

Transformation optics and its applications to antennas

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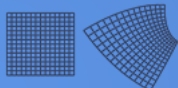
Stockholm, Sweden.

1



Outline

- **Theory:**
 - Transformation optics concept.
 - Types:
 - Analytical transformation.
 - Quasi-conformal transformation.
 - Non-Euclidean transformation.
- **Practice:**
 - Lens design.
 - Compressed lenses.
 - Planar Lenses.
 - Collimated lenses.
 - Bespoke lenses.
 - Surface propagation.
 - Cloaking.
 - Surface waves lensing.
- **Conclusions.**



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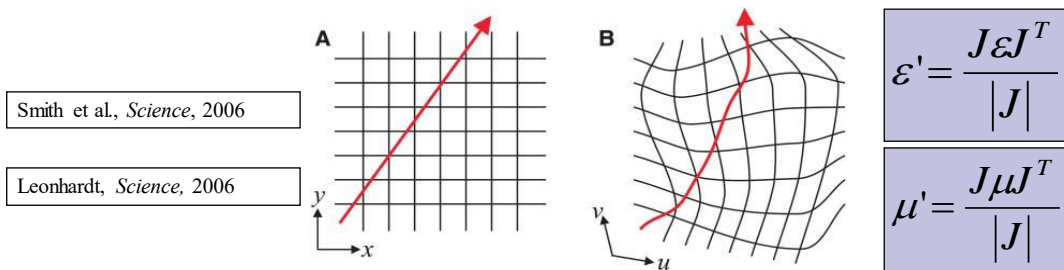
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Transformation Optics Concept

- In 2006 two pioneering papers were published in *Science* defining the concept of **transformation optics**:
 - According to the theory, any given electromagnetic device can be transformed into an infinite number of new ones with same electromagnetic responses.
- This tool has incredible possibilities to redesign classic devices.



3



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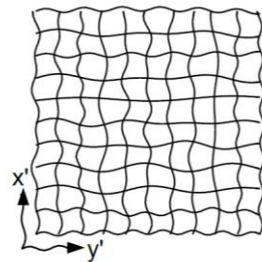
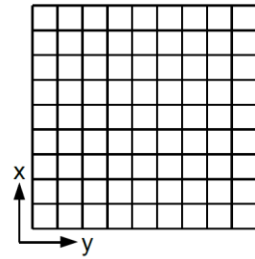


4



1) Analytical Transformation

- To make an analytical transformation taking into account all the components in Maxwell's equations.
- No approximations:
 - The physical space has the same response as the original virtual space.
- In practice, this idea is unaffordable:
 - Dispersive materials.
 - Anisotropic materials.



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5

5



1) Analytical Transformation: Dispersive materials and anisotropy

- Lets assume the simplest transformation:
 - Compression in one of coordinate axes.

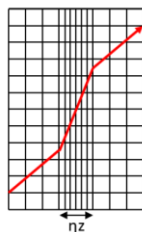
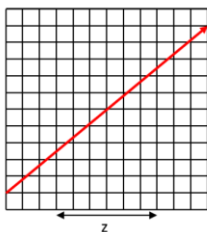
D. A. Roberts, N. Kundtz, and D. R. Smith, *Opt. Express*, 2009.

$$x'(x) = \begin{cases} ax + c, & l_1 < x < l_2 \\ l_1' + (x - l_1), & x \leq l_1 \\ l_2' + (x - l_2), & x \geq l_2 \end{cases}$$

- The new permittivity and permeability maps will be:

$$\epsilon' = \epsilon \begin{Bmatrix} a & 0 & 0 \\ 0 & 1/a & 0 \\ 0 & 0 & 1/a \end{Bmatrix}$$

$$\mu' = \mu \begin{Bmatrix} a & 0 & 0 \\ 0 & 1/a & 0 \\ 0 & 0 & 1/a \end{Bmatrix}$$



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6

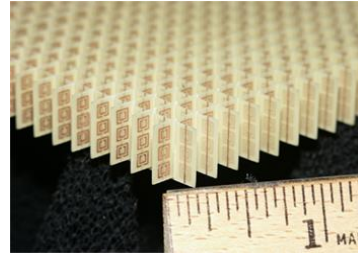
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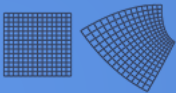
Dispersive materials

- Materials with refractive indexes lower than 1 can be only obtained with the use of metamaterials.
- Metamaterials are strongly dispersive:
 - Very limited bandwidth of operation.

$$\epsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - j\omega\gamma}$$



- To avoid the use of these materials is always an advantage for practical applications.



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7

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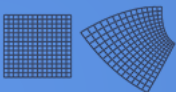
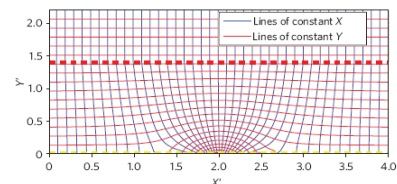
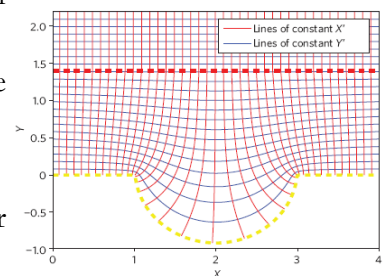


2) Discrete Transformation

- To make a transformation based on graphical coordinates.
- Importance of the coordinate lines to be orthogonal to the metallic boundaries.
- Transformation based on areas (not in shapes).
 - It does not take into consideration non-linear effects.

$$n' = n_0 \sqrt{\frac{\Delta x_0 \Delta y_0}{\Delta x'_0 \Delta y'_0}}$$

N. Kundtz and D.R. Smith,
Nature Materials, 2010.



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8

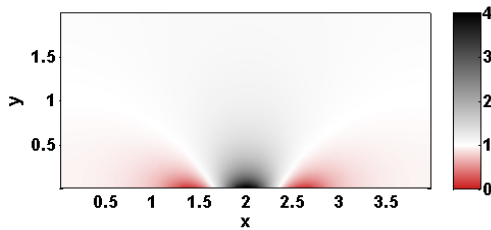
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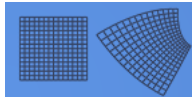
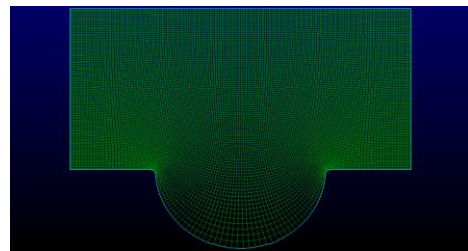
2) Discrete Transformation: Dispersive materials (I)

- Depending on the geometry, the required refractive index for the new map will have some lower than one index regions.
 - Lower than 1 refractive indexes require metamaterials implementations.
 - Dispersive materials and narrow band.

Physical Space



Virtual Space



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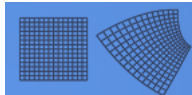
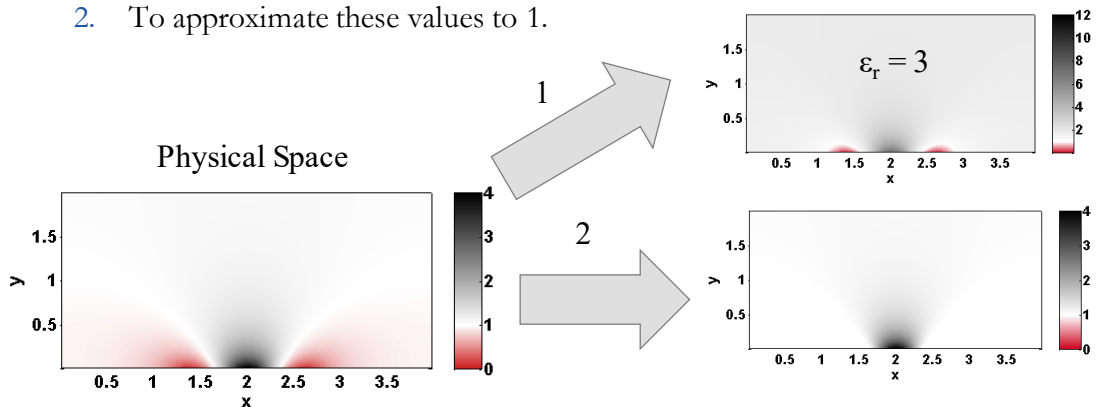
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2) Discrete Transformation: Dispersive materials (II)

- Two possible solutions:
 1. To develop the transformation over an original dense material.
 2. To approximate these values to 1.



