Aeolian and subaqueous bedforms in shear flows
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Phil. Trans. R. Soc. A 2013 371, 20120364, published 4 November 2013

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A sediment bed sheared by an unbounded flow is unconditionally unstable towards the growth of bedforms called ripples under water and dunes in the aeolian case. We review here the dynamical mechanisms controlling this linear instability, putting the emphasis on testing models against field and laboratory measurements. We then discuss the role of nonlinearities and the influence of finite size effects, namely the depth of the atmospheric boundary layer in the aeolian case and the water depth in the case of rivers.

1. Introduction

Geomorphology is an active field of research focused on the understanding of the dynamical mechanisms responsible for the formation of geological patterns and on the identification of their related time and length scales [1]. In this context, we discuss here the emergence, the growth and the propagation of bedforms from a flat sand bed submitted to a shear flow. Typical examples are dunes formed by the wind or ripples under water, which we address in the first two sections. In §4, we are also interested in situations where finite size effects influence the growth of bedforms, such as chevrons, river bars or anti-dunes, and the nonlinear pattern coarsening, leading, for example, to giant dunes. We end the paper with a summary and raise related open issues.

2. Aeolian dunes

The formation and growth of aeolian sand dunes and subaqueous sand ripples originate from the very same dynamical mechanisms. However, the state of the art is not the same in both problems: in the aeolian case, all these mechanisms have been investigated, modelled and calibrated independently so that the linear instability can be fairly considered as a closed issue. We therefore
start by reviewing this situation in this first section; we show that the corresponding predictions for the scaling properties of the most unstable mode are in quantitative agreement with field and laboratory measurements. We then present the subaqueous case where several elements of understanding are still missing.

(a) Aerodynamics

The wind flow around aeolian sand dunes can be considered as fully turbulent, which, surprisingly, makes the problem much simpler than the subaqueous case. The averaged turbulent flow over a flat surface is characterized by a logarithmic velocity profile, function of the distance \( z \) to the surface

\[
  u(z) = \frac{u_*}{\kappa} \ln \left( \frac{z}{z_0} \right),
\]

where \( \kappa \simeq 0.4 \) is the von Kármán constant. \( z_0 \) is the aerodynamic roughness length, typically of the order of a fraction of a millimetre. \( u_* \) is the shear velocity, which is built on the shear stress \( \rho_f u^2_* \), where \( \rho_f \) is the fluid density. In the case of an undulated bottom \( Z(x) = Z_{\text{ref}} + \xi \cos(kx) \), where \( x \) is the direction of the flow and \( k \) is the wavenumber of the undulation, the wind velocity is also modulated along \( x \). In the limit of a small aspect ratio \( k \xi \ll 1 \), the linear response of the flow to the topography can be computed using asymptotic matching [2–4] or numerical techniques [5]. The flow presents a layered structure. In the outer layer far from the surface, fluid inertia dominates and is balanced by a pressure gradient. As a consequence, the wind velocity increases on the upwind (stoss) slope of the dune and decreases on the downwind (lee) side, with a maximum in phase with topography. In the inner layer located closer to the surface, however, turbulent shear stress becomes dominant and the velocity field becomes locally logarithmic and controlled by a modulated basal shear velocity \( u_b \). The matching between the inner and outer regions leads to a phase advance of the basal shear stress with respect to \( Z \):

\[
  u_b(x) = u_* + \delta u_b \cos(kx + \psi_b),
\]

where \( \psi_b \) is the phase shift and \( \delta u_b \) the amplitude of the modulation, of the order of \( u_* k \xi \). For the purpose of sand dune initiation and growth, we rewrite the modulation of the basal shear stress \( \tau_b = \rho_f u^2_b \) under the form

\[
  \tau_b(x) \propto \rho_f u^2_* \{1 + k \xi [A \cos(kx) - B \sin(kx)]\},
\]

where the two coefficients \( A \) and \( B \) are the dimensionless stress coefficients that must be retained from aerodynamics and used as inputs of the linear stability analysis described below. Theoretical computations of these coefficients in the case of an infinite turbulent boundary layer show that \( A \) and \( B \) are weak (logarithmic) functions of \( k z_0 \) [6]. They turn out to be rather robust with respect to the choice of turbulent closure [7].

