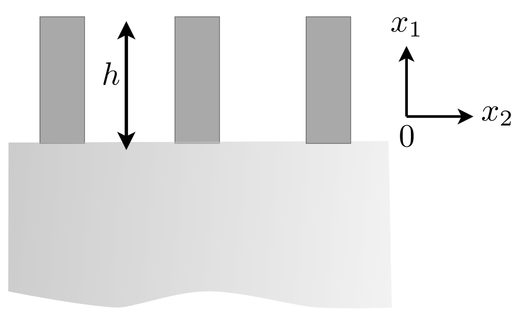

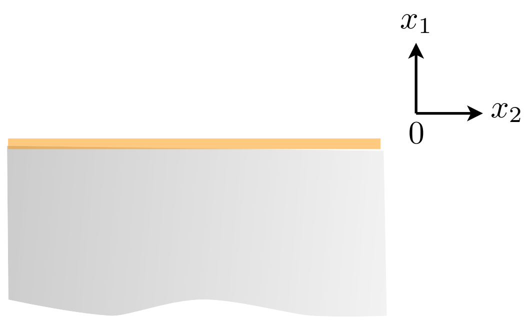

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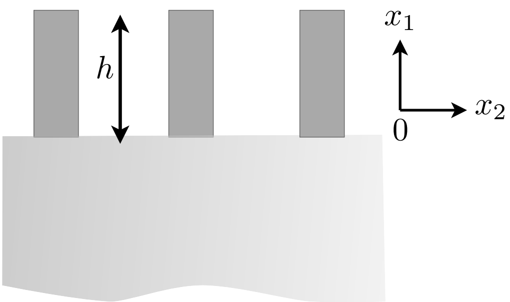

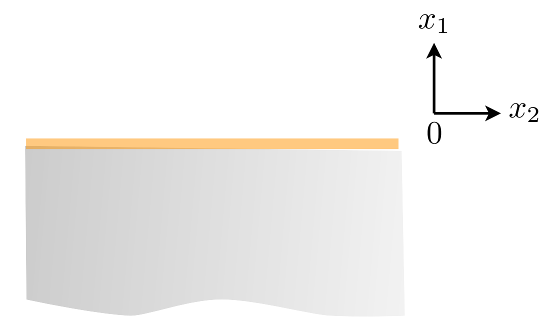

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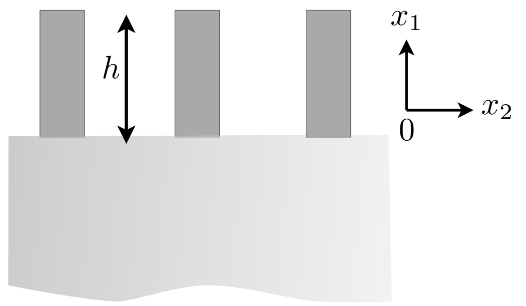
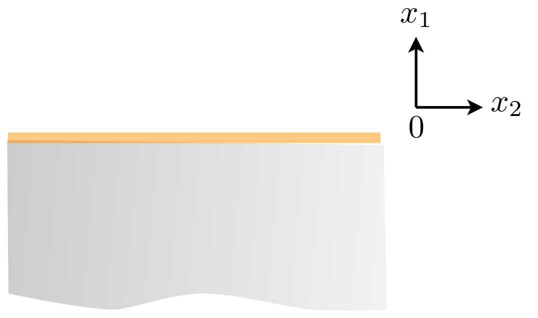


For an incident plane wave



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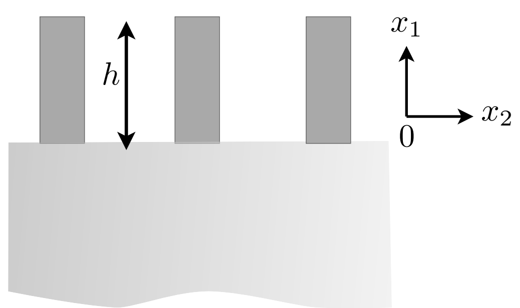

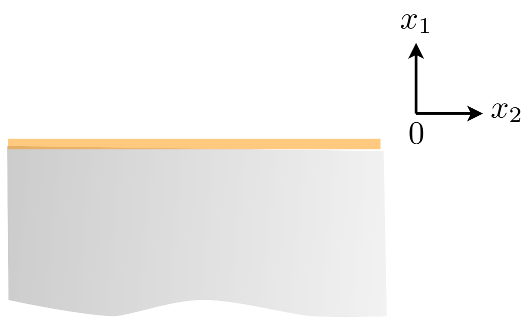

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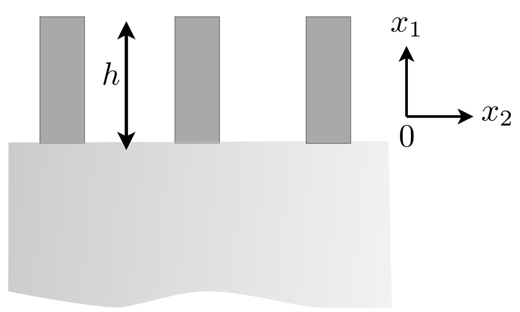

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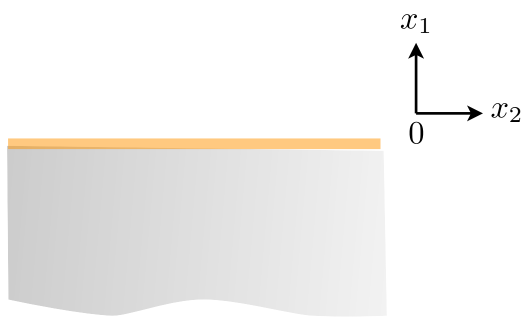

The homog. problem can be solved explicitly
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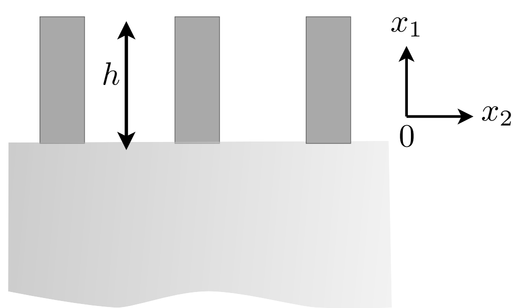

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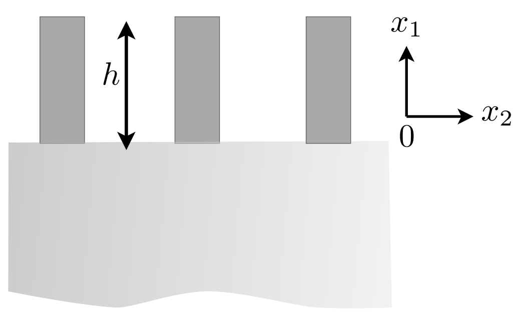

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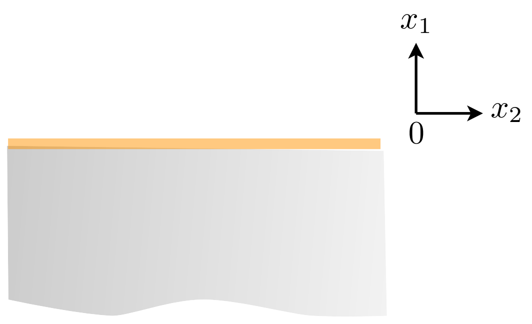

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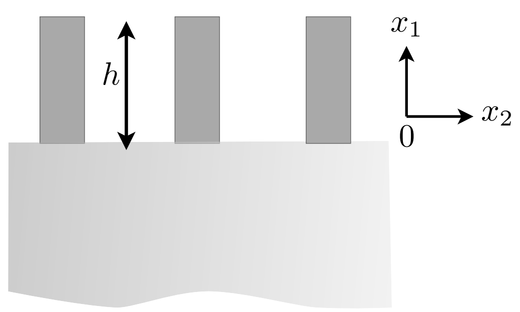

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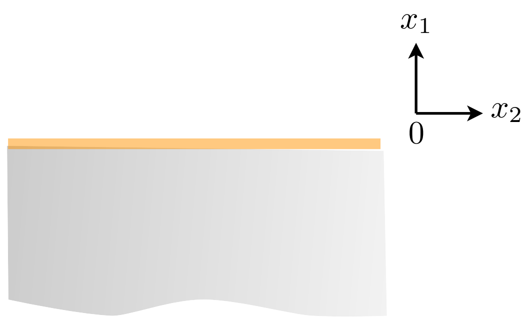

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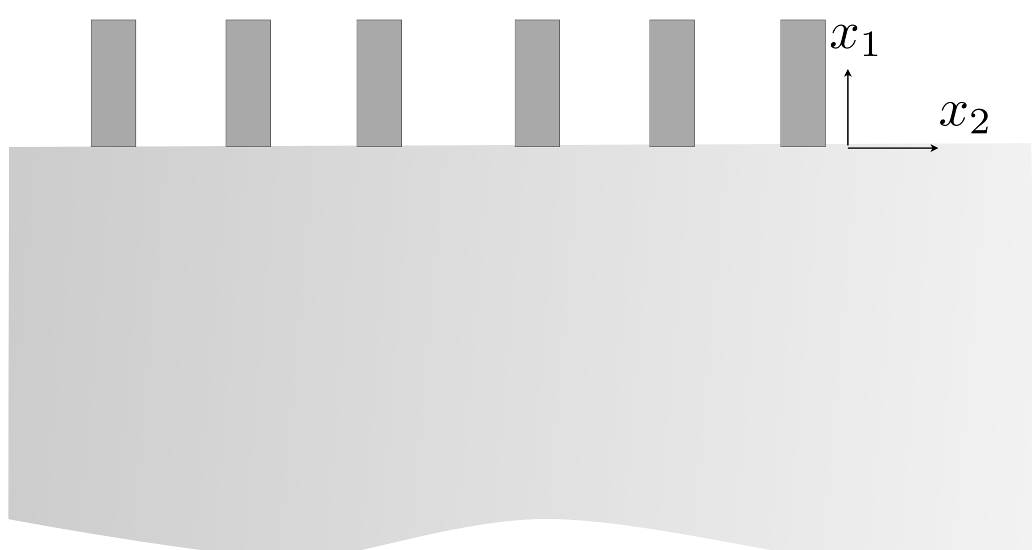

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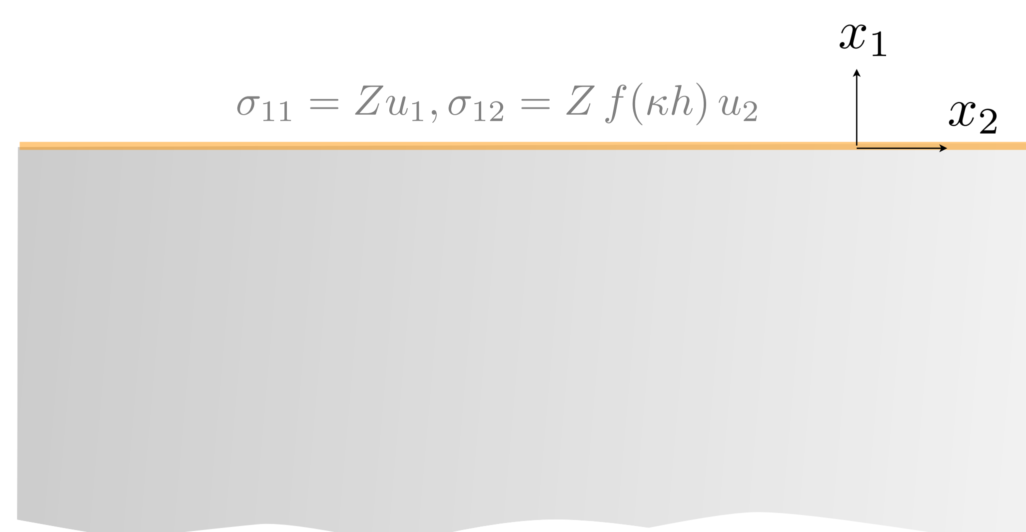

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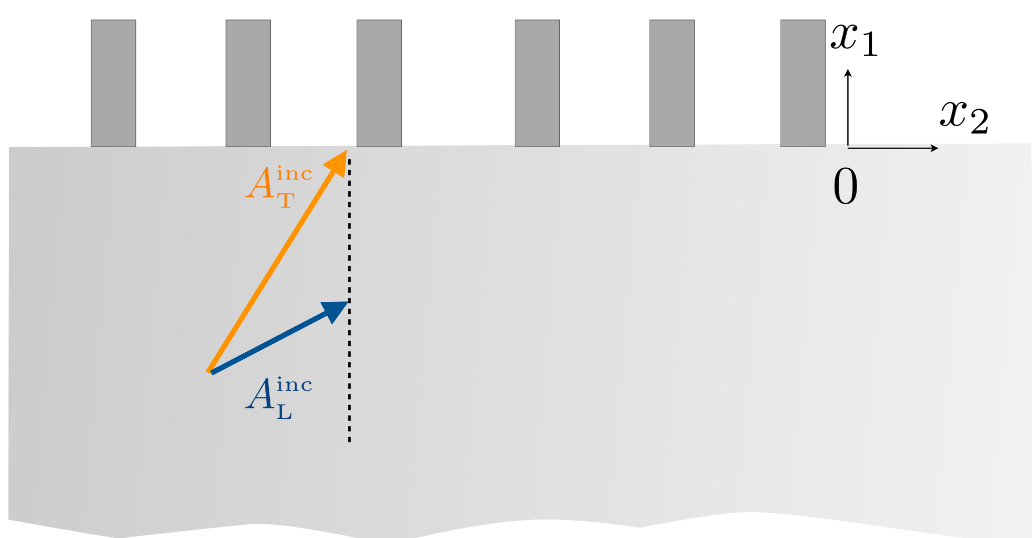