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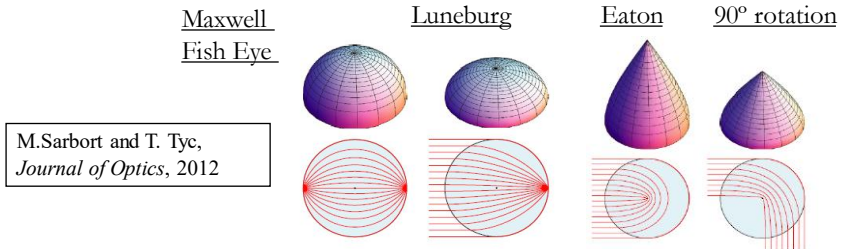
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3) Non-Euclidean transformation

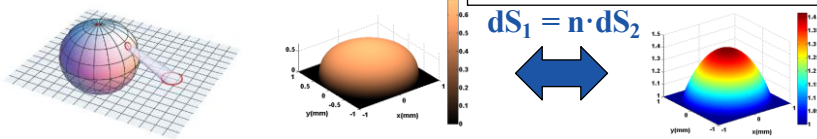
- To analyse the ray paths in a surface and to obtain the equivalent 2D plane which remains the same properties.



M.Sarbort and T. Tyc, *Journal of Optics*, 2012

U. Leonhardt and T. G. Philbin, *Geometry and Light: The Science of Invisibility* 2010.

- One method: Stereographic projection



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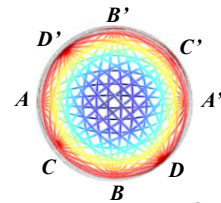
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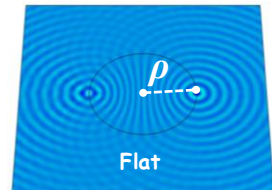
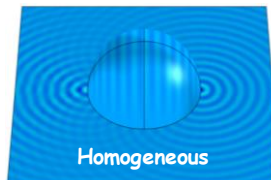
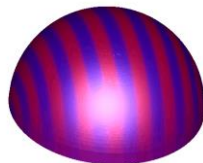
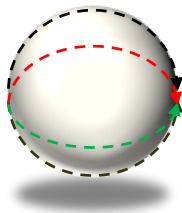
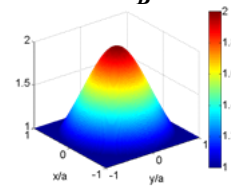


Maxwell Fish Eye Lens

- It is a lens in which a source in any excited point of the circle surface will converge exactly at the opposite size of the circle.
- It is a rotationally symmetric lens.
- Equivalent to an homogeneous sphere.



$$n(\rho) = \frac{2n_0}{1 + \left(\frac{\rho}{a}\right)^2}$$



R. C. Mitchell–Thomas, O. Quevedo–Teruel, T.M. McManus, S.A.R. Horsley, Y. Hao, *Optics Letters*, 2014.

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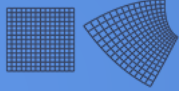
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12



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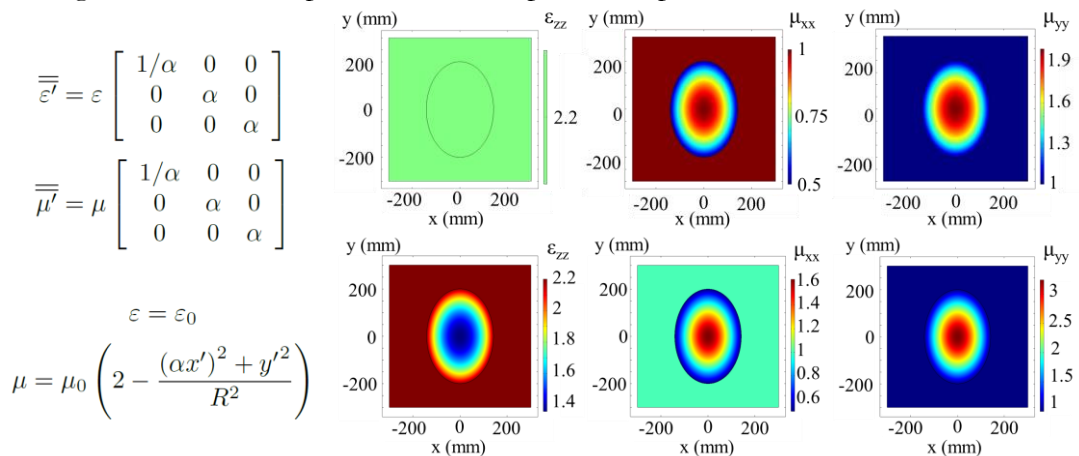
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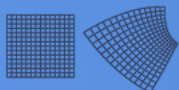


Compressed lenses: Transformation optics

- Using transformation optics, we can compress the space:



M. Ebrahimpouri, O. Quevedo-Teruel, "Ultra-wideband Anisotropic Glide-symmetric Metasurfaces", *IEEE Antennas and Wireless Propagation Letters*, vol. 18, no. 8, pp. 1547-1551, Aug. 2019.



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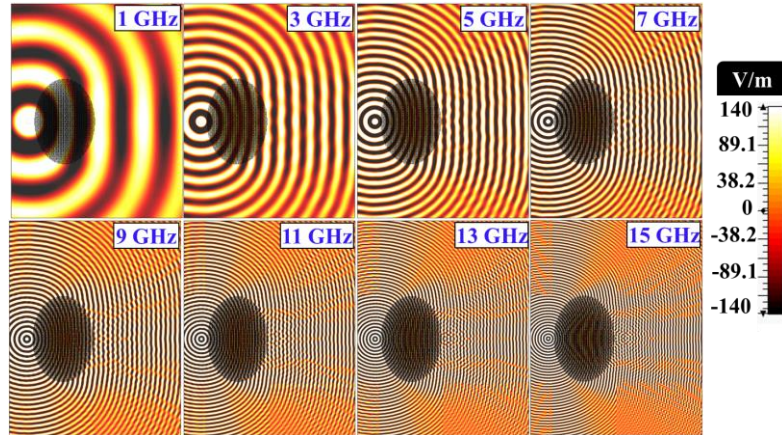
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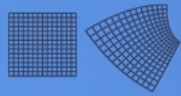


Compressed Luneburg lens

- Main direction:



M. Ebrahimpouri, O. Quevedo-Teruel, "Ultra-wideband Anisotropic Glide-symmetric Metasurfaces", *IEEE Antennas and Wireless Propagation Letters*, vol. 18, no. 8, pp. 1547-1551, Aug. 2019.



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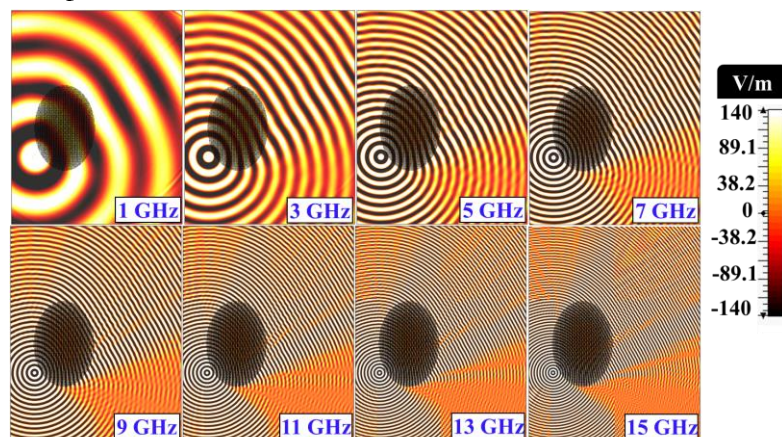
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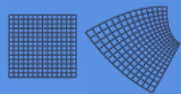


Compressed Luneburg lens

- Oblique radiation pattern:



M. Ebrahimpouri, O. Quevedo-Teruel, "Ultra-wideband Anisotropic Glide-symmetric Metasurfaces", *IEEE Antennas and Wireless Propagation Letters*, vol. 18, no. 8, pp. 1547-1551, Aug. 2019.