The thickness \( \ell \) of the inner layer is related to the horizontal wavelength \( \lambda = 2\pi/k \) of the relief by the scaling law [8]:

\[
  \ell \sim \frac{1}{k^2} \ln^2 \frac{\ell}{z_0} = O(1).
\]

On a typical aeolian dune such as that shown in figure 1, \( \ell \) is typically of the order of a few tens of centimetres, which allows one to determine \( u_b(x) \) from velocity measurements at \( z < \ell \). The field data of figure 1 directly show the phase advance of the wind speed with respect to topography (\( \psi_b \simeq 0.43 \), i.e. around 26°, or \( \psi_b/k \simeq 3.2 \text{ m} \)). The measured parameters \( A \simeq 3.4 \pm 0.4 \) and \( B \simeq 1.55 \pm 0.05 \) are in reasonable agreement with the theoretical predictions for \( k z_0 \) is in the range 10^{-5}–10^{-4}: \( A \simeq 4 \) and \( B \simeq 2 \) [5], see also figure 7.

(b) Sediment transport

Let us consider aeolian transport over a flat uniform bed. In situations relevant for dune formation, the dominant mode of transport is saltation, so-called because the grains, dragged by
the wind flow, experience small jumps. When they collide with the static bed, they can rebound or get trapped, and eject other particles—a process called ‘splash’. Steady transport is reached when each colliding grain eject, on average, a single other one. In this steady homogeneous situation, one defines the ‘saturated’ flux of particles \( q_{\text{sat}} \), as the volume of the grains crossing per unit time an infinite vertical surface of unit transverse length. For convenience, this volume is rescaled by the bed packing fraction \( \phi \approx 0.6 \). The sediment flux depends on the wind strength, characterized by the shear velocity \( u^* \). Below the threshold value \( u_{\text{th}} \), there is no sustained transport possible (\( q_{\text{sat}} = 0 \)). Above it, \( q_{\text{sat}} \) increases with the shear velocity as \( u^* \) (figure 2). Importantly, the threshold velocity is an increasing function of the bed angle \( \alpha \). Introducing an effective friction coefficient \( \mu \), this dependence can be modelled as \( u_{\text{th}}^2 = (u_{\text{th}}^0)^2 (\cos \alpha + \sin \alpha / \mu) \). Experiments in tilted wind tunnels give \( \mu \approx \tan 37^\circ \), i.e. a value slightly larger than the avalanche angle [10].

When grains are dragged by the wind, they exert in turn a resistive stress on the flow. The wind velocity inside the transport layer is therefore lower than that in the absence of transport. The steady state is reached when one impacting grain on the sediment bed is replaced, on the average by a single transported grain (by rebound or ejection). Then, the number \( n \) of grains moving per unit surface must be such that the reduced wind leads to a characteristic impact velocity \( \bar{u}_{\text{p}} \) independent of \( u^* \). Therefore, the wind velocity in the transport layer must be independent of \( u^* \). The wind reduction can be quantified by partitioning the total shear stress \( \rho_f u^* \) into the fluid-borne shear stress \( \tau_f \) and the grain-borne shear stress \( \tau_p \) [12–14]. In the steady saturated state, \( \tau_f \) has decreased to its threshold value \( \tau_{\text{th}} = \rho_f u_{\text{th}}^2 \), whereas \( \tau_p = nF \), where \( F \) is the average drag force exerted on the grains, which is again independent of \( u^* \). The sediment flux is the product of \( n \) by the average velocity \( \bar{u}_{\text{p}} \) of moving grains. It finally gives the observed scaling for the transport law \( q_{\text{sat}}(u^*) \propto u^2 - u_{\text{th}}^2 \) shown in figure 2. This whole scenario is supported by semidirect [15,16] or direct [17] numerical simulations.
Figure 2. (a) Wind tunnel measurement of the dimensionless saturated flux \( q_{\text{sat}} \) [10] for sand grains of size \( d = 242 \mu \text{m} \) (filled circles). The flux is normalized by \( u^* \) in order to highlight the asymptotic behaviour. The solid curve corresponds to a flux proportional to \( u^* - u_{\text{th}}^2 \). (b) Dimensionless saturated flux obtained from simulations of aeolian transport (filled circles, numerical data of Durán et al. [11]).