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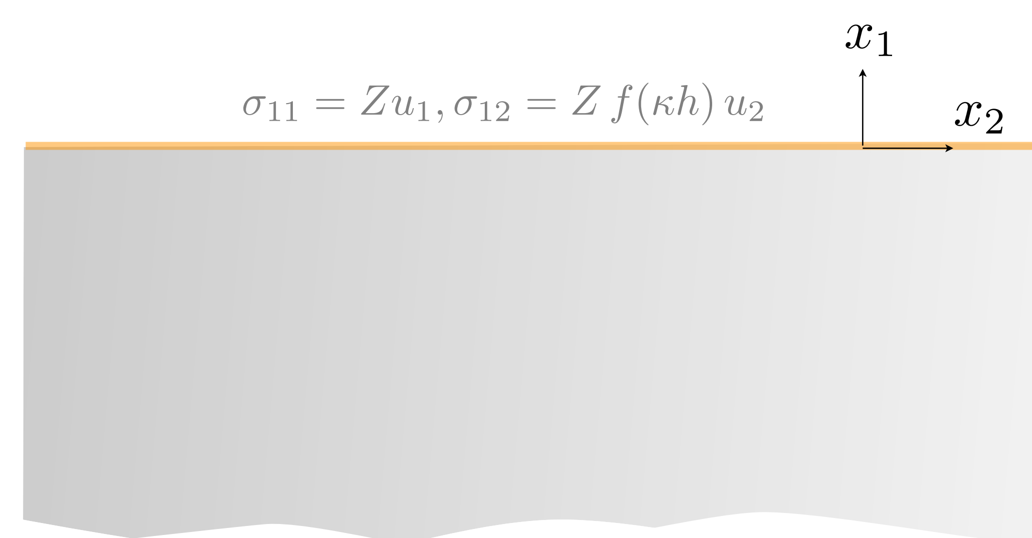

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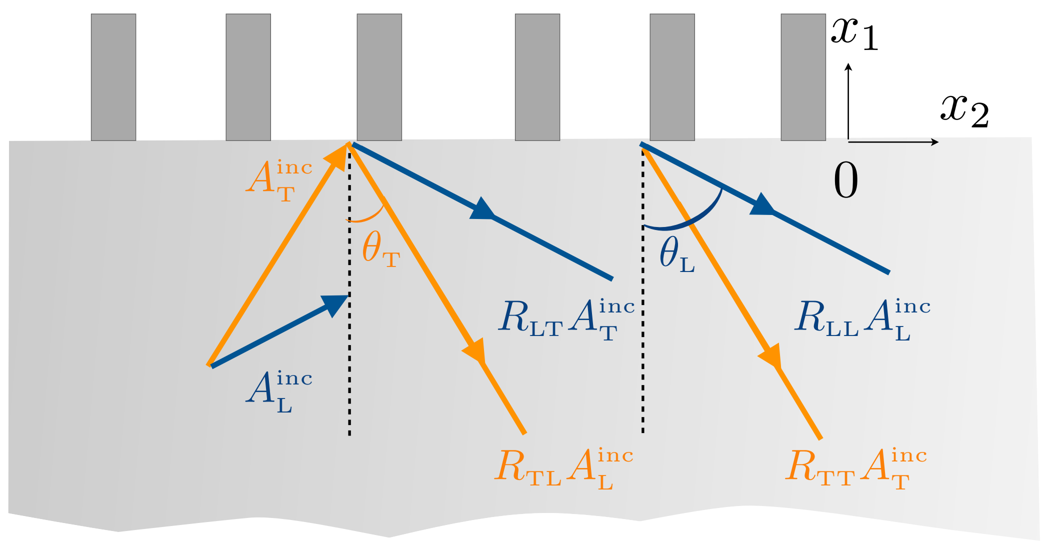

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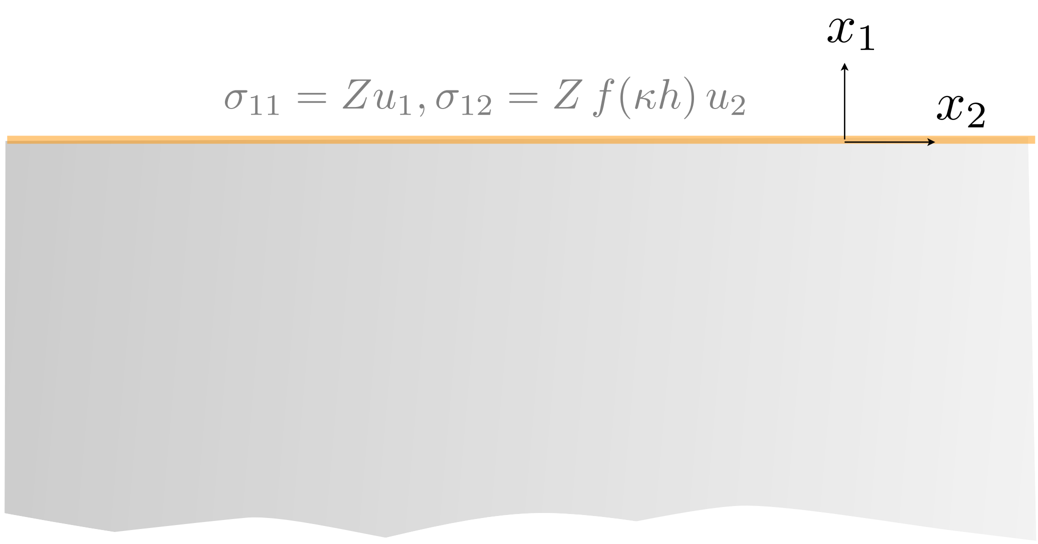
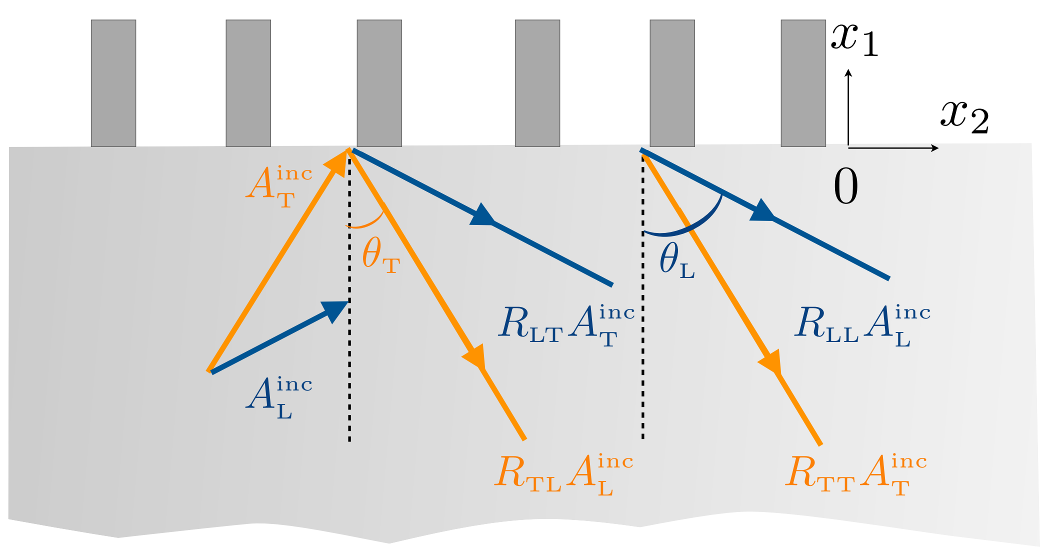


For an incident plane wave

The actual problem can be solved numerically



The homog. problem can be solved explicitly
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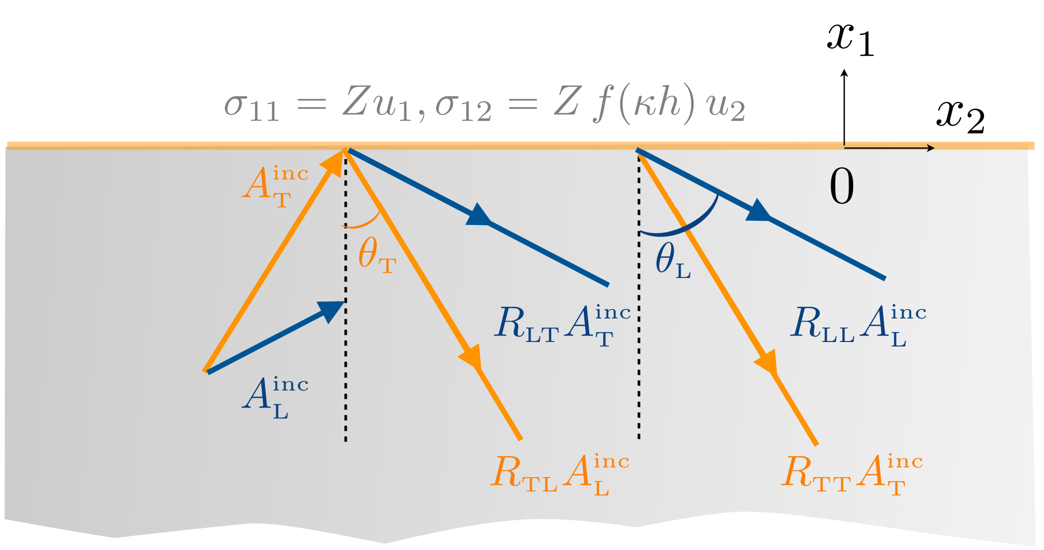

For an incident plane wave

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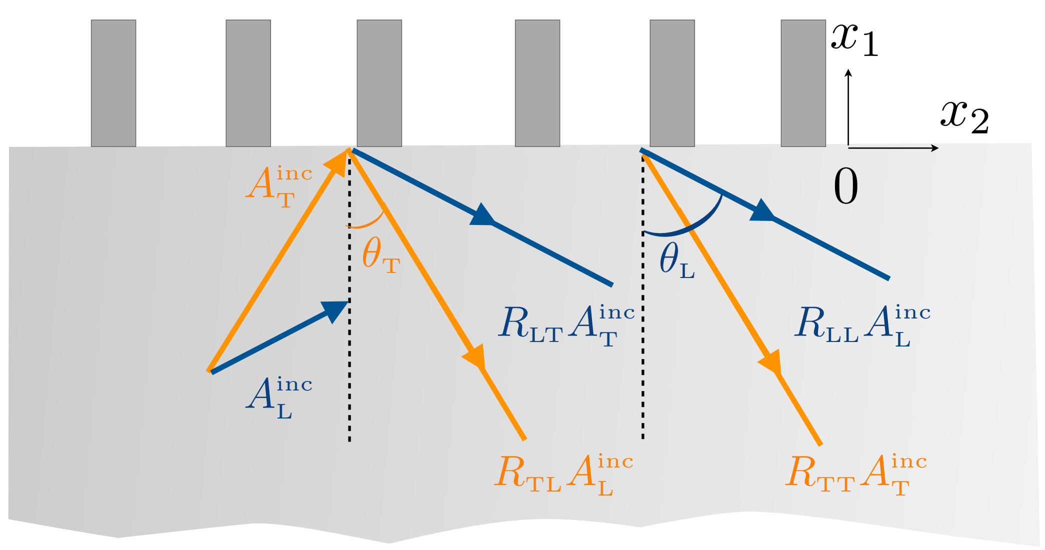

The homog. problem can be solved explicitly
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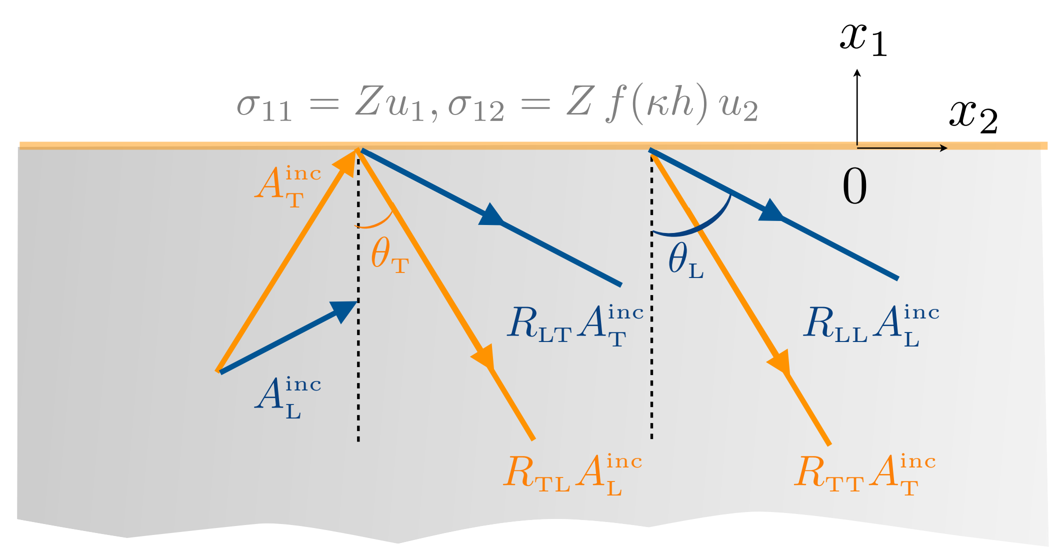
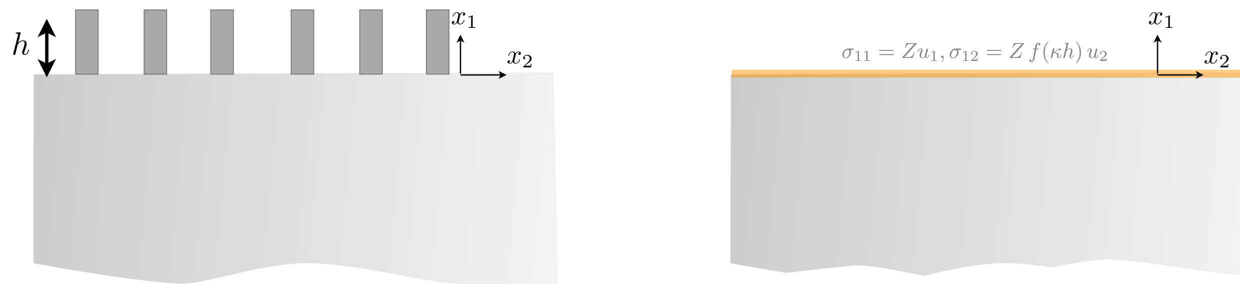
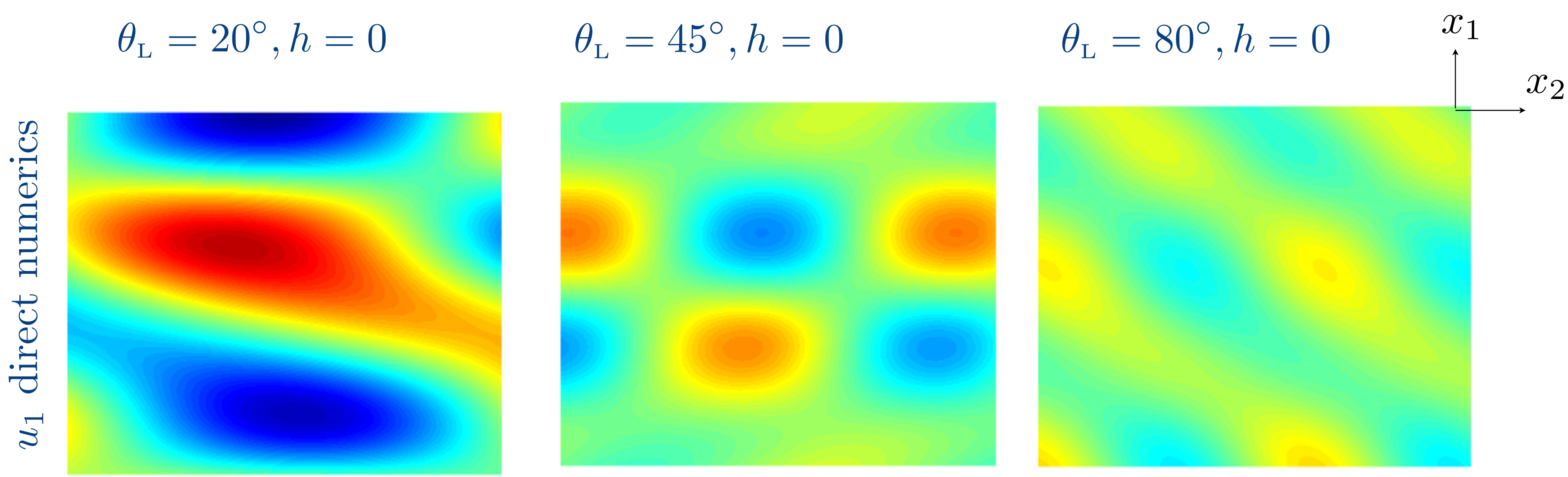