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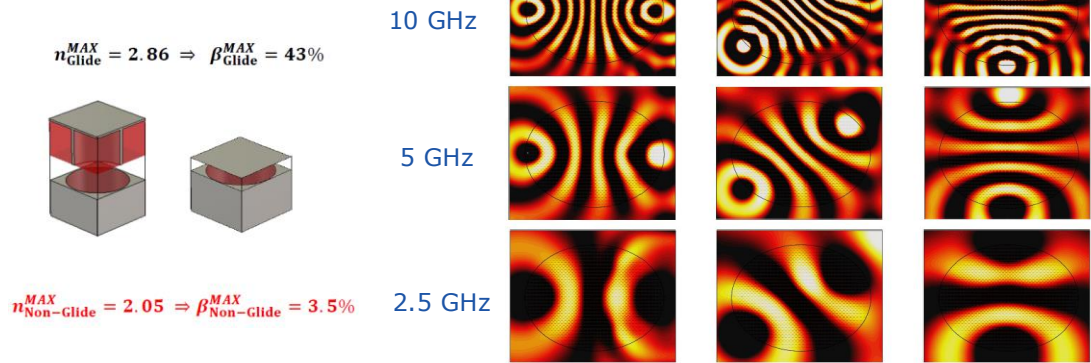
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16



Mode-matching: Anisotropy

- Lens compression:



- A. Alex-Amor, F. Ghasemifard, G. Valerio, P. Padilla, J. M. Fernandez-Gonzalez, O. Quevedo-Teruel, "Glide-Symmetric Metallic Structures with Elliptical Holes", *submitted to IEEE Transactions on Microwave Theory and Techniques*.



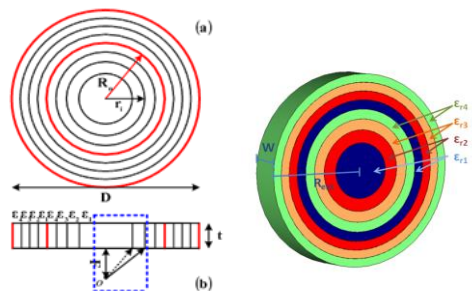
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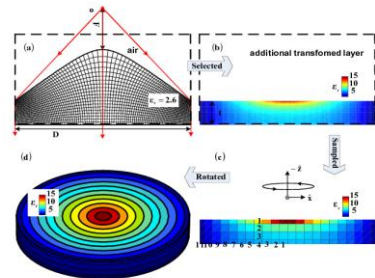
Planar lenses

- Option 1:
 - Fresnel Lenses
 - Single frequency of operation

A. Petosa, et al. *IEE Proc., Microw. Antennas Propag.* 2006.



- Option 2:
 - Transformation Electromagnetics
 - UWB solution.

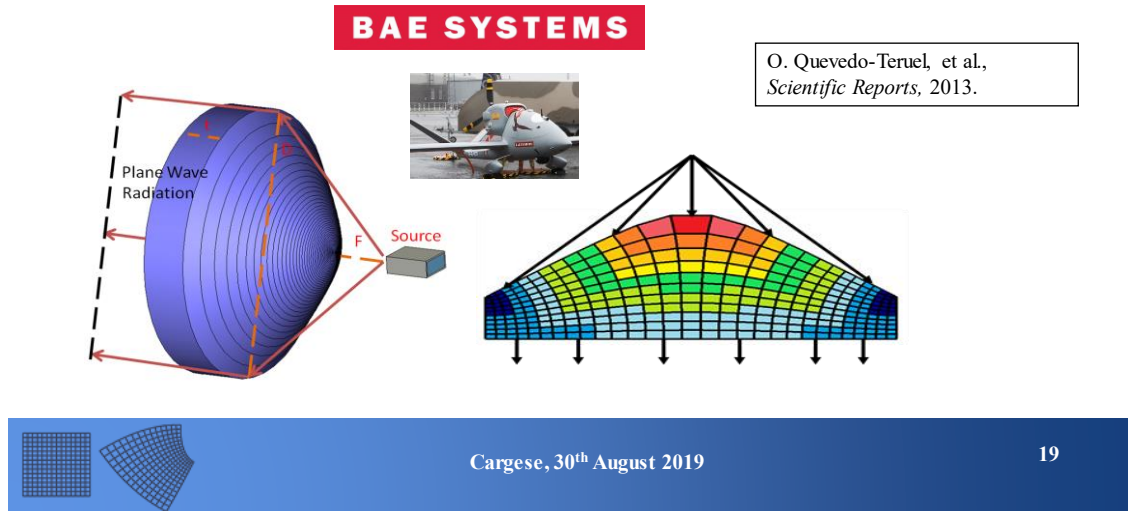


18



Design Process

- Quasi-conformal Transformation Optics.
- A discretization for the manufacturing process is required.



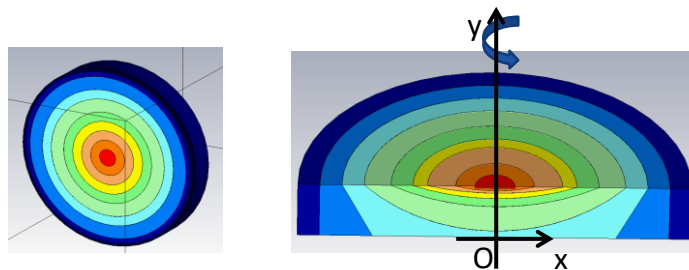
19



Discretization process

O. Quevedo-Teruel, et al., *Scientific Reports*, 2013.

- Spheres/Ellipsoids:
 - Ellipsoidal/spherical discretization along the iso-permittivity lines.
 - Zones of different permittivities:
 - $2 < \epsilon_r < 14.5$
 - 10-15 zones



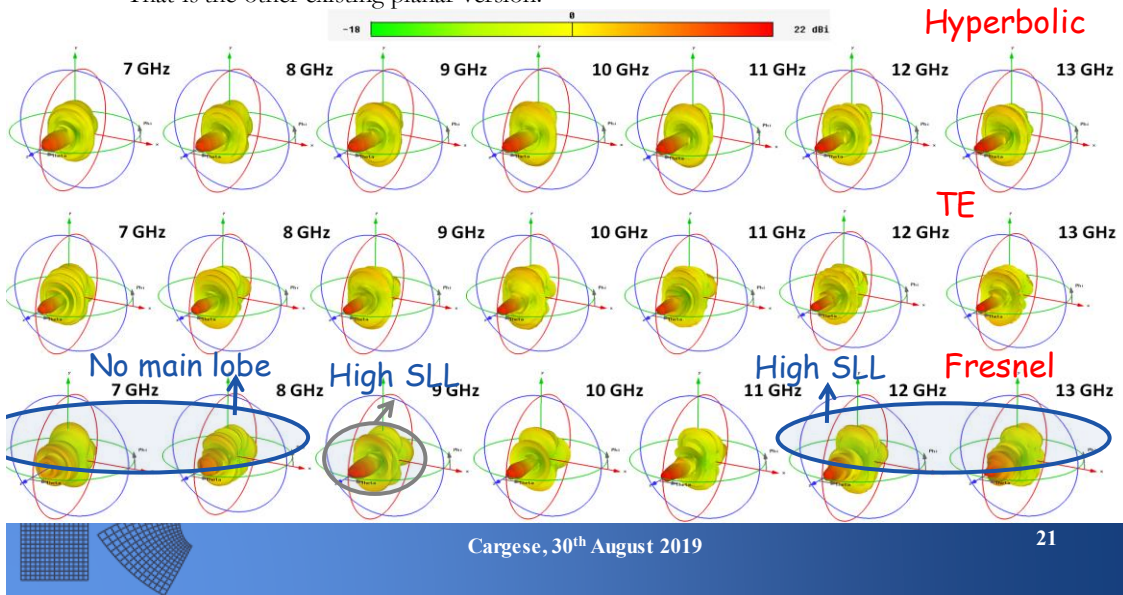
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O. Quevedo-Teruel, et al., *Scientific Reports*, 2013.

Comparison with Fresnel lens

- Our solution overcomes Fresnel lenses in Bandwidth.
 - That is the other existing planar version.



21

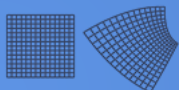
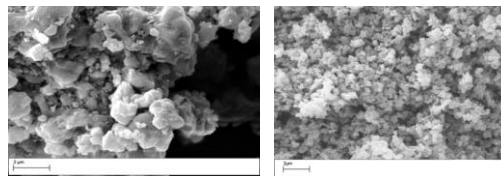


O. Quevedo-Teruel, et al., *Scientific Reports*, 2013.

Manufacturing

- Alternative permittivity regions have been produced through a combination of tailoring:
 1. Particle size.
 2. Dispersion and volume fraction of materials.
- The particle sizes were obtained using particle size reduction methods such as milling to achieve the distribution of sizes required, and these varied from nano- to micron size.