Figure 3. (a) Measured aerodynamic roughness length \( z_s \) above the transport layer (black circles, sand grains of size \( d = 242 \mu \text{m} \)). Solid line denotes prediction with the focal point hypothesis. Data are from Rasmussen et al. [18]. (b) Wind tunnel measurements of the vertical profiles of the volume fraction \( \phi(z) \) of sand grains (\( d = 242 \mu \text{m} \)) in saltation for different shear velocities \( u_s \) in the range 0.24 (filled circles) to 0.67 m s\(^{-1} \) (filled diamonds). The volume fraction decreases exponentially with a characteristic height of about 10 mm, independently of the shear velocity (solid lines are exponential fits). Data are from Creyssels et al. [19].

The first experimental proof that a negative feedback of transport on the flow indeed takes place derives from measurements of the velocity profile above the transport layer. Owing to saltation, the roughness length \( z_s \) seen far from the transport layer (see equation (2.1)) is not a constant, but an increasing function of \( u_s \) ([18]; figure 3a). More precisely, the reduction of the wind in the transport layer leads to the existence of an altitude \( z = H_f \) at which the velocity profiles obtained for different \( u_s \) cross each other [12]. \( H_f \) can be directly interpreted as the thickness of the transport layer and the ‘focal point’ velocity \( U_f \) can be related to the threshold velocity. Then, the roughness length can be expressed as \( z_s = H_f \exp(-\kappa U_f/u_s) \), which nicely fits the experimental data. Figure 3b shows measurements of the decay of the grain volume fraction \( \phi(z) \) with height [19]. Fitting these profiles to an exponential, one obtains a direct measurement of the transport layer thickness, which turns out to be independent of \( u_s \), as expected from the above argument.

In the case of an undulated bed surface, the modulation of the basal shear stress (equation (2.3)) induces a modulation of the saturated flux, through the transport law. It is then important to
describe the relaxation of the sediment flux $q$ towards its saturated value $q_{\text{sat}}$ in such a non-homogeneous situation. The calibration of transient effects can be performed by considering a sand bed starting at $x = 0$, the surface being non-erodible in the region $x < 0$. Downwind of $x = 0$, the flux increases with the distance to the beginning of the sand bed, and progressively saturates to $q_{\text{sat}}$, which depends on the imposed $u_s/u_{th}$. The final part of this saturation transient is well described by a first-order relaxation equation of the form

$$L_{\text{sat}} \partial_x q = q_{\text{sat}} - q,$$  \hspace{1cm} (2.5)

where $L_{\text{sat}}$ is the relaxation length called the saturation length in this context (figure 4a). $L_{\text{sat}}$ encodes the space lag of flux response to a change of basal shear stress. Measurements of $L_{\text{sat}}$ show that it is independent of the wind velocity. This suggests that the saturation is limited by the inertia of the grains [20] and that saturation length scales on the drag length which is proportional to the grain to fluid density ratio $\rho_s/\rho_f$ and to the grain size $d$ (figure 4b).