compare the 4 scattering coefficients



The reference case: $h = 0$



Asymptotic Analysis of Arrays of Beams on the Top of an Elastic Half-Space

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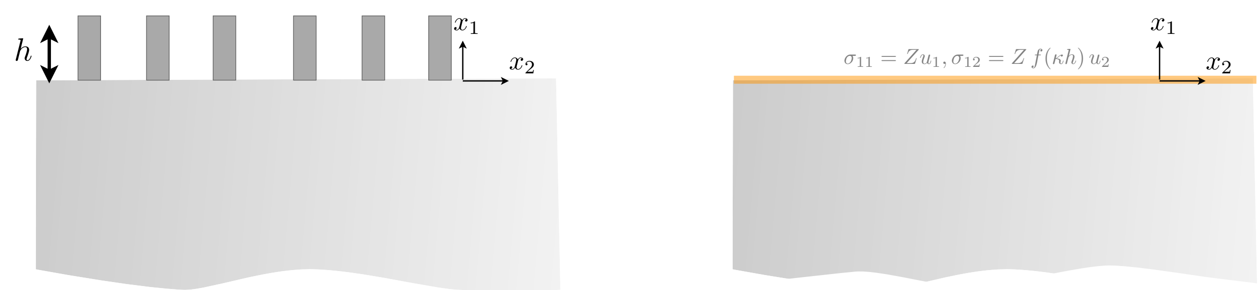

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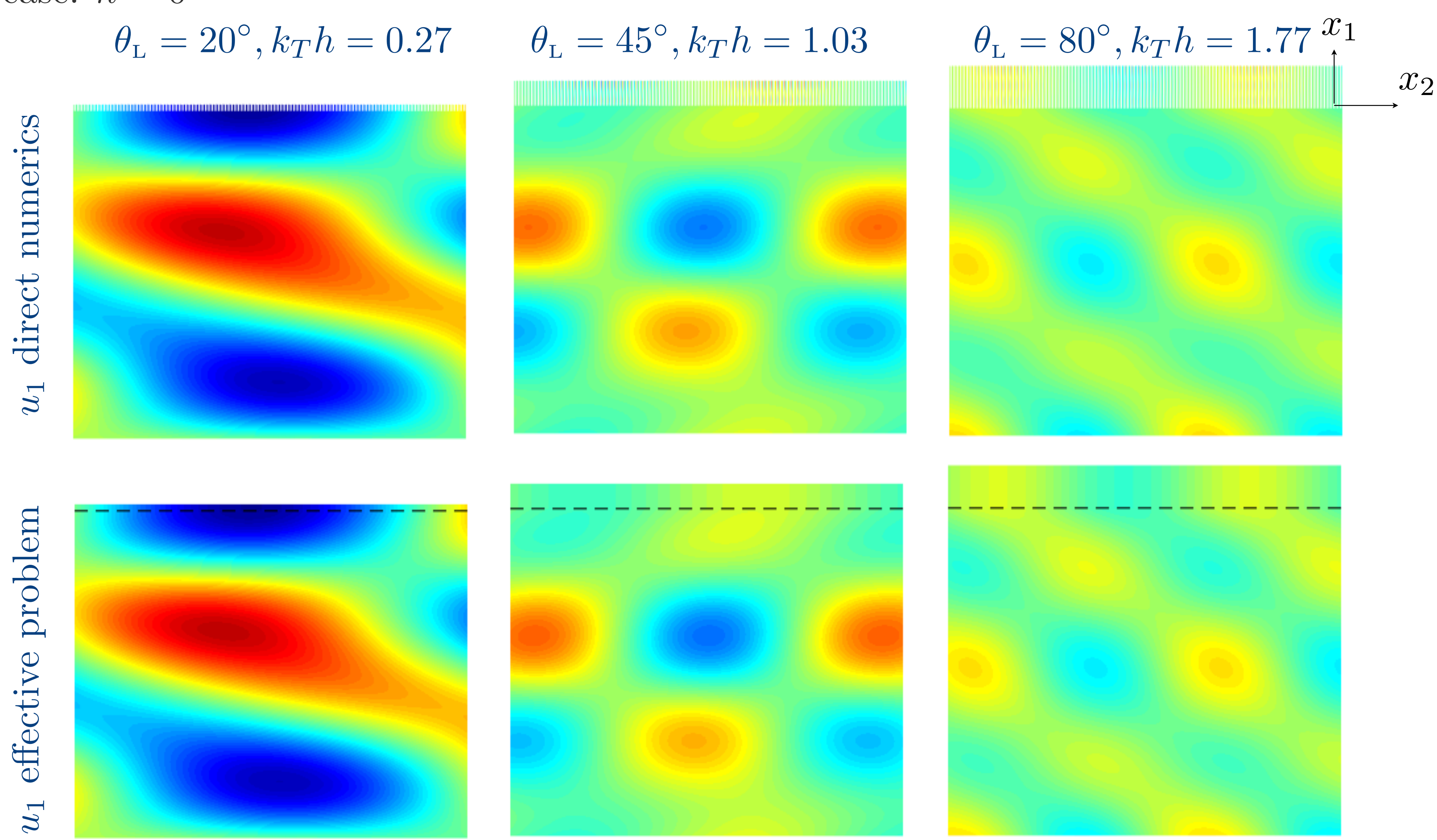

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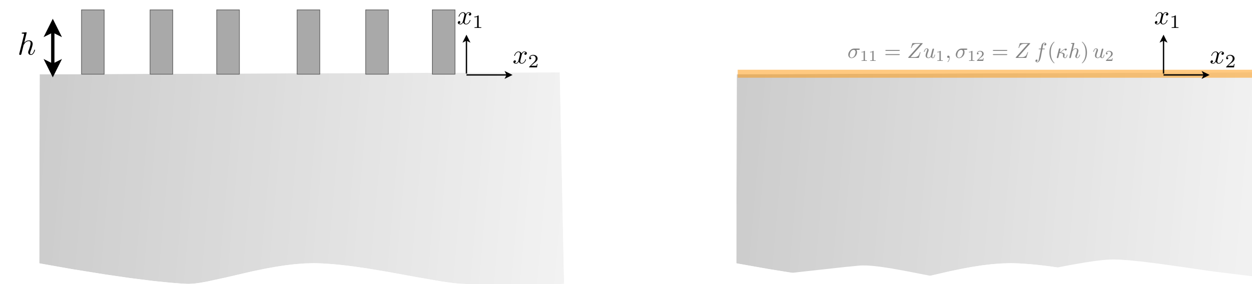

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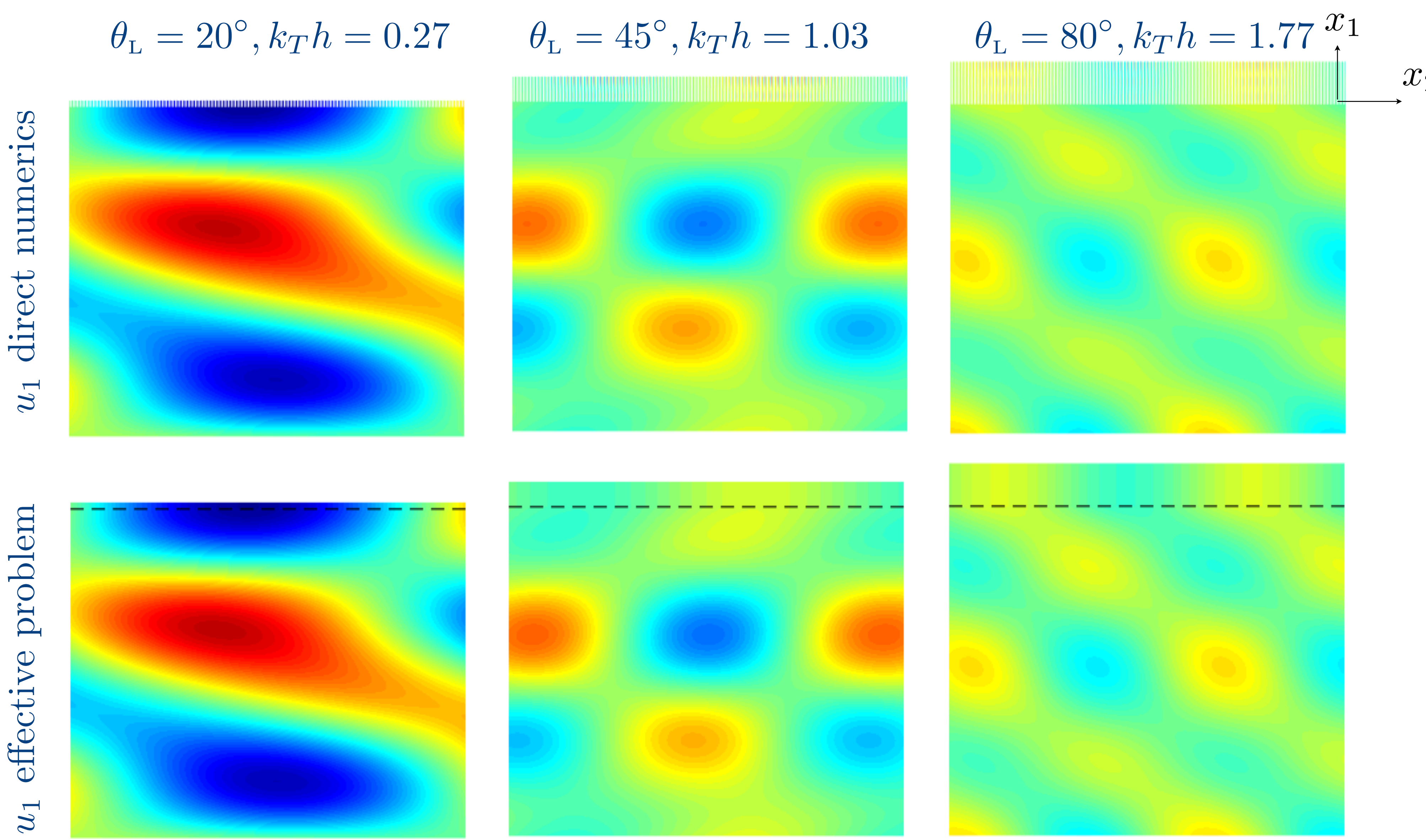

No interaction between the plates and the substrate

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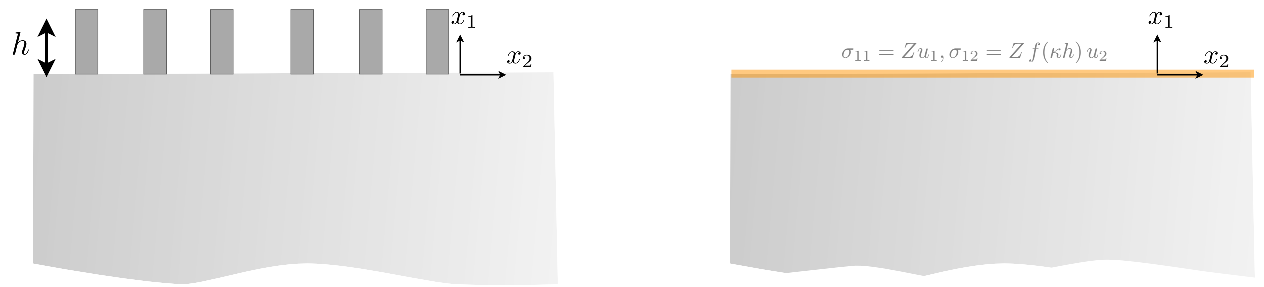