BAE SYSTEMS



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22

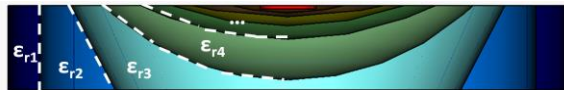
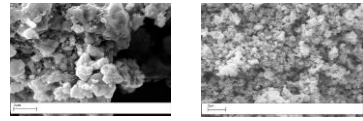
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Stability with Frequency

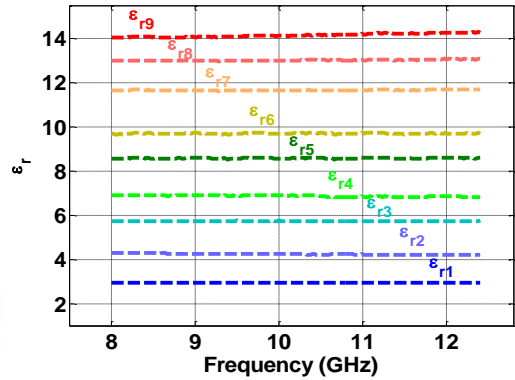
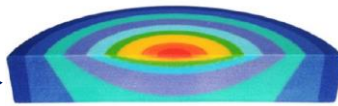
O. Quevedo-Teruel, et al., *Scientific Reports*, 2013.

- The materials have a constant response with frequency (from 8GHz to 13GHz).



Layers distributions

Prototype



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23

23

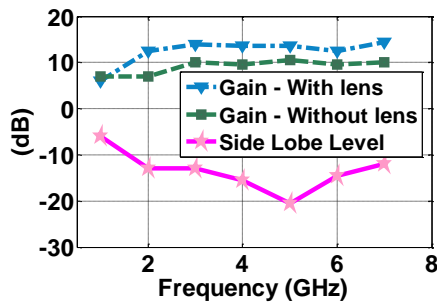
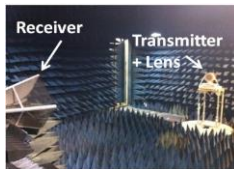


Measurements

O. Quevedo-Teruel, et al., *Scientific Reports*, 2013.

- Low frequency (1-7GHz):

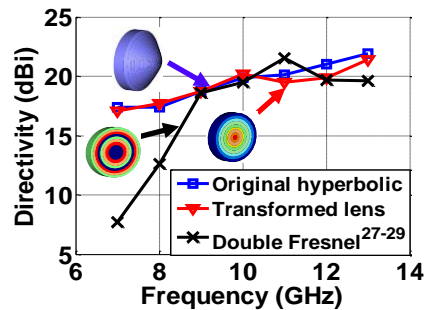
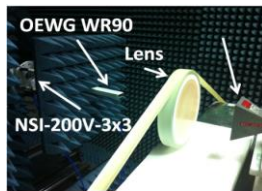
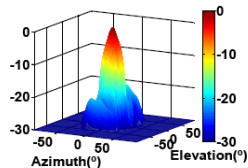
- Anechoic chamber.



- High frequency (7-14GHz):

- Near Field Scanner

- NSI-200V-3x3



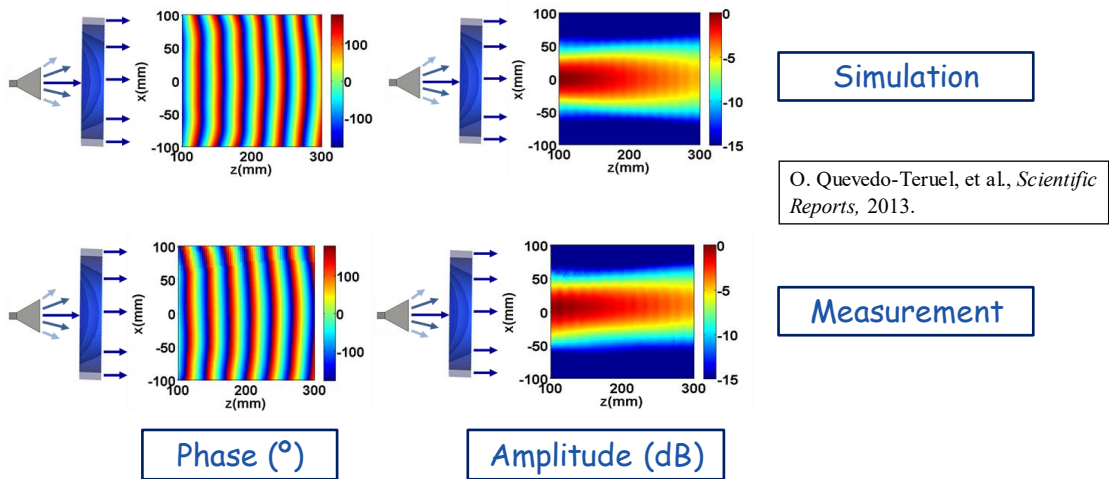
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24

24



Simulation vs Measurements

- Good agreement in terms of phase and amplitude (10 GHz)



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25

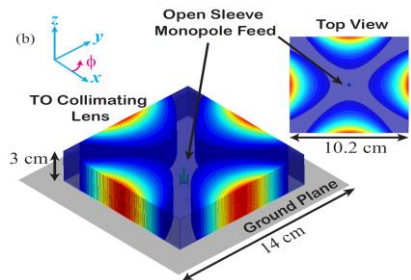
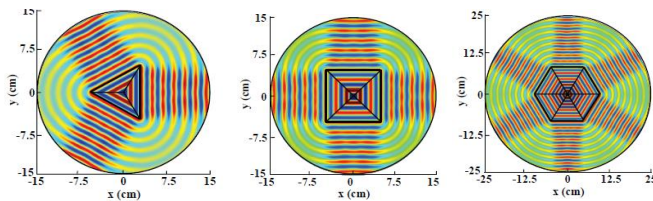
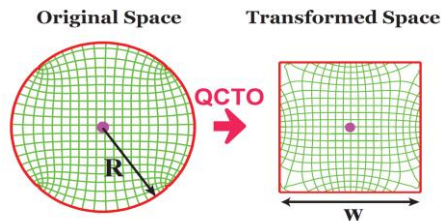
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Collimated lenses

- Quasi-conformal transformation to obtain collimated lenses: multi-beams.

Q. Wu, Z. H. Jiang, O. Quevedo-Teruel, J. P. Turpin, W. Tang, Y. Hao, D. H. Werner, *IEEE Transactions on Antennas and Propagation*, vol.61, no.12, pp.5910-5922, Dec. 2013.



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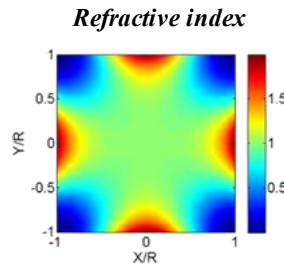
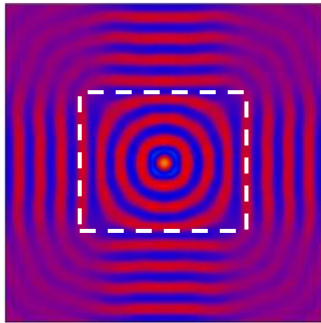
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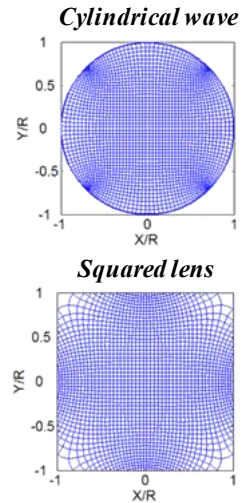


From cylindrical to squared waves

- Optical transformations can be used to create completely new type of lenses.
 - Transformation of a cylindrical wave in four directive beams.



R. C. Mitchell-Thomas, M. Ebrahimpouri and O. Quevedo-Teruel, *Eucap 2015*.



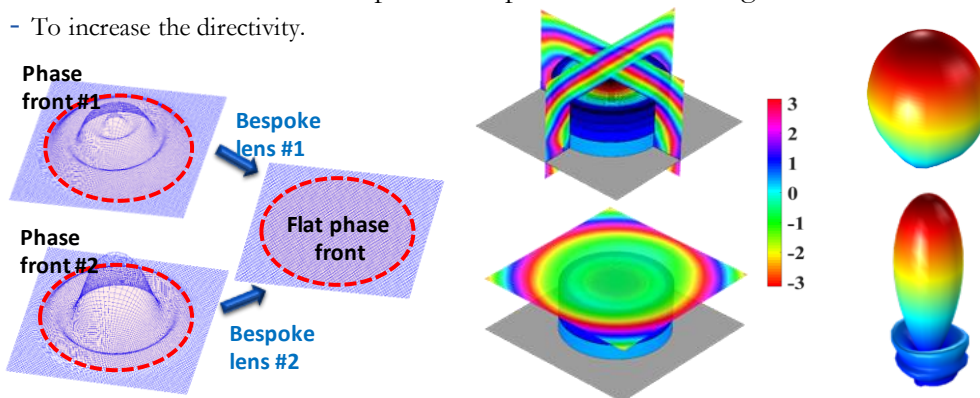
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27

27



Bespoke lenses

- To produce ad-hoc lens for practical feedings.
- Most common situation is to produce a plane wave from a given radiator:
 - To increase the directivity.