(c) Temporal linear stability analysis

We can now proceed to the linear stability analysis of a flat bed submitted to steady wind conditions [5,21–29]. The time evolution of the bed elevation $Z(x,t)$, here assumed much slower with respect to hydrodynamics and transport time scales, is governed by the sediment mass conservation equation:

$$\partial_t Z + \partial_x q = 0.$$  \hspace{1cm} (2.6)

Taking all processes described above into account and looking for modes of the form $\exp[\sigma t + i(kx - ct)]$, we find for the growth rate $\sigma$ and the phase velocity $c$ the following expressions:

$$\sigma = Qk^2 \frac{(B - S) - AkL_{\text{sat}}}{1 + (kL_{\text{sat}})^2} \quad \text{and} \quad c = Qk \frac{A + (B - S)kL_{\text{sat}}}{1 + (kL_{\text{sat}})^2},$$  \hspace{1cm} (2.7)

where $Q$ is the susceptibility with respect to the shear stress and $S$ the susceptibility with respect to the slope:

$$Q = \frac{\tau}{\phi_b} \frac{\partial q_{\text{sat}}}{\partial \tau} \bigg|_{\tau} \quad \text{and} \quad S = -\frac{1}{Q} \frac{\partial q_{\text{sat}}}{\partial \alpha} \bigg|_{\alpha=0}.$$  \hspace{1cm} (2.8)
Figure 5. Measured initial wavelength of aeolian dunes as a function of the rescaled wind velocity (circles). Solid line: prediction of the model without any adjustable parameter. Grey zone denotes effect of the uncertainty on the value of $L_{\text{sat}}$ (in the range 0.5–0.9 m). Inset: longitudinal profiles of the relative amplitude $\delta h$ of waves on the flank of a large barchan dune (circles). Data are from Elbelrhiti et al. [25] and Andreotti et al. [20]. (Online version in colour.)

Figure 6. Measured wavelength $\lambda$ of elementary dunes, formed by linear instability, as a function of the grain to fluid density ratio multiplied by the grain size. Solid line: $\lambda = 50(\rho_s/\rho_f)d$. Data are from Claudin & Andreotti [27].

The phase advance of the basal shear stress with respect to the topography (the positive coefficient $B$) is the destabilizing mechanism, which is therefore of hydrodynamical origin. The space delay of sediment transport behind the basal shear stress (the length $L_{\text{sat}}$) is stabilizing. The slope term $S$ has a further stabilizing effect, which decays as $u_s^2/u_{\text{th}}^2$. Overall, small wavenumbers are unstable, whereas large ones are stabilized by the transport saturation transient. The most unstable mode can be computed from the dispersion relation—recall that $A$ and $B$ are functions of $k_0$. In figure 5, we show the corresponding prediction for the wavelength as a function of the wind velocity.
There is a quantitative agreement with field measurements performed on growing undulations observed during storms on the flank of large barchan dunes (inset), i.e. a situation close to the initial development of the instability. Interestingly, $\lambda$ diverges at small winds. This is due to the term $B - S$ that can vanish, leading to an instability threshold slightly above the threshold for sediment transport $u_{\text{th}}$.

For a typical value of $u_*/u_{\text{th}} \gtrsim 1.5$, $\lambda$ does not depend much on the wind speed and it purely scales on the saturation length, with a pre-factor of hydrodynamical origin of the order of $4\pi A/B \simeq 25$. Accounting for the fact that $L_{\text{sat}}$ is related to the drag length (figure 4b), we end up with a scaling law $\lambda \sim 8\pi A/B(\rho_s/\rho_f) d$ [27,30]. This prediction has been successfully tested to compare terrestrial aeolian dunes with martian dunes, with snow dunes in Antarctica, with subaqueous ripples and with dunes formed in dense CO₂, close to the critical point (figure 6).

3. Subaqueous bedforms

Although the scaling law displayed in figure 6 is also verified for ripples under water within a factor of two, the case of subaqueous bedforms differs from the aeolian case in several places.