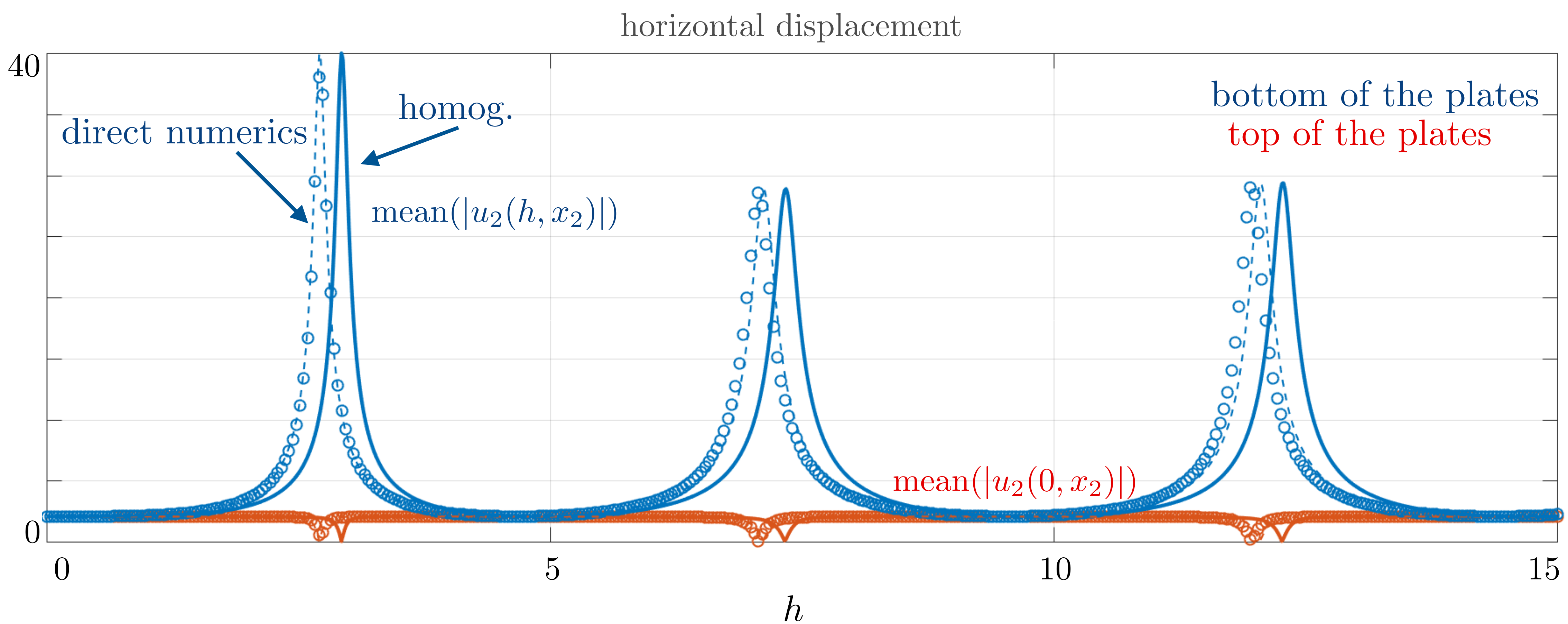

The interaction is weak except near the flexural resonances

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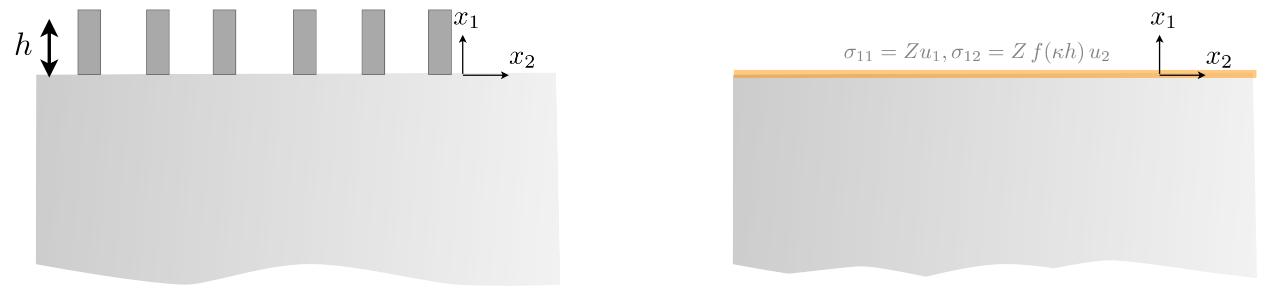

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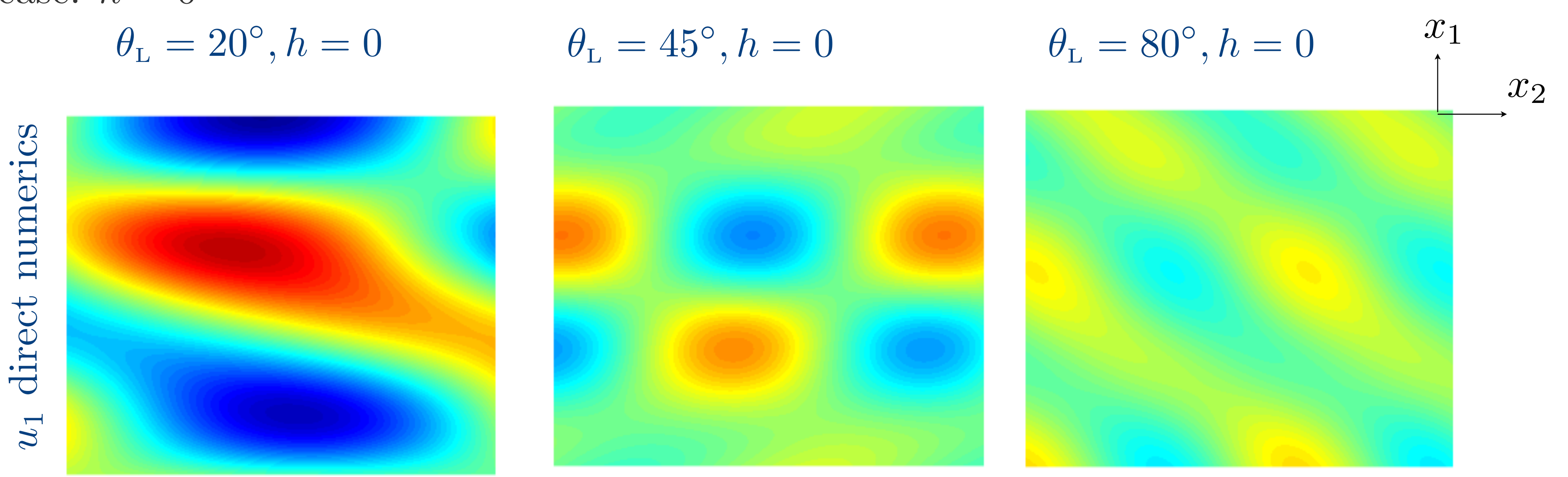

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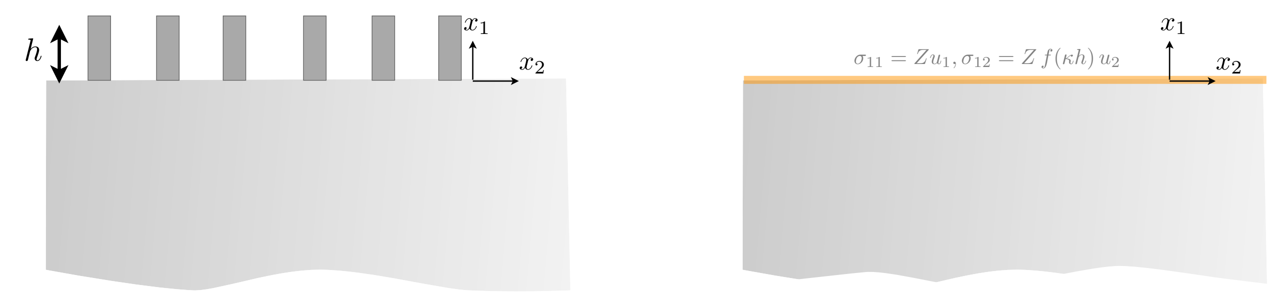

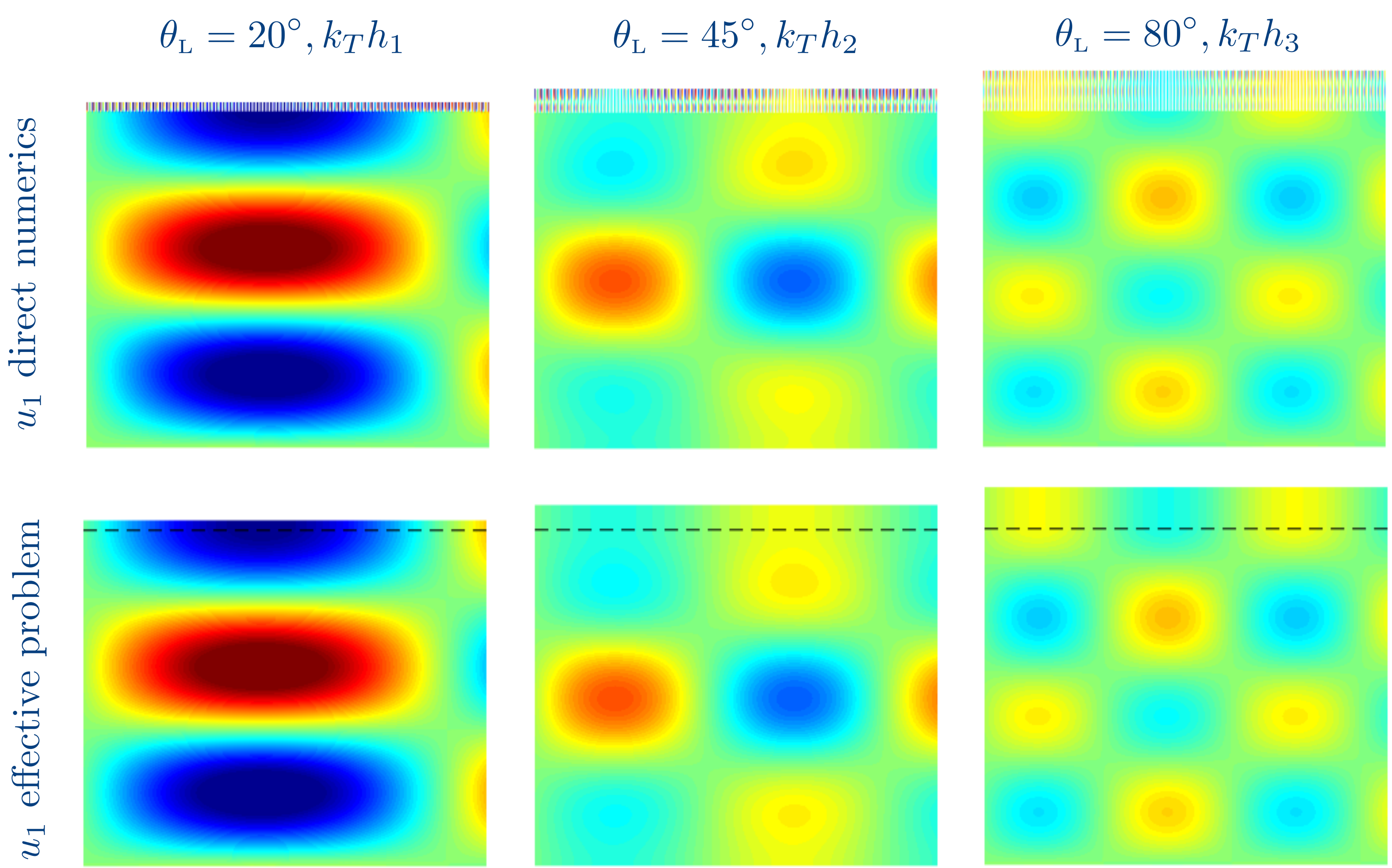

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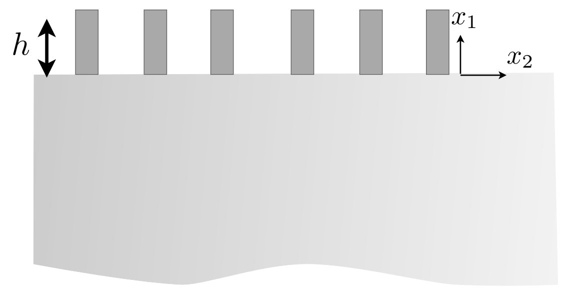
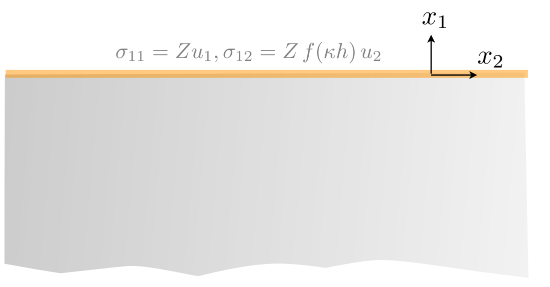
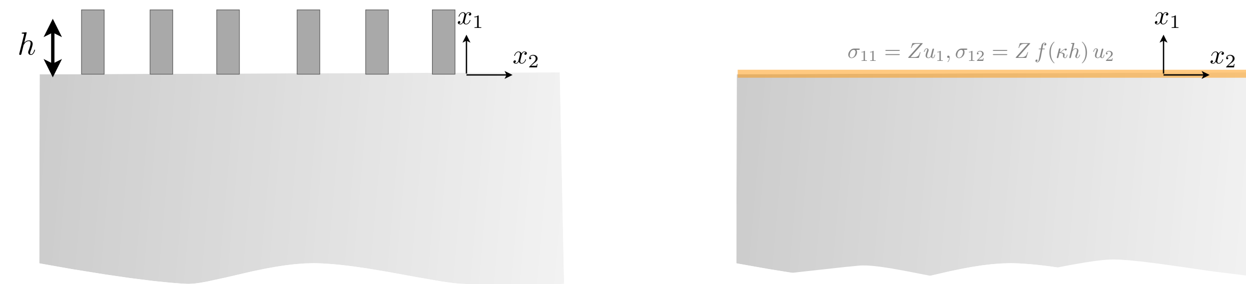