M. McCall, J. Pendry, V. Galdi, Y. Lai, S. Horsley, J. Li, J. Zhu, R. C. Mitchell-Thomas, O. Quevedo-Teruel, P. Tassin, V. Ginis, E. Martini, G. Minatti, S. Maci, M. Ebrahimpouri, Y. Hao, P. Kinsler, J. Gratus, J. Lukens, A. M. Weiner, U. Leonhardt, I. Smolyaninov, V. Smolyanova, R. Thompson, M. Wegener, M. Kadic, S. Cummer, "Roadmap on transformation optics", *Journal of Optics*, Volume 20, Number 6, May 2018.

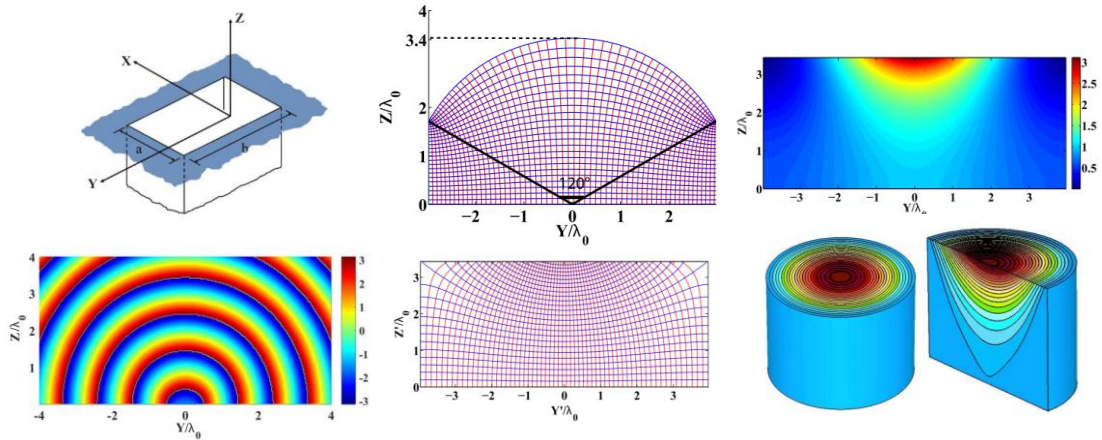
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28

28



Bespoke lenses: Aperture antenna (I)

- Technique: Obtaining the map for the lens.



M. Ebrahimpouri, O. Quevedo-Teruel, "Bespoke Lenses Based on Quasi Conformal Transformation Optics Technique", *IEEE Transactions on Antennas and Propagation*, vol. 65, no. 5, pp. 2256-2264, May 2017.

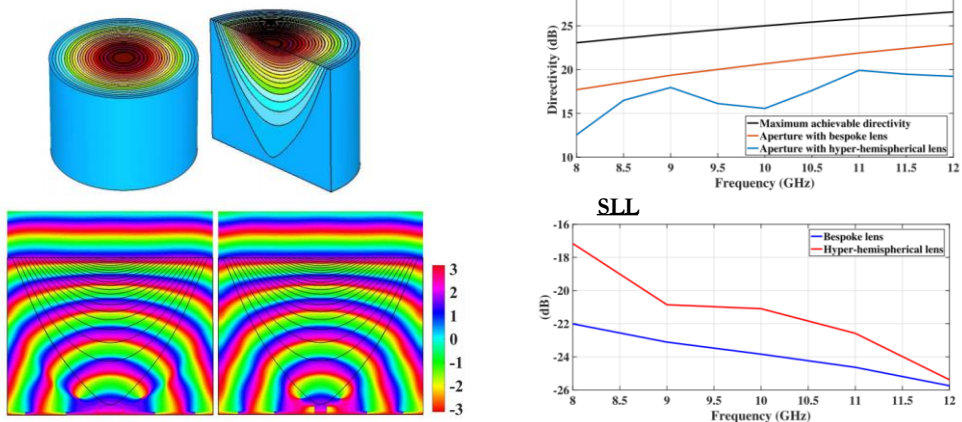
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29

29



Bespoke lenses: Aperture Antenna (II)

- Technique: Final design and results.



M. Ebrahimpouri, O. Quevedo-Teruel, "Bespoke Lenses Based on Quasi Conformal Transformation Optics Technique", *IEEE Transactions on Antennas and Propagation*, vol. 65, no. 5, pp. 2256-2264, May 2017.

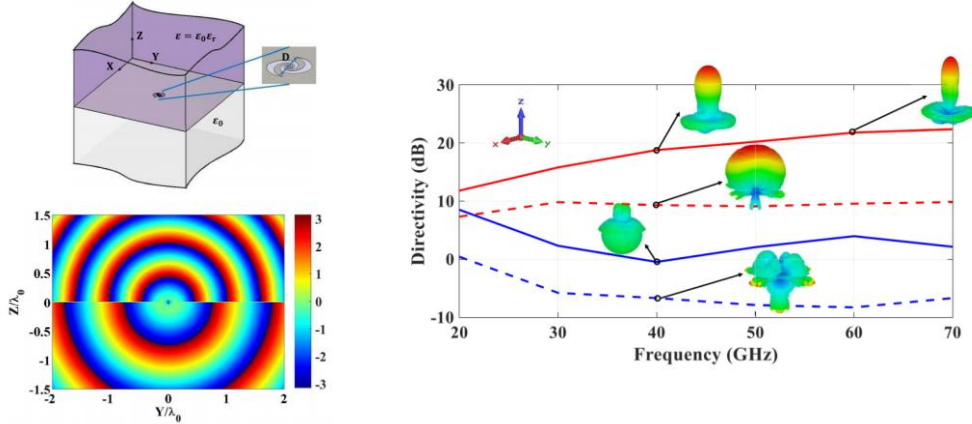
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30

30



Bespoke lenses: Spiral antenna

- The lens is not limited in bandwidth and it applies to both polarizations:



M. Ebrahimpouri, O. Quevedo-Teruel, "Bespoke Lenses Based on Quasi Conformal Transformation Optics Technique", *IEEE Transactions on Antennas and Propagation*, vol. 65, no. 5, pp. 2256-2264, May 2017.

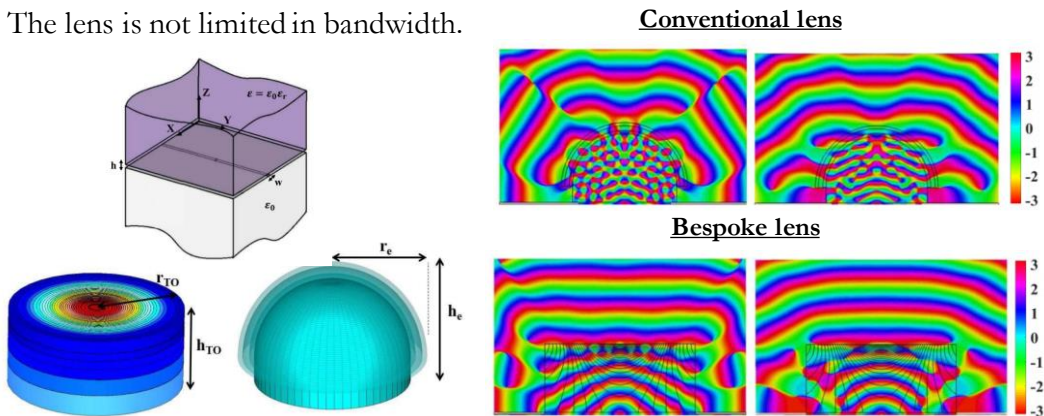
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31

31



Bespoke lenses: Slot lines

- The lens is not limited in bandwidth.



M. Ebrahimpouri, O. Quevedo-Teruel, "Bespoke Lenses Based on Quasi Conformal Transformation Optics Technique", *IEEE Transactions on Antennas and Propagation*, vol. 65, no. 5, pp. 2256-2264, May 2017.