(a) Hydrodynamics

A first difference concerns hydrodynamics—for more details on the fluid flow analysis, readers should refer to Charru et al. [29]. Like aeolian dunes, subaqueous ripples are transverse patterns that form by linear instability of a flat bed, when sediment transport occurs. They emerge at a wavelength $\lambda$ typically of the order of a few centimetres for submillimetric grains (figure 8a), independently of the water depth $D$ as long as $\lambda \ll D$. At this scale, the flow disturbance induced by bedforms is neither laminar nor fully turbulent. This has strong consequences on the behaviour of the basal shear stress, as illustrated in figure 7, where the stress coefficients $A$ and $B$ are displayed as a function of $kz_0$. Contrary to the aeolian case, $z_0$ is not controlled by sediment transport but by the thickness of the viscous sublayer: $z_0 \simeq 0.1v/u_*$. Low values of $kz_0$ correspond to a disturbance in the fully turbulent regime; as expected from the asymptotic analysis [6], $A$ and $B$ then show a weak dependence on $kz_0$. For $kz_0$ in the range from $10^{-4}$ to $10^{-2}$, however, $A$ and $B$ present variations over one order of magnitude, which have a notable effect on the dispersion relation (2.7). In the laminar–turbulent transitional zone, represented in grey in figure 7, $B$ presents a sharp increase while $A$ becomes very small, which enhances the instability. At lower values of $kz_0$, $B$ changes sign in a narrow range of wavenumbers, which are therefore stable. These variations have been accounted for by a dependence of the turbulent closure on the pressure gradient, with a space lag ([32,35], see also [29]). Although this model provides a convenient parametrization, a true understanding of the interplay between a wavy bottom and the modulation of the viscous sublayer remains to be achieved.

(b) Sediment transport

A second difference with the aeolian case concerns sediment transport, and the saturation length in particular. For small Rouse number $u_*/V_{\text{fall}}$, where $V_{\text{fall}}$ is the settling velocity of the particle, the transport mode is called bed-load, as all particles move within a thin layer of thickness comparable with the grain size $d$. In contrast to saltation and its splash process, the grains move with an average velocity scaling with $u_*$, so that the saturated sediment flux scales as $q_{\text{sat}} \propto u_*^3$ far enough above the threshold $u_{\text{th}}$. More importantly, the saturation length associated to bed-load has never been directly measured. Indirect estimates gives an order of magnitude of $10d$, but its scaling properties are still debated [26,29,36,37]. In the near future, numerical simulations will probably enlighten its dependence on grain and flow characteristics [17]. At large Rouse numbers, particles can remain trapped in turbulent eddies for a long time, and sediment transport occurs over the whole water depth $D$. This mode of transport is called suspension. Both the sediment
Figure 7. Rescaled shear stress components in-phase \(A\) (a) and in-quadrature \(B\) (b) as a function of the rescaled wavenumber \(kz_0\) (data and curves gathered/calculated by [29]). Solid line denotes ‘Hanratty’ model. Dashed lines denote asymptotic calculations. Grey region denotes transition between laminar and turbulent regimes. Superimposed points: data from electrochemical measurements (filled symbols) of Hanratty et al. [31,32]; data from velocity measurements on a 40-m-long dome dune (open triangle, see figure 1) and in flumes (open diamond [33]; open circle [34]).

Figure 8. (a) Formation of ripples in a natural river, starting at \(t = 0\) from a flat sand bed. Amplitude \(H\) (squares in mm) and wavelength \(\lambda\) (circles in cm) of the bed disturbances as a function of time \(t\). Inset: two longitudinal profiles of the ripples at \(t = 25\) and \(t = 150\) s. Adapted from Fourrière et al. [5]. (b) Dataset of measurements of the initial wavelength \(\lambda\), rescaled by \(d\) as a function of \(d/z_0\), for \(u_*/u_{th}\) in the range \(1.8 \pm 0.2\) (squares). Predictions for the most unstable wavelength are added in solid line (with ‘Hanratty’ hydrodynamical description), using the slope susceptibility \(S = 0.5\) and \(L_{sat} = 12 U(d)/V_{fall}\). Grey region denotes transition between laminar and turbulent regimes. Data are collected by Charru et al. [29]. (Online version in colour.)

flux and the saturation length are much larger than that of bed-load, by a factor \(D/d\). It has been measured \([38–41]\) and shown to scale as \(L_{sat} \propto u_*/D/V_{fall}\) \([42]\).