more systematically, looking at the reflection coefficients



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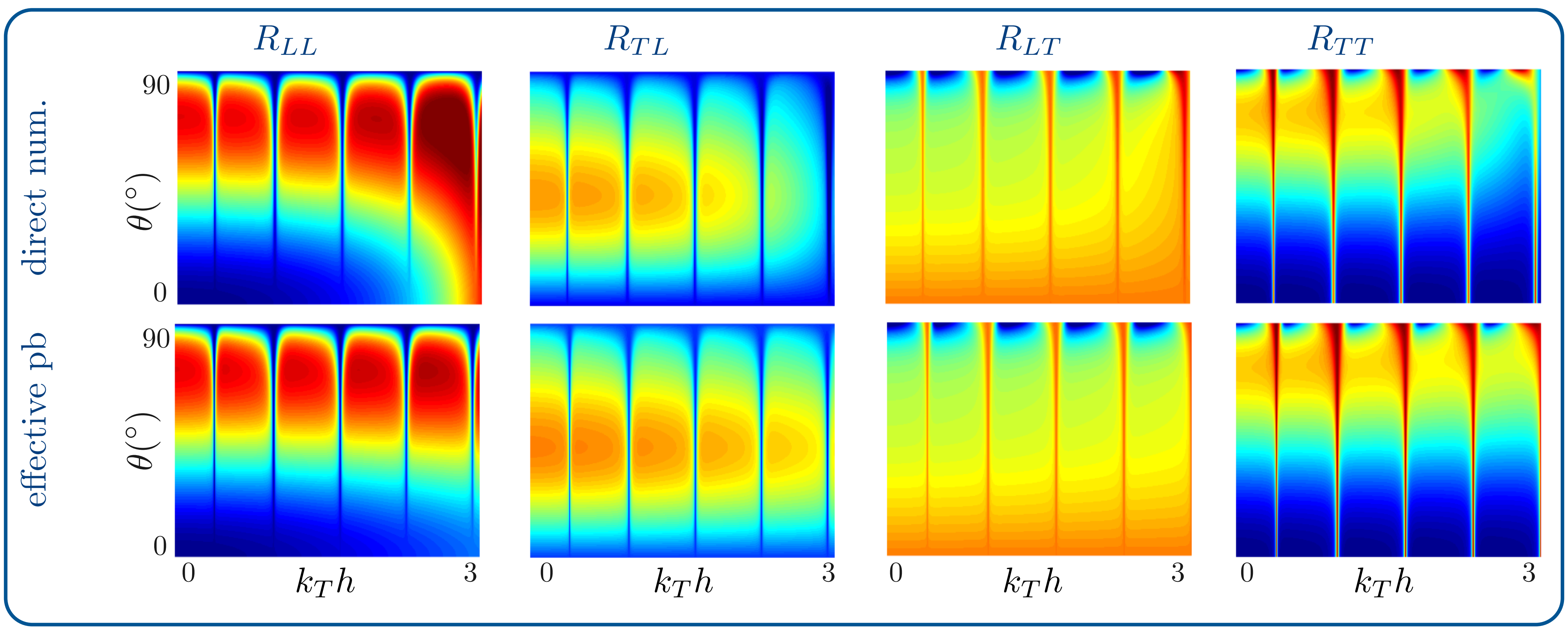
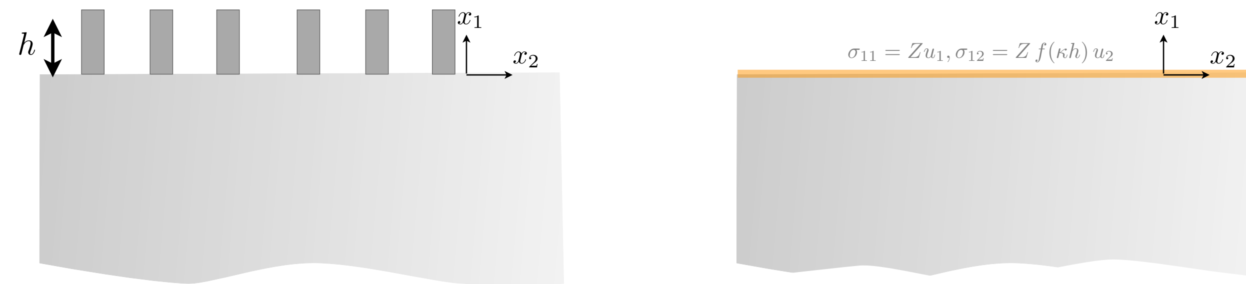
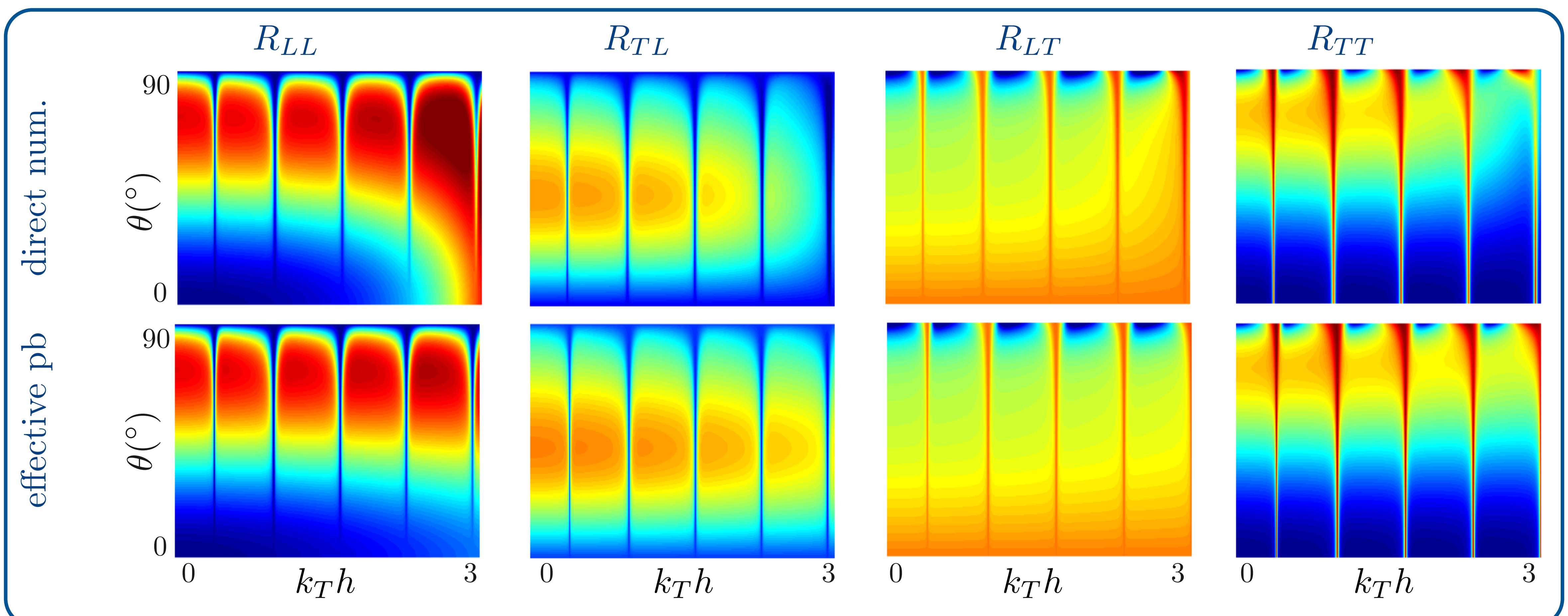
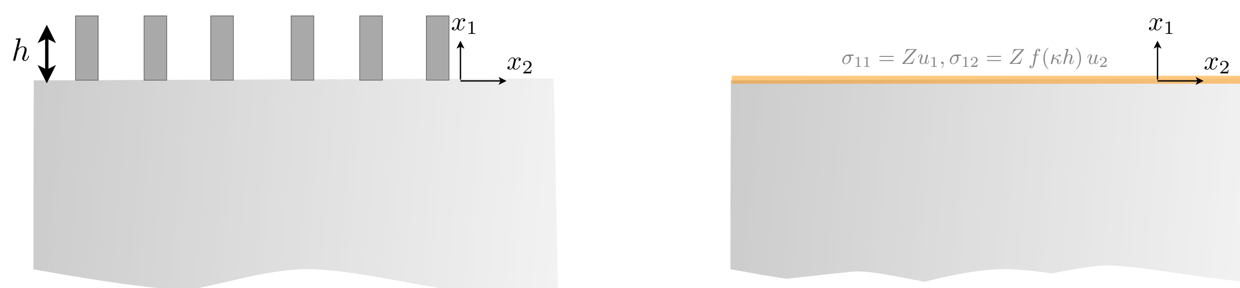




more systematically, looking at the reflection coefficients

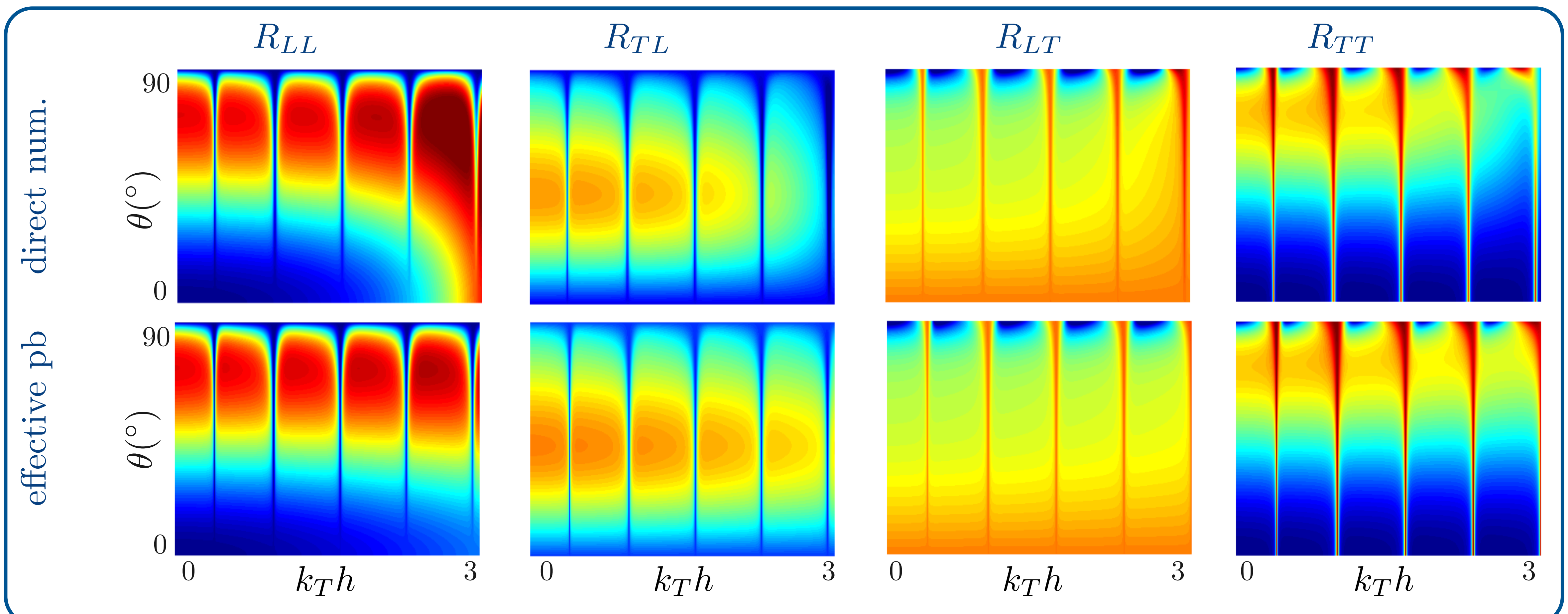
we find an overall good agreement

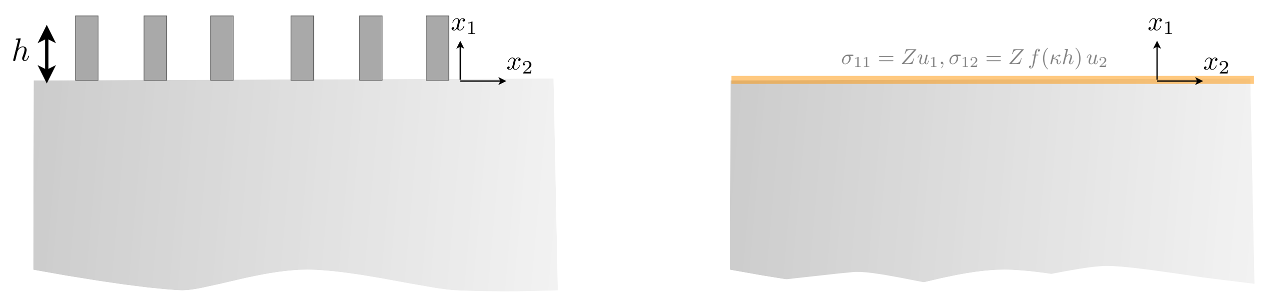




more systematically, looking at the reflection coefficients

we find an overall good agreement , except that





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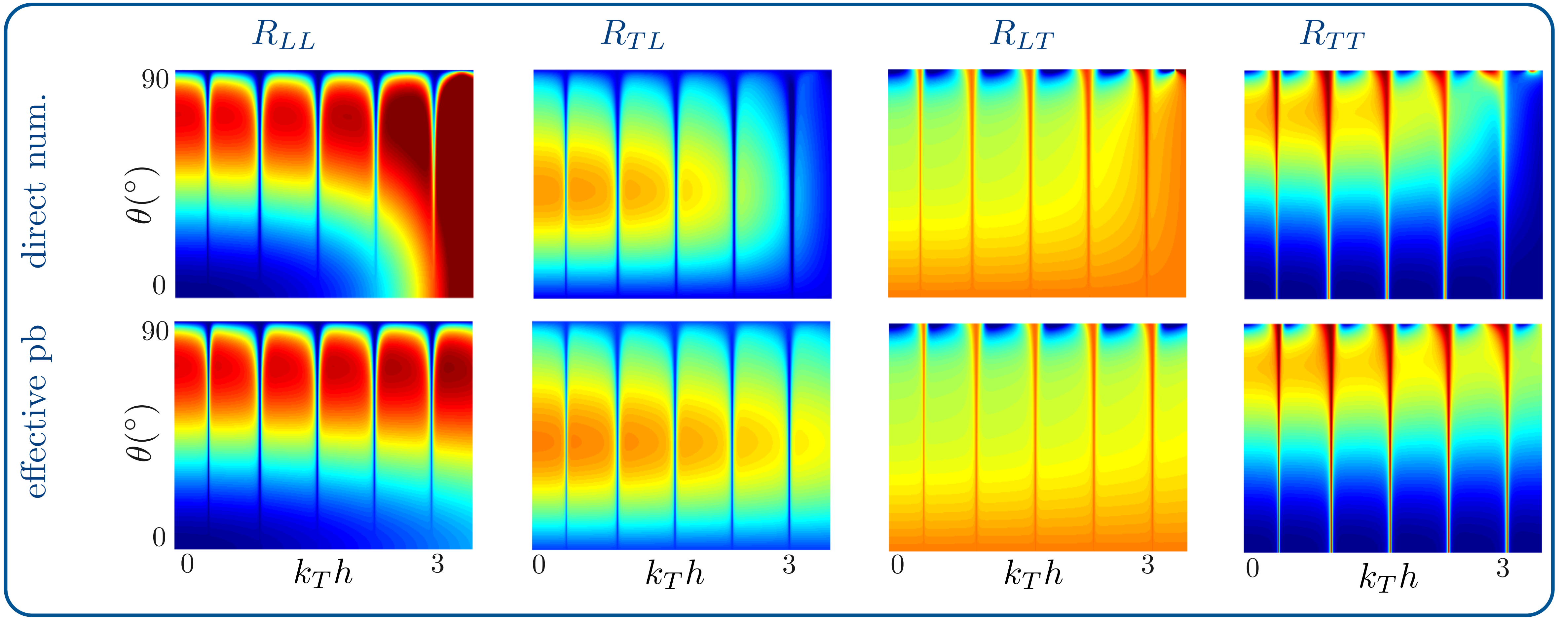