A. Neto, "UWB, non dispersive radiation from the Planarly fed leaky lens antenna—Part 1: Theory and design," *IEEE Trans. Antennas Propag.*, vol. 58, no. 7, pp. 2238-2247, Jul. 2010.

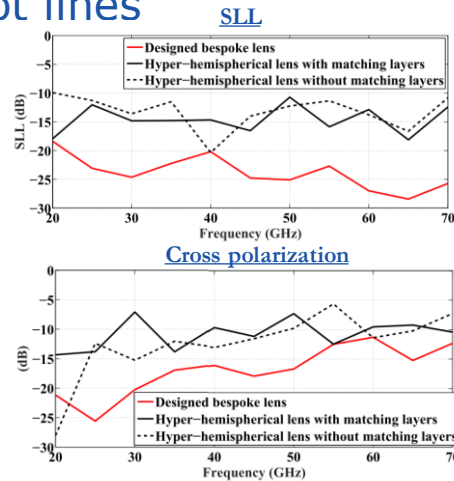
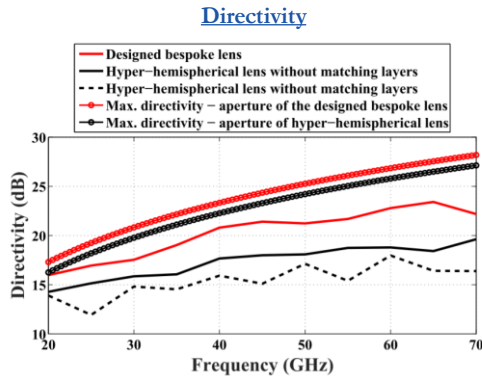
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32

32



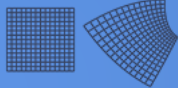
Bespoke lenses: Slot lines

- Figures of merit.



M. Ebrahimpouri, O. Quevedo-Teruel, "Bespoke Lenses Based on Quasi Conformal Transformation Optics Technique", *IEEE Transactions on Antennas and Propagation*, vol. 65, no. 5, pp. 2256-2264, May 2017.

A. Neto, "UWB, non dispersive radiation from the Planarly fed leaky lens antenna—Part 1: Theory and design," *IEEE Trans. Antennas Propag.*, vol. 58, no. 7, pp. 2238–2247, Jul. 2010.



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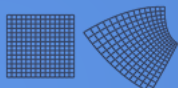
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33



Outline

- Theory:
 - Transformation optics concept.
 - Types:
 - Analytical transformation.
 - Quasi-conformal transformation.
 - Non-Euclidean transformation.
- Practice:
 - Lens design.
 - Conformal lenses.
 - Compressed lenses.
 - Planar Lenses.
 - Collimated lenses.
 - Bespoke lenses.
 - Surface propagation.
 - Cloaking.
 - Surface waves lensing.
- Conclusions.



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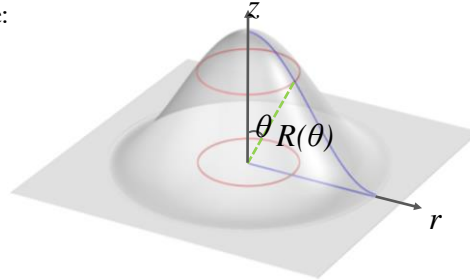
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Theory

- The refractive index distribution of a rotationally symmetric curved surface can mimic an equivalent flat homogeneous surface:

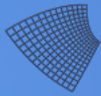


$$\int_0^{\pi/2} n(\theta) \sqrt{R(\theta)^2 + R'(\theta)^2} d\theta = \int_0^a dr$$

$$2\pi R(\theta) \sin(\theta) n(\theta) = 2\pi r$$

$$\frac{n'(\theta)}{n(\theta)} = \frac{\sqrt{R(\theta)^2 + R'(\theta)^2} - R'(\theta) \sin \theta - R(\theta) \cos \theta}{R(\theta) \sin \theta}$$

R. C. Mitchell-Thomas, T.M. McManus, O. Quevedo-Teruel, S.A.R. Horsley, Y. Hao, "Perfect Surface Wave Cloaks", *Physical Review Letters*, vol. 111, p. 213901, Nov 2013.



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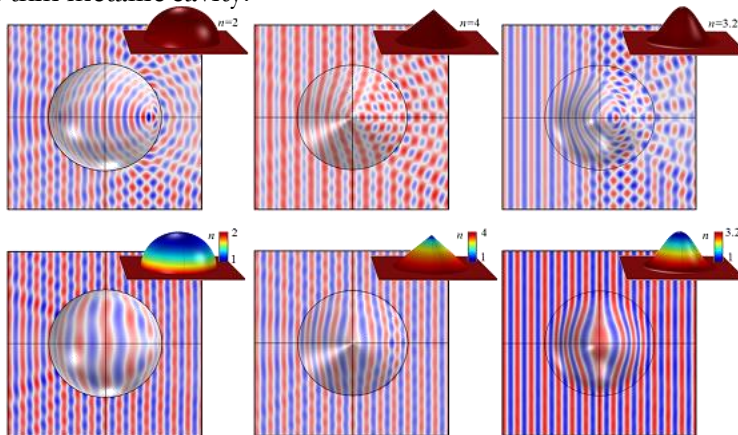
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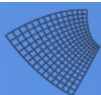


Results

- Cloaking in a thin metallic cavity:



R. C. Mitchell-Thomas, T.M. McManus, O. Quevedo-Teruel, S.A.R. Horsley, Y. Hao, "Perfect Surface Wave Cloaks", *Physical Review Letters*, vol. 111, p. 213901, Nov 2013.



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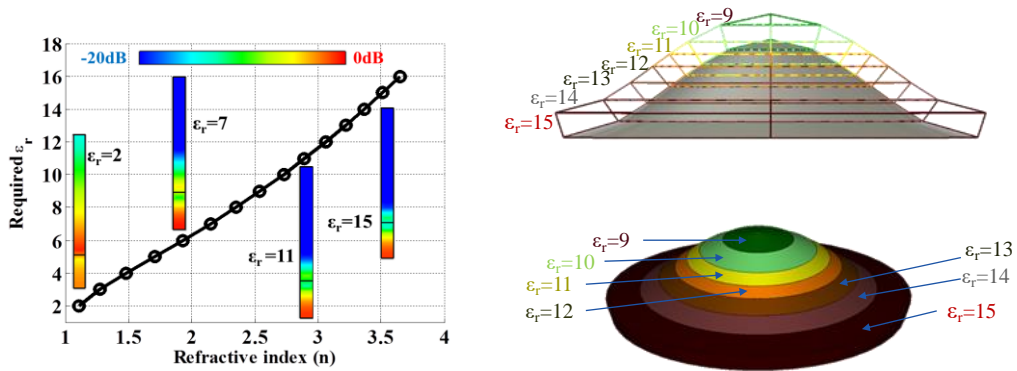
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36

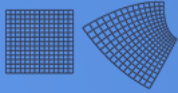


Surface waves implementation

- Slabs of different dielectric constants with constant thickness.



R. C. Mitchell-Thomas, T.M. McManus, O. Quevedo-Teruel, S.A.R. Horsley, Y. Hao, "Perfect Surface Wave Cloaks", *Physical Review Letters*, vol. 111, p. 213901, Nov 2013.



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37

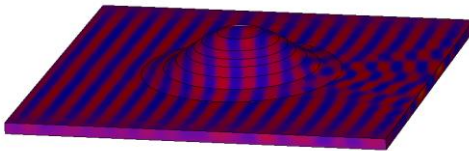
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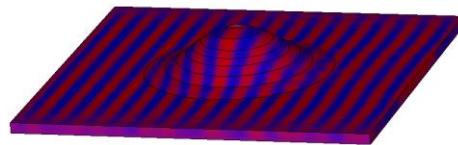
Surface waves results

- Very robust design.
- Only 7 layers have been used.
- Small deviations in the manufacturing process would not seriously influence the performance.

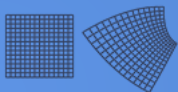
Uniform material



After cloaking



R. C. Mitchell-Thomas, T.M. McManus, O. Quevedo-Teruel, S.A.R. Horsley, Y. Hao, "Perfect Surface Wave Cloaks", *Physical Review Letters*, vol. 111, p. 213901, Nov 2013.