Bed-load is the dominant mode of transport when ripples form. Their initial wavelengths \(\lambda\) are typically in the range \(100–800\ d\), both in water \([5,43–47]\) and in viscous fluids \([48,49]\). The general
trend that emerges is the decrease in $\lambda/d$ with the particle Reynolds number $d/z_0$ (figure 8b). Because these data fall in the transitional regime, $\lambda$ depends both on the saturation length and on the viscous length. Only Hanratty’s model reproduces the correct trend, with the saturation length fitted to $L_{\text{sat}} \simeq 12(U/V_{\text{fall}})d$, consistently with the measurements of the deposition length by Lajeunesse et al. [37].

4. Finite depth effect

The linear regime, characterized by an exponential growth of the bedform amplitude, is left when the aspect ratio reaches $\simeq 1/20$ (see figure 8a for the case of subaqueous ripples). Then, hydrodynamical nonlinearities make the amplitude saturate, and lead to the development of an avalanche slip-face on the lee side, associated to a separation of the boundary layer. Then, other nonlinear processes start: as small bedforms propagate faster than large ones, collisions occur, leading to an amalgamation or merging, and thus a progressive increase in the bedform average wavelength (figure 9a,b). This pattern coarsening is eventually stopped at longer times by further stabilizing processes. One of them is the presence of a free surface, and subaqueous ripples can coarsen into dunes when their wavelength $\lambda$ is of the order of the water depth $D$ (figure 9b). Subaqueous ripples can also stop their evolution when they reach the edge of the laminar–turbulent transitional region, in the range of wavenumber for which $B$ becomes negative (figure 9c).

(a) Giant aeolian dunes

Dune coarsening is also observed in the aeolian case and is limited by the thickness of the convective boundary layer. The atmosphere in deserts has a well-defined layered structure, described by the vertical profile of the virtual potential temperature $\Theta(z)$ [52]. At large altitudes, the atmosphere is stably stratified, with a constant gradient $\gamma = d\Theta/dz$, whereas, close to the surface, one observes a well-mixed layer ($\Theta = \text{Cte}$) because of convection owing to heating of the air by the ground. In between, there is a so-called inversion capping layer, which plays a role analogous to the free surface of a river (inset of figure 10). The typical thickness $D$ of the convective layer depends on the seasonal variations of the surface temperature $\delta \Theta$, and one can write in first approximation $D \simeq \delta \Theta/\gamma$. It varies from a few hundreds of metres in coastal deserts to several kilometres in continental regions. The progressive amalgamation of dunes leads to giant dunes, whose size $\lambda$ precisely scales with $D$, independently of their shapes (figure 10a).