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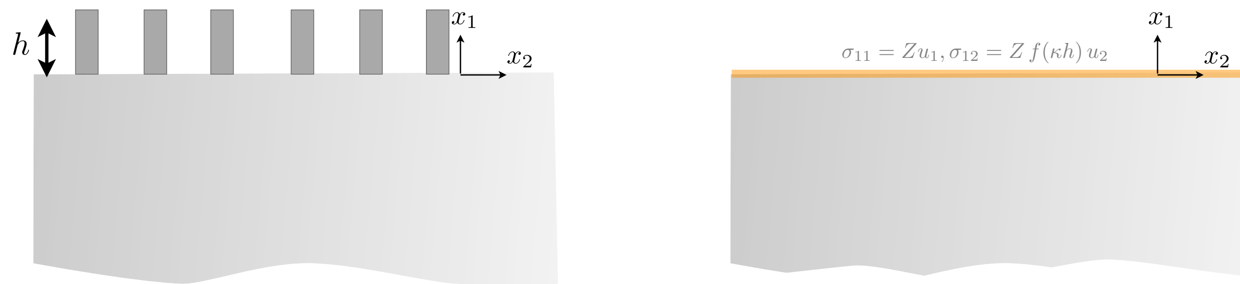

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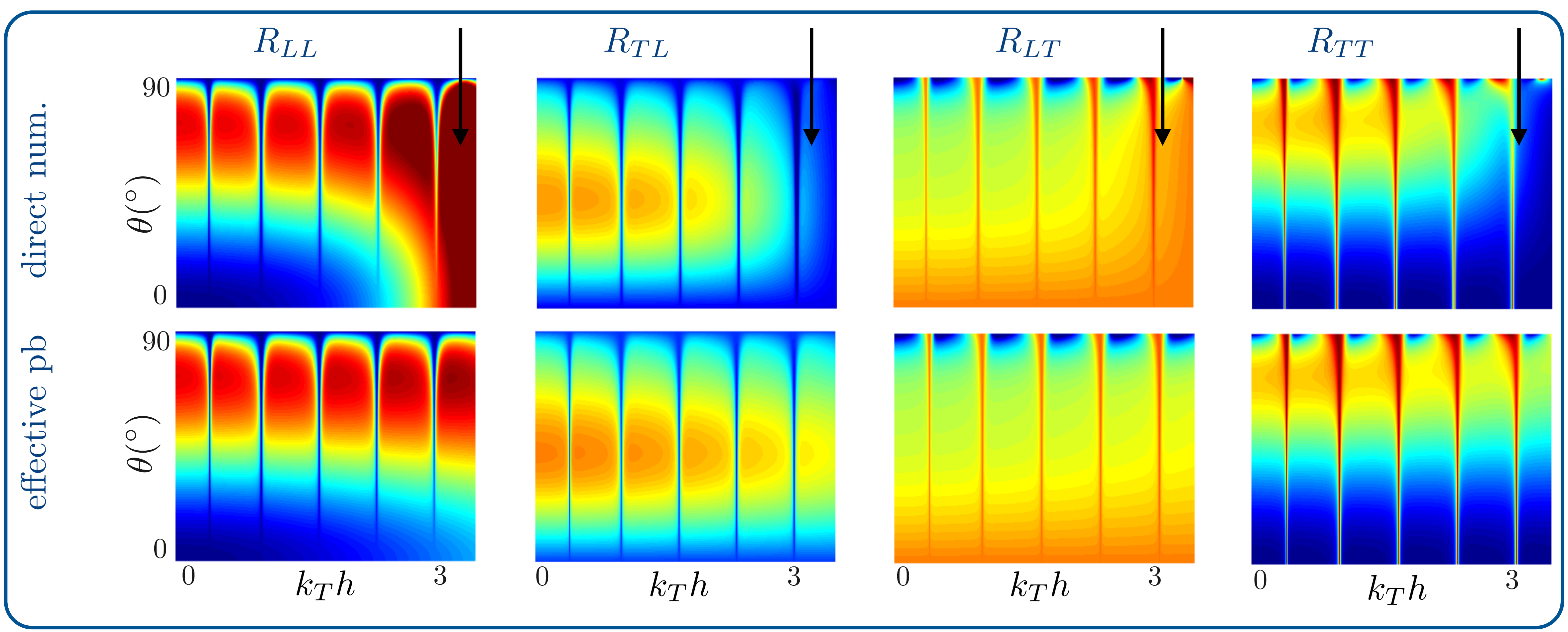

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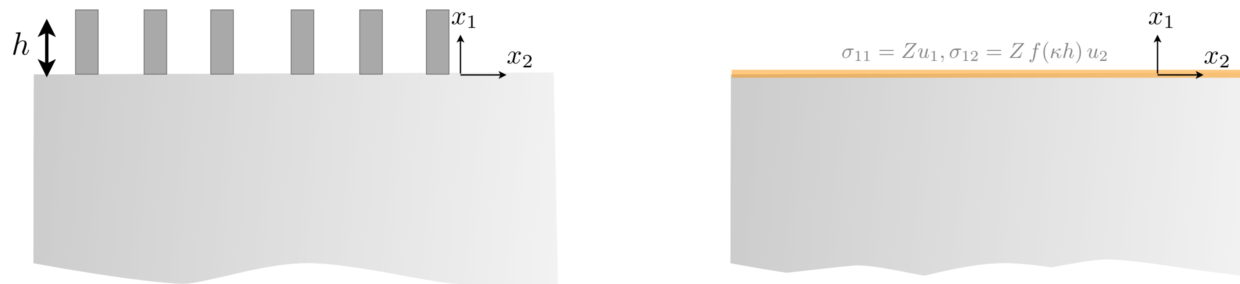

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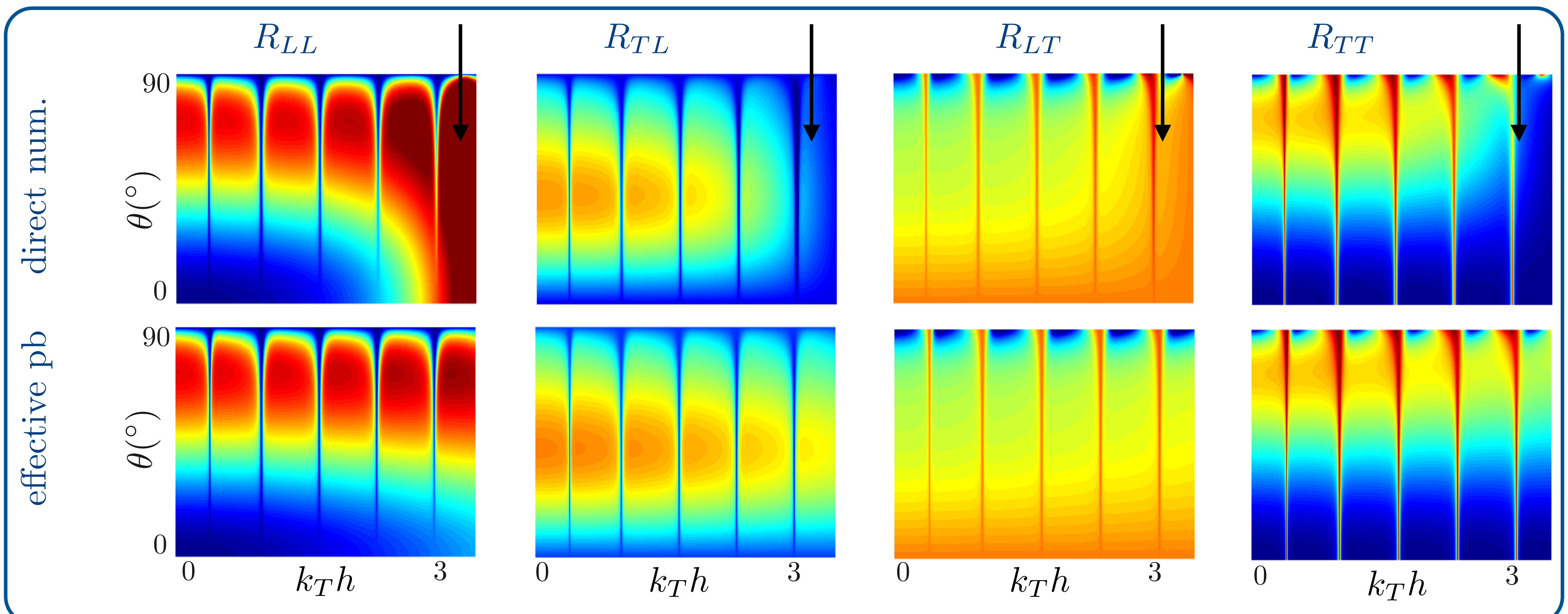

expected

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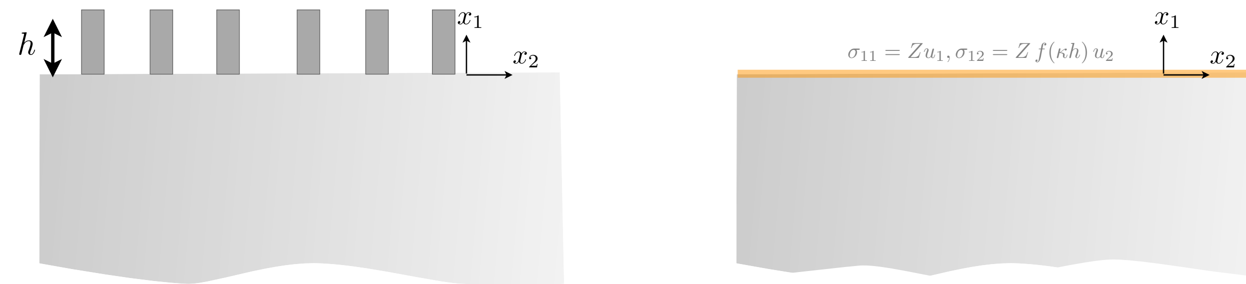

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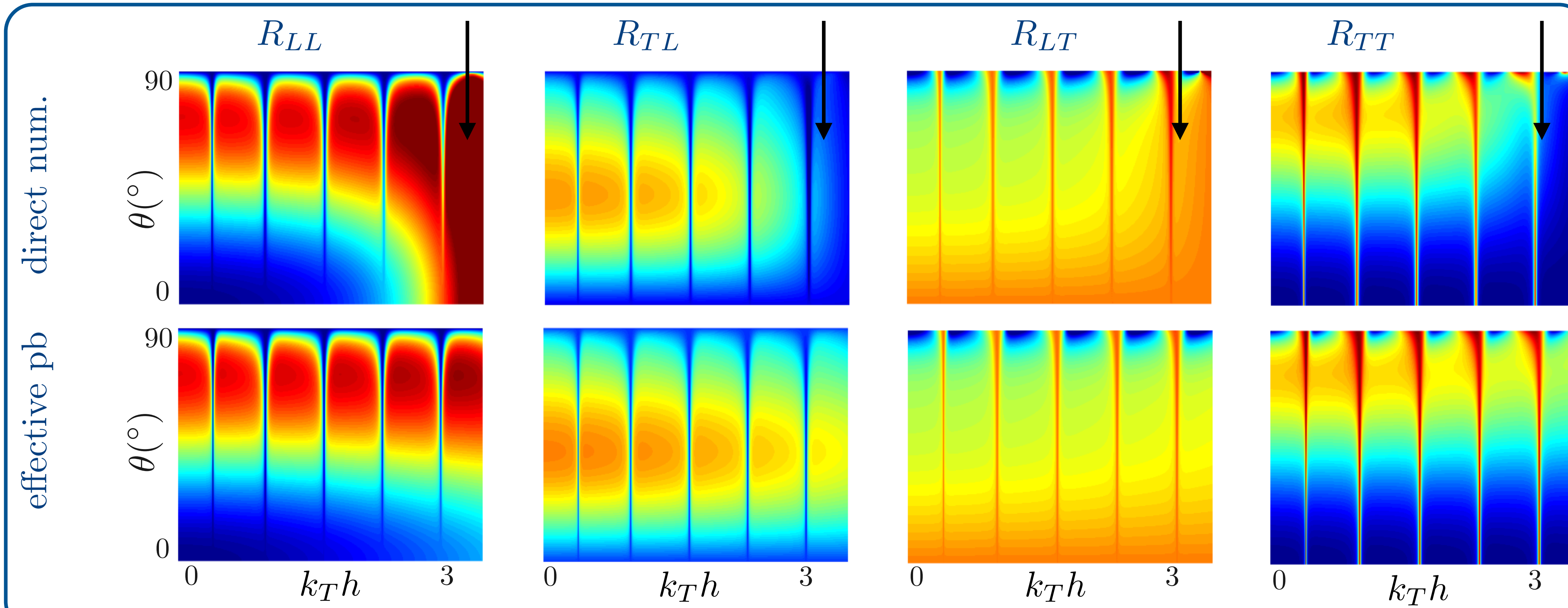

expected because of the occurence of longitudinal resonances

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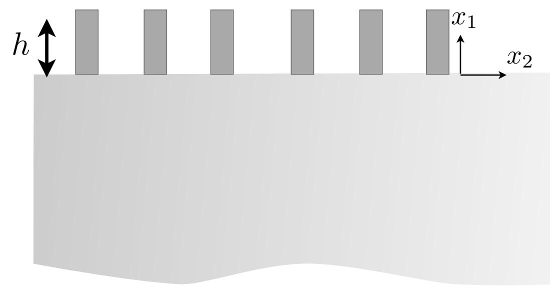

