Buckling cascades in free sheets

Wavy leaves may not depend only on their genes to make their edges crinkle.

The edge of a torn plastic sheet forms a complex three-dimensional fractal shape. We have found that the shape results from a simple elongation of the sheet in the direction along its edge. Natural growth processes in some leaves, flowers and vesicles could lead to a similar elongation and hence to the generation of characteristic wavy shapes.

We used rectangular plastic sheets pulled from the sides (in the y-direction) to generate a steadily travelling crack (in the x-direction). The high stresses near the crack tip produce an irreversible plastic deformation of the sheet and, as they are relieved, the deformed sheet is free to relax and to adopt a new shape in space.

Surprisingly, the equilibrium shape of the sheet consists of a cascade of waves upon waves along the newly formed edge. The waveform along the edge examined at six levels of magnification is self-similar, with a scaling factor of 3.2 (Fig. 1a). The amplitudes, A, of the waves in the cascades are simply related to their wavelengths λ by $A = 0.15\lambda$ (Fig. 1b); the edge of the sheet is therefore a fractal. The fractal scaling spans 2.5 orders of magnitude and stops at a small length scale that is 6.5 times the sheet's thickness. Measurements of the deformation field revealed an increase in the length of the sheet along its edge — elasticity takes care of the rest.

Thin films

Wrinkling of an elastic sheet under tension

Here we consider the wrinkling of a stretched, slender elastic sheet and derive scaling laws for the wrinkling wavelength and amplitude that are valid far from the onset of buckling. Our results, which can be generalized to stretched and sheared anisotropic and inelastic sheets, could form the basis of a sensitive assay for the mechanical characterization of thin solid films.

When such a thin isotropic elastic sheet of thickness $t$, width $W$ and length $L$ ($t \ll W < L$), composed of a material with Young's modulus $E$ and Poisson's ratio $\nu$, is subjected to a longitudinal stretching strain, $\gamma$, in its plane, it remains flat for $\gamma < \gamma_c$, a critical stretching strain. Further stretching causes the sheet to wrinkle (Fig. 1a).