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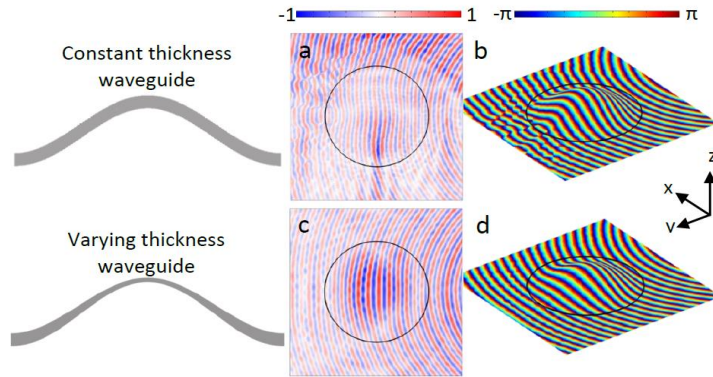
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38



Experimental results

- Implementation with a single dielectric slab which changes with the position to achieve different equivalent refractive indexes.



R. Mitchell-Thomas, O. Quevedo-Teruel, J. R. Sambles, A. P. Hibbins, "Omnidirectional surface wave cloak using an isotropic homogeneous dielectric coating" *Scientific Reports*, vol. 6, article number 30984, 2016.

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39

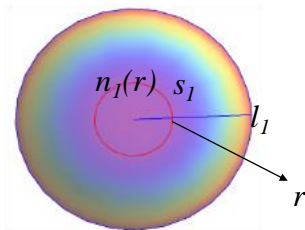
39



Non-Euclidian mapping

- By equating optical path lengths, it is possible to calculate the refractive index of a lens on a curved surface.

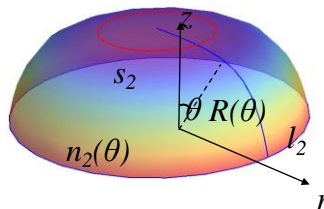
Flat space



$$s_1 = 2\pi n_1(r)$$

$$l_1 = \int_0^a n_1(r) dr$$

Arbitrarily curved space



R.C. Mitchell-Thomas, O. Quevedo-Teruel, T.M. McManus, S.A.R. Horsley, and Y. Hao, "Lenses on Curved Surfaces", *Optics Letters*, vol. 39, pp. 3551-3554, 2014.

$$\int_0^{\pi/2} n_2(\theta) \sqrt{R(\theta)^2 + R'(\theta)^2} d\theta = \int_0^a n_1(r) dr$$

$$2\pi \cdot R(\theta) \cdot n_2(\theta) \sin(\theta) = 2\pi \cdot r \cdot n_1(r)$$

$$\frac{n_2(\theta)}{n_2(\theta)} = \frac{\left(1 + \frac{n_1'(r)}{n_1(r)}\right) \sqrt{R(\theta)^2 + R'(\theta)^2} - R'(\theta) \sin(\theta) - R(\theta) \cos(\theta)}{R(\theta) \sin(\theta)}$$

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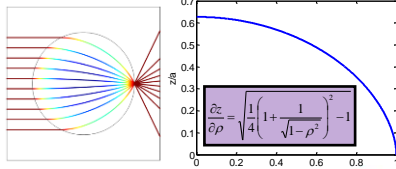
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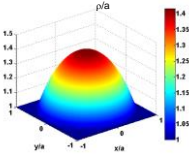


Luneburg lens examples

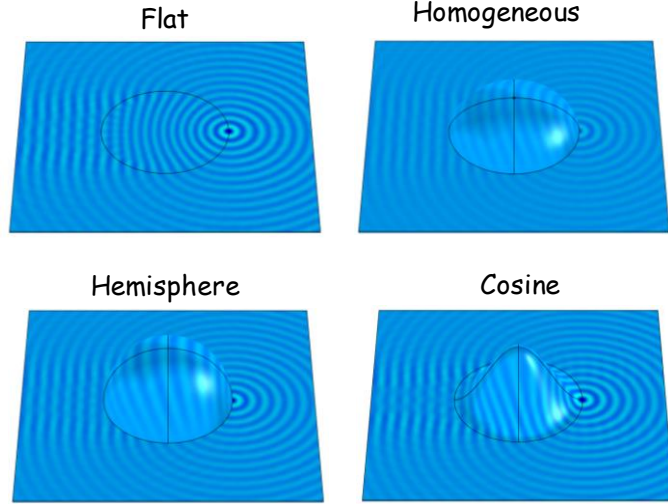
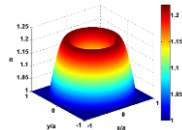
M. Sarbort and T. Tyc, *Journal of Optics*, 2012.



$$n(r) = \sqrt{2 - \left(\frac{\rho}{a}\right)^2}$$



$$n(\theta) = \frac{\sqrt{1+3\cos(\theta)}}{(1+\cos(\theta))^{3/2}}$$



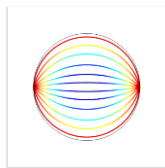
R.C. Mitchell-Thomas, O. Quevedo-Teruel, T.M. McManus, S.A.R. Horsley, and Y. Hao, "Lenses on Curved Surfaces", *Optics Letters*, vol. 39, pp. 3551-3554, 2014.

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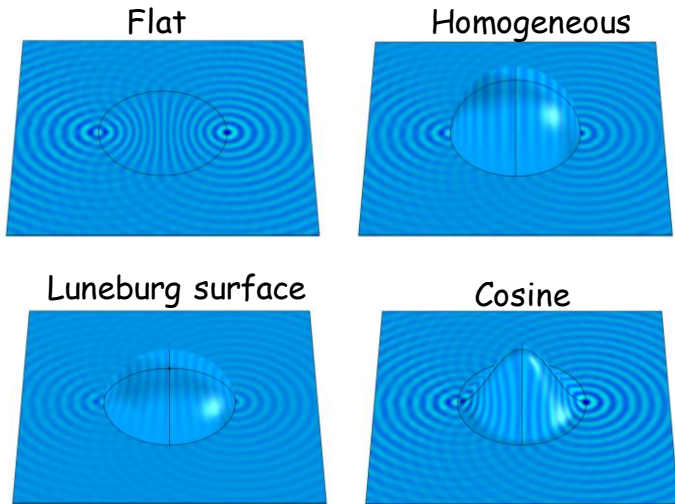
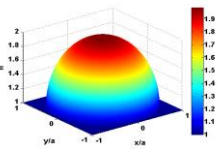
41



Maxwell fish eye lens examples



$$n(r) = \frac{n_0}{1+r^2}$$



R.C. Mitchell-Thomas, O. Quevedo-Teruel, T.M. McManus, S.A.R. Horsley, and Y. Hao, "Lenses on Curved Surfaces", *Optics Letters*, vol. 39, pp. 3551-3554, 2014.

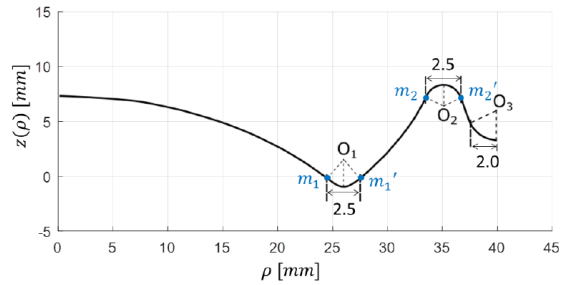
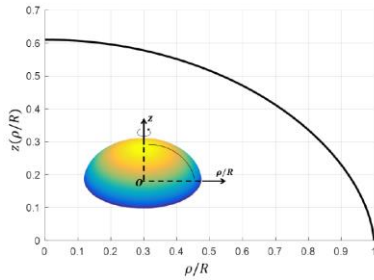
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42

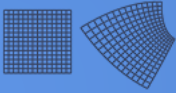


Lens implementation

- A Luneburg lens can be implemented with this technique.
- The surface can be bended to reduce the vertical dimension.



Q. Liao, N.J.G. Fonseca, O. Quevedo-Teruel, "Compact Multibeam Fully Metallic Geodesic Luneburg Lens Antenna Based on Non-Euclidean Transformation Optics", *IEEE Transactions on Antennas and Propagation*, vol. 66, no. 12, pp. 7383-7388, Dec. 2018.