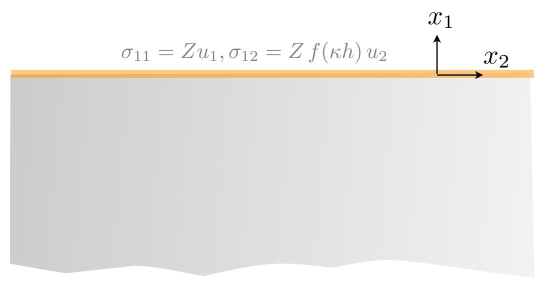

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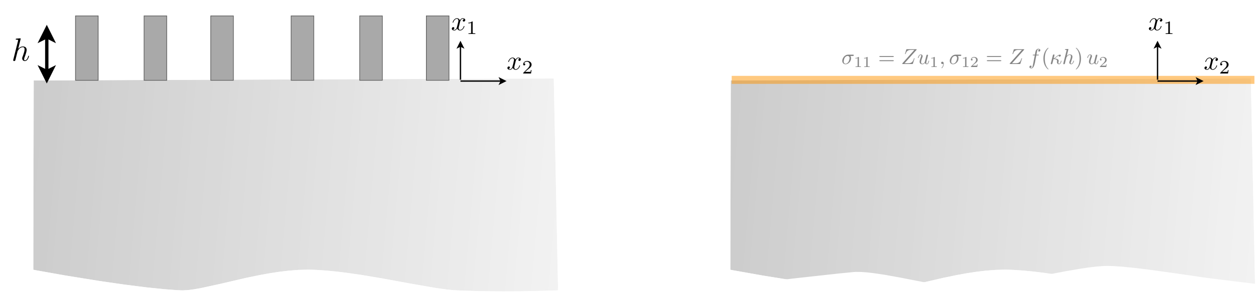

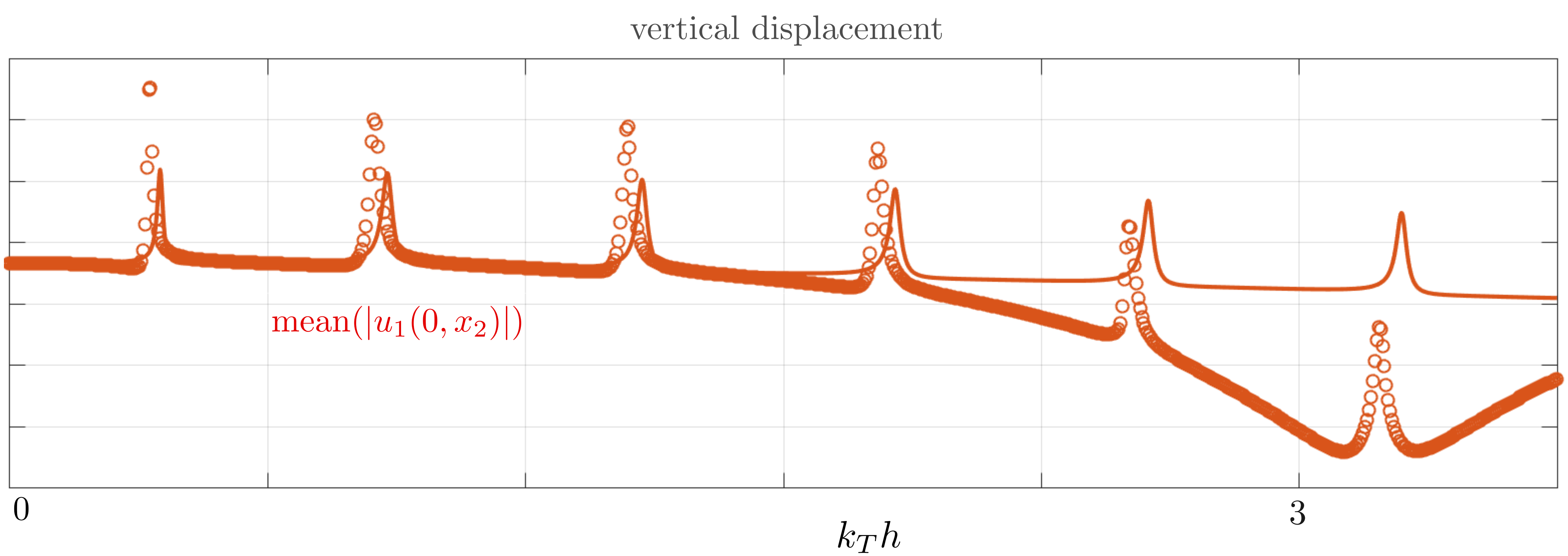

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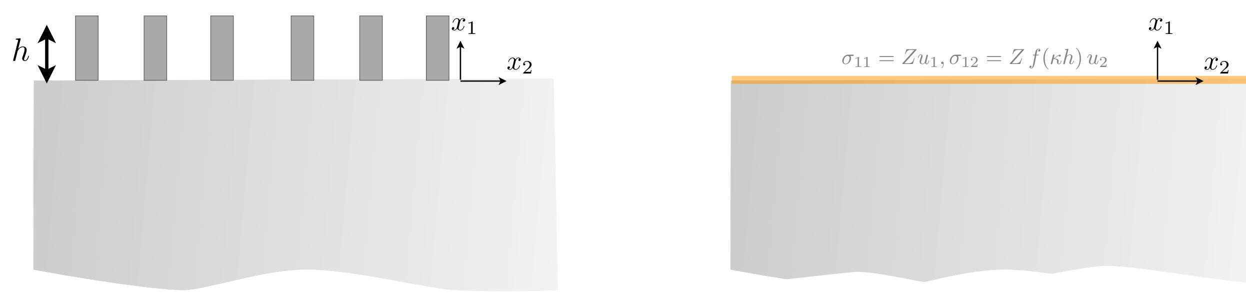
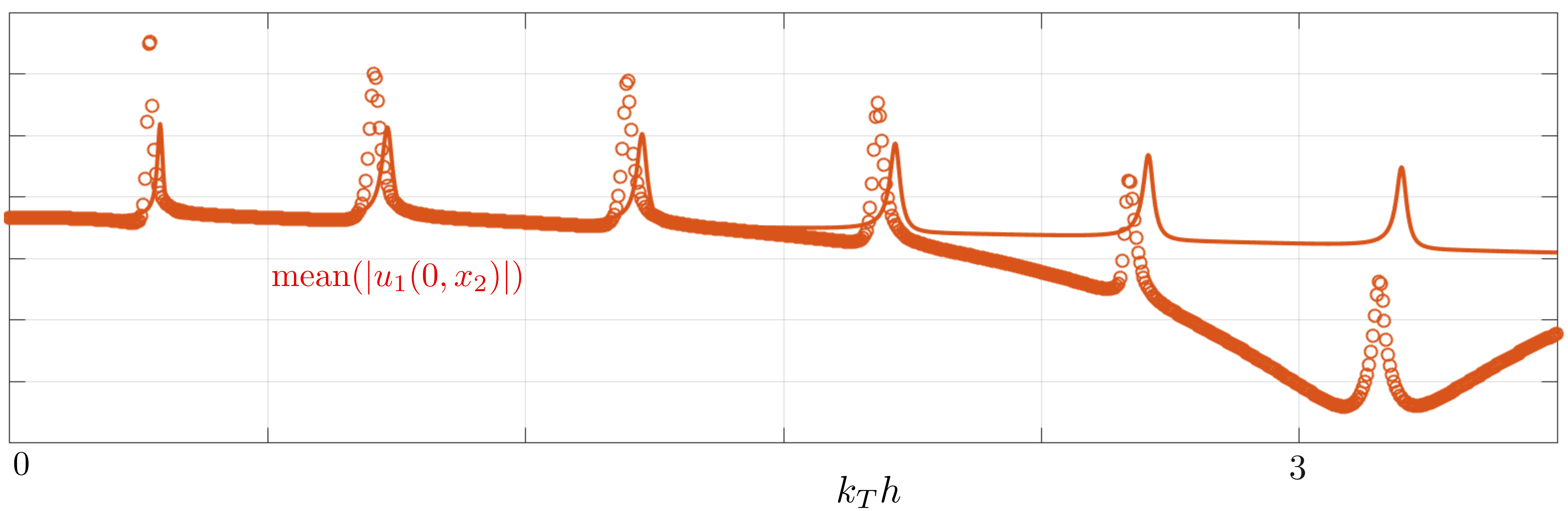


vertical displacement



superposition of flexural resonances

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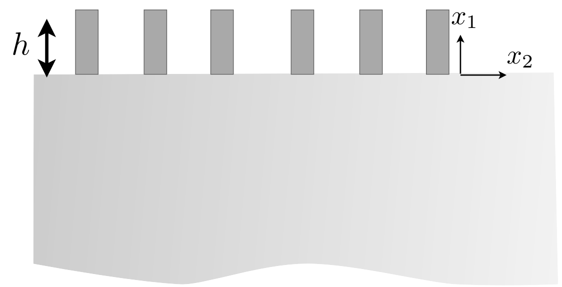

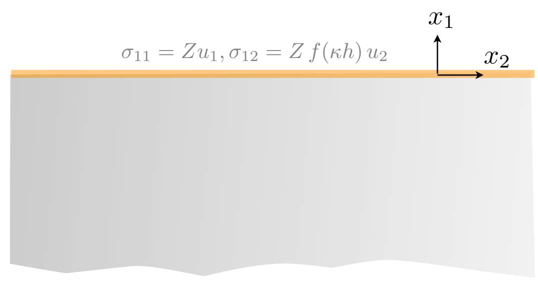

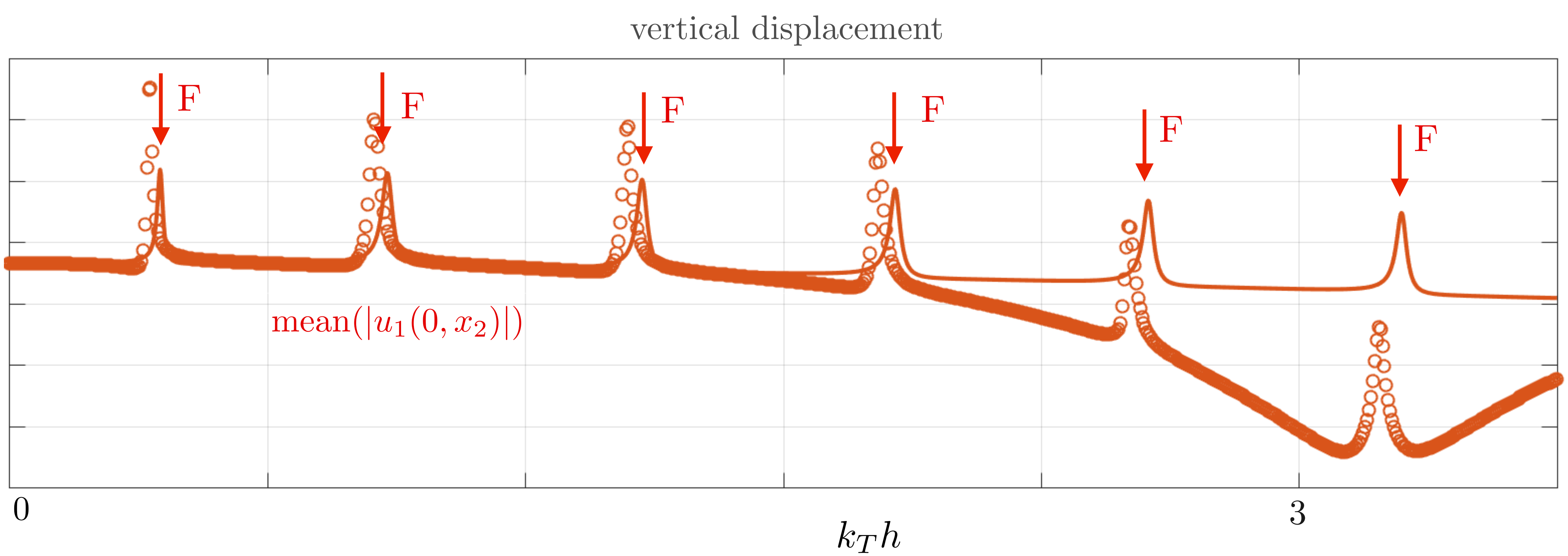

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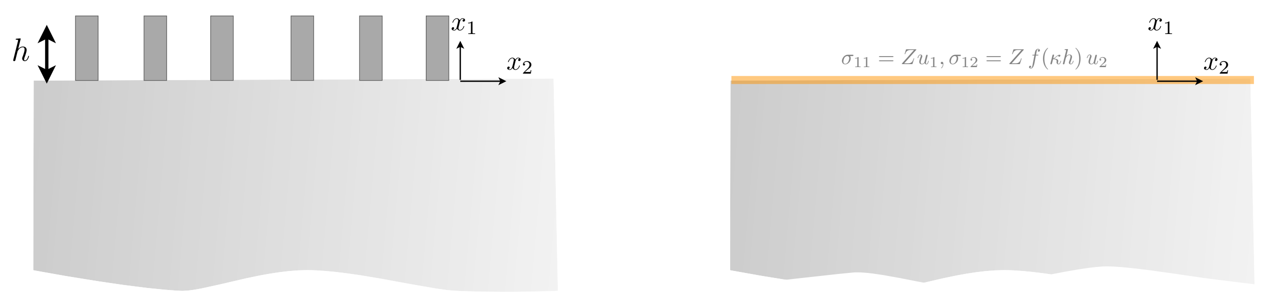
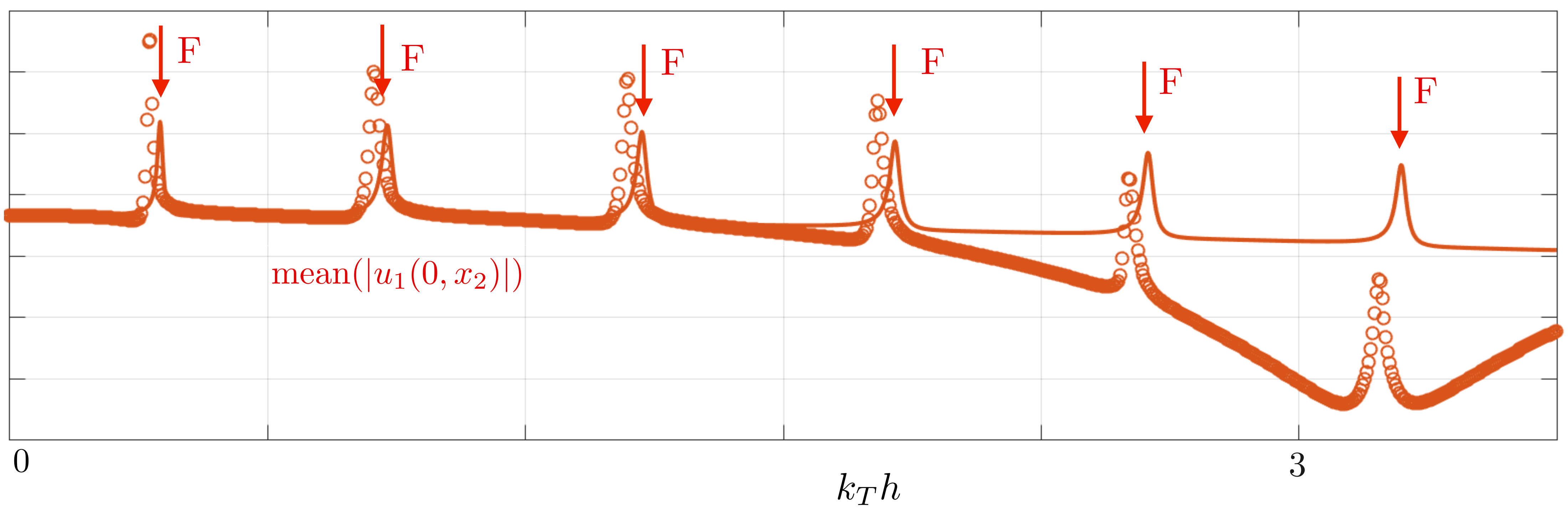


vertical displacement



superposition of flexural resonances   with the first longitudinal resonance

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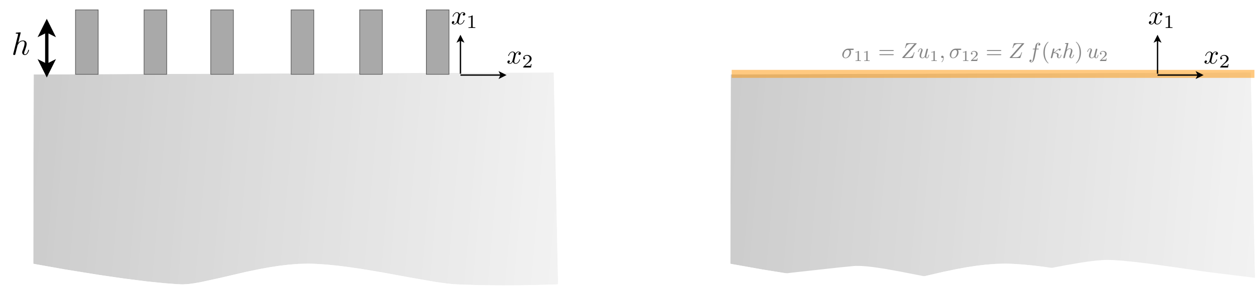

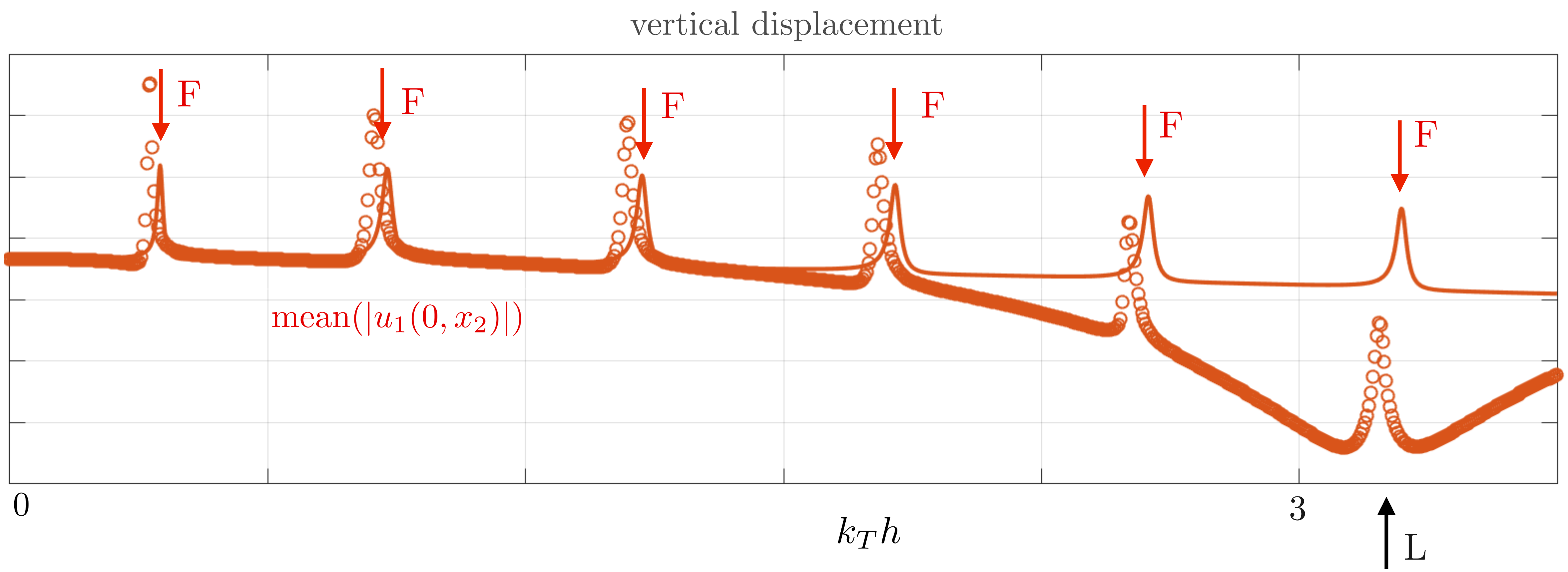

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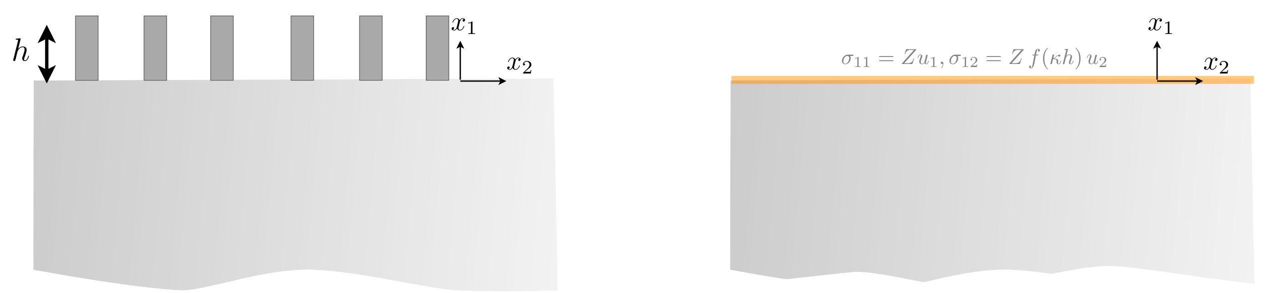

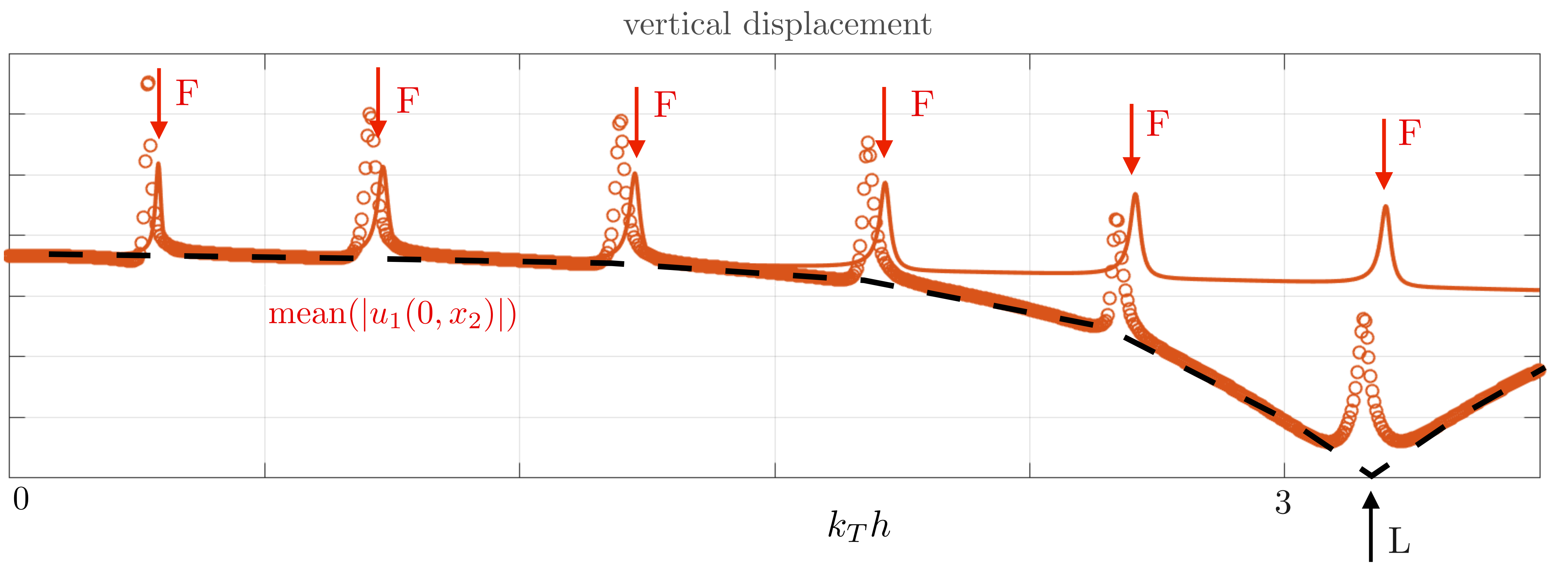

superposition of flexural resonances with the first longitudinal resonance

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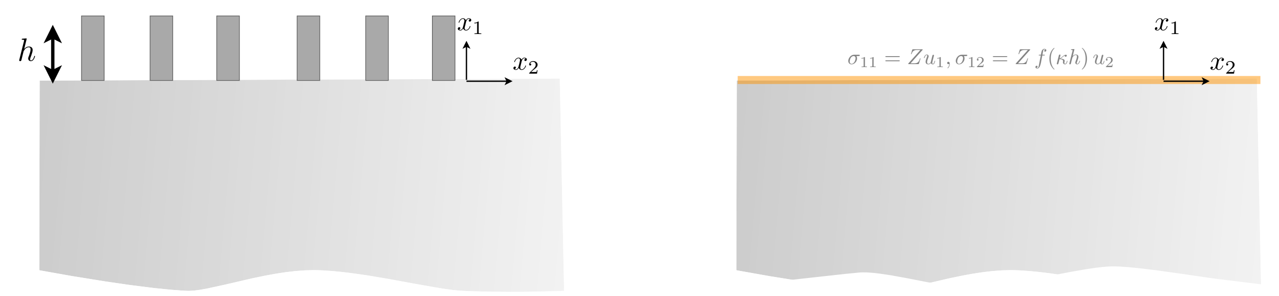

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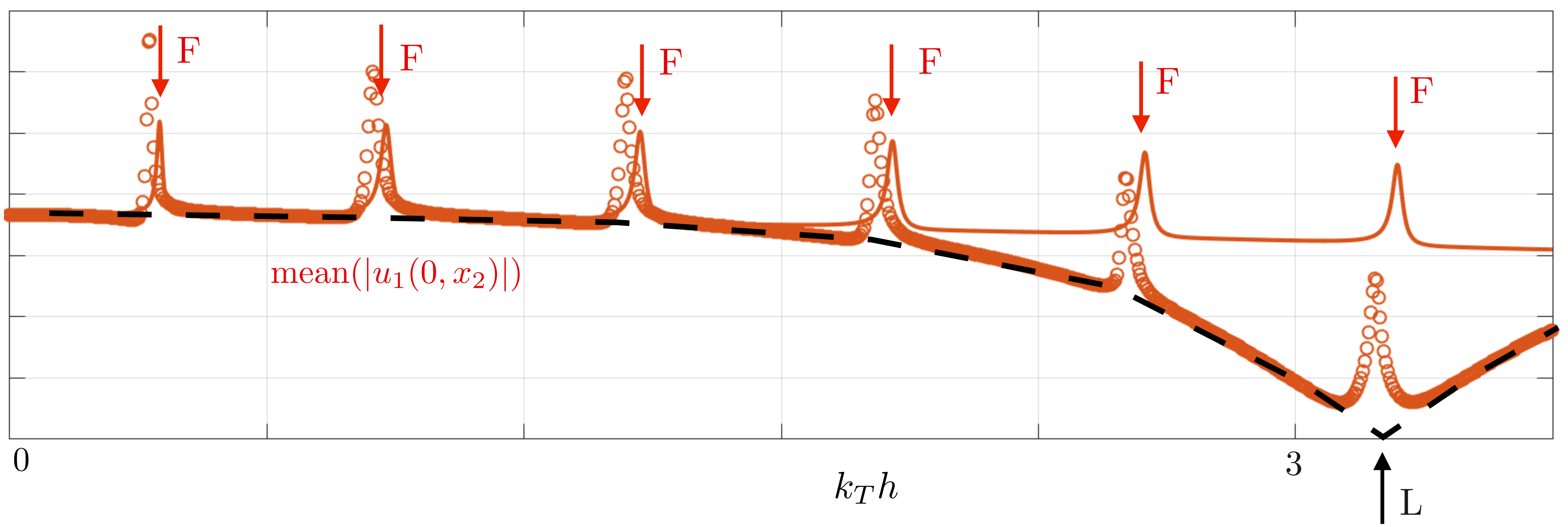

superposition of flexural resonances with the first longitudinal resonance

and to capture both F and L resonances, other scalings are required, slightly more demanding