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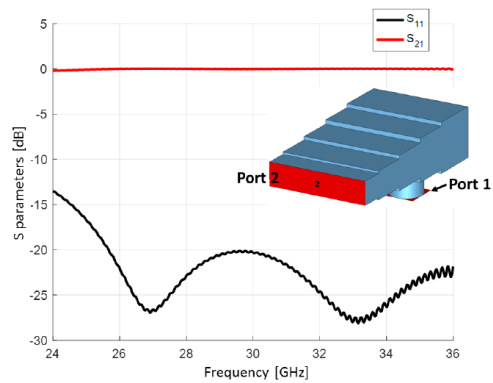
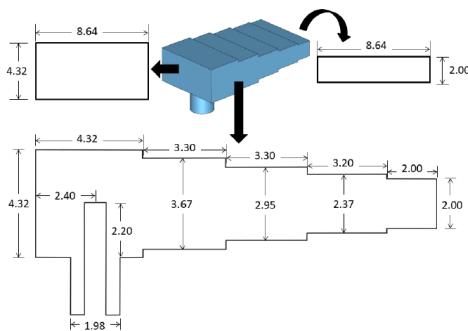
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43

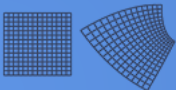


Antenna design

- Waveguides as connectors.
- Optimized flare design.



Q. Liao, N.J.G. Fonseca, O. Quevedo-Teruel, "Compact Multibeam Fully Metallic Geodesic Luneburg Lens Antenna Based on Non-Euclidean Transformation Optics", *IEEE Transactions on Antennas and Propagation*, vol. 66, no. 12, pp. 7383-7388, Dec. 2018.



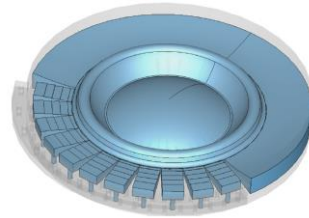
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44

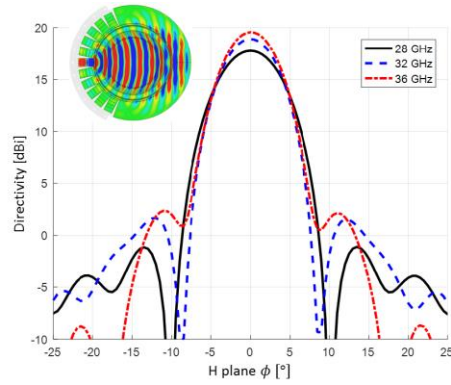
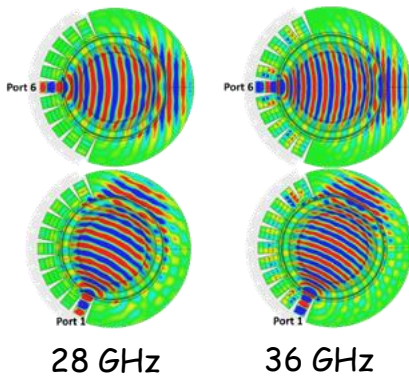
44



Luneburg lens antenna



- The antenna is fed with coaxial ports.
- Each port excitation produces a different beam.



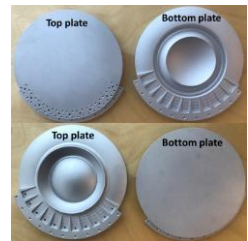
Q. Liao, N.J.G. Fonseca, O. Quevedo-Teruel, "Compact Multibeam Fully Metallic Geodesic Luneburg Lens Antenna Based on Non-Euclidean Transformation Optics", *IEEE Transactions on Antennas and Propagation*, vol. 66, no. 12, pp. 7383-7388, Dec. 2018.

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45

45

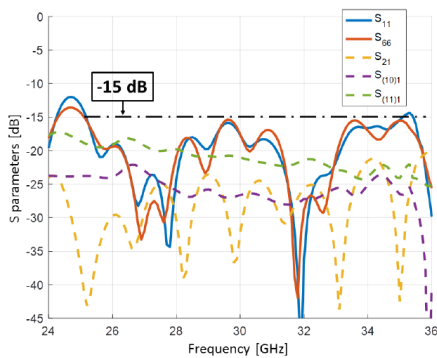


Matching

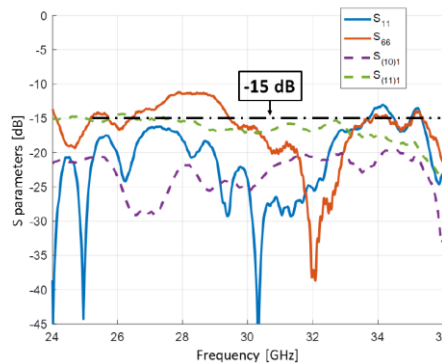


- Slightly higher mismatch in measurements

Simulations



Measurements



Q. Liao, N.J.G. Fonseca, O. Quevedo-Teruel, "Compact Multibeam Fully Metallic Geodesic Luneburg Lens Antenna Based on Non-Euclidean Transformation Optics", *IEEE Transactions on Antennas and Propagation*, vol. 66, no. 12, pp. 7383-7388, Dec. 2018.

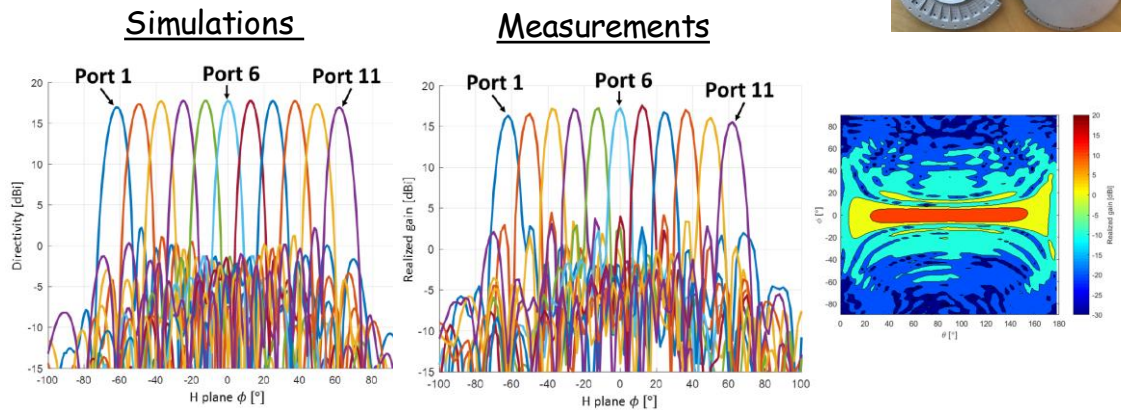
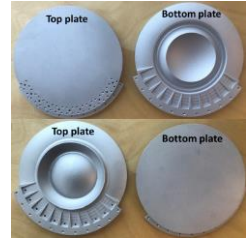
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46

46

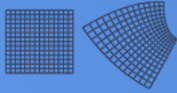


Radiation patterns

- Good agreement with simulations



Q. Liao, N.J.G. Fonseca, O. Quevedo-Teruel, "Compact Multibeam Fully Metallic Geodesic Luneburg Lens Antenna Based on Non-Euclidean Transformation Optics", *IEEE Transactions on Antennas and Propagation*, vol. 66, no. 12, pp. 7383-7388, Dec. 2018.



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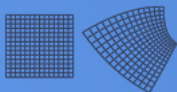
47

47



Outline

- Theory:
 - Transformation optics concept.
 - Types:
 - Analytical transformation.
 - Quasi-conformal transformation.
 - Non-Euclidean transformation.
- Practice:
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 - Conformal lenses.
 - Compressed lenses.
 - Planar Lenses.
 - Collimated lenses.
 - Bespoke lenses.
 - Surface propagation.
 - Cloaking.
 - Surface waves lensing.
- **Conclusions.**



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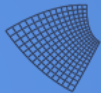
48

48



Conclusions

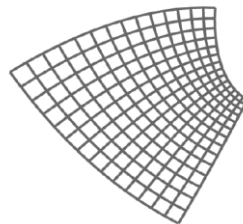
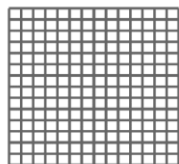
- The concept of Transformation Optics has been introduced.
- Three possible methodologies: **Euclidean** (**analytical** and **discrete**) and **non-Euclidean** have been drawn, and their advantages and disadvantages have been summarized.
- Few examples of design has been introduced:
 1. Conformal lenses:
 - They can be used to design lenses which **bespoke** surfaces.
 2. Planar lenses:
 - The use of metamaterials is not necessary for this design.
 - Measurements corroborate the original results.
 3. Surface propagation:
 - **Cloaking** has been demonstrating to be obtain with only full dielectric materials.
 - This technique can be employed to produce **lenses conformal** to surfaces and to **eliminate singularities** of lenses.



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49

49



Transformation optics and its applications to antennas

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<http://www.etk.ee.kth.se/person/oscarqt>

50