(b) Importance of the Froude number

A different scenario occurs when the wavelength at which bedforms would emerge in an unbounded flow is actually comparable or larger than the finite thickness $D$. Then, the influence of the latter does not appear at a late nonlinear stage to interrupt the pattern coarsening process but already modifies the linear instability. In this situation, the Froude number $F = U/\sqrt{gD}$, where $U$ is the surface velocity, becomes important. The linear stability analysis can be performed as before, but the basal stress coefficients $A$ and $B$ now depend nonetheless on $kz_0$ but also on the rescaled wavenumber $kD$ and on the Froude number $F$. Sediment transport remains unchanged and essentially imposes the length scale $L_{\text{sat}}$. Different regimes are found, depending on $F$ and on the saturation number $L_{\text{sat}}/D$ [28]. The transverse ripples discussed in the previous section correspond to small $L_{\text{sat}}/D$, where the most unstable mode is not influenced by the free surface. In particular, there is no secondary maximum of the growth rate that could be associated with dunes [22–24], as soon as all the modes (including inclined ones) are taken into account. For $L_{\text{sat}}/D \gtrsim 0.1$ and subcritical flows ($F < 1$), patterns inclined at an angle with respect to the flow are more unstable than transverse ones. This corresponds to rhomboid patterns or chevrons in shallow water [54–57] as well as oblique or alternate bars observed in flumes with coarse grains [58–64]. Large values of $L_{\text{sat}}/D$ are also reached when suspension is the dominant mode of transport. Some
Figure 9. (a) Space–time diagram of ripple nonlinear evolution. Data are from Coleman & Melville [50]. (b) Long time evolution of the wavelength by pattern coarsening after the initial stage shown in figure 8a and saturation owing to the influence of the water free surface (grey circles). (c) Data collected by Yalin [51] for final subaqueous ripple wavelength (triangles), for \( \frac{u_\ast}{u_{th}} < 1.5 \), in the graph \( \lambda/d \) versus \( d/z_0 \). The black line of figure 8b, showing the initial wavelength, is displayed for comparison. Grey region denotes transition between laminar and turbulent regimes.

Figure 10. Average wavelength of giant dunes in all deserts of the Earth as a function of the average height \( D \) of the capping inversion layer. Data are from Andreotti et al. [53]. Inset: schematic of the vertical profile of virtual potential temperature in desertic regions.
alternate bars observed in flume experiments are in this regime [65] as well as the bars formed in rivers during floods, where most of the sediment is suspended. Finally, for supercritical Froude numbers \( F > 1 \), the free surface has a destabilizing effect owing to the change of sign of \( A \) at the resonance of the surface waves with the bed undulations. This situation corresponds to anti-dunes propagating upstream [21,66,67]. The range of unstable wavenumbers \( kD \) is so narrowly related to the resonance that the most unstable mode is selected by hydrodynamics, \( \lambda \propto D \). Measured wavelengths corresponding to the patterns described above are displayed in the \((Fr, kD)\) diagram of figure 11.

\((c)\) Alternate bars

The case of alternate bars deserves to be elaborated more. In the case of a channel of finite width \( W \), the aspect ratio \( W/D \) becomes a new important dimensionless number. Like in a wave guide, the modes are guided by the banks and take the form \( \exp[\sigma t + i(kx – ct)] \sin(m\pi y/W) \), where \( y \) is the transverse direction and \( m \) an integer labelling the mode number. As shown in figure 12b, at small \( L_{sat}/D \), only transverse \((m = 0)\) modes can develop, which correspond to the ripples discussed in the previous section. Above a critical aspect ratio, \( L_{sat}/D \) controls the transition from transverse to inclined patterns. The first non-transverse mode \((m = 1)\) corresponds to alternate bars. Larger values of \( W/D \) lead to the selection of higher order modes \((m > 1)\). For a given (and large enough) value of \( L_{sat}/D \), a stability diagram can be plotted in the plane \((W/(mD), kD)\) (figure 12a). The flat bed configuration is unstable in the central region above the marginal stability curve \((\sigma = 0)\). The most unstable bar wavelength typically scales as \( \lambda \propto W^{2/3} L_{sat}^{1/3} \). The crossing of the marginal stability curve with that corresponding to non-propagating bars \((c = 0)\) is called the ‘resonant’ point and is currently assumed to play an important role in the development of river meanders [68,70].
5. Conclusion

To sum up, the formation of subaqueous ripples and aeolian dunes in an unbounded flow results from the combined action of fluid inertia and shear stress (turbulent or not), which induce a phase advance of the shear stress relative to the bed disturbance. This is balanced by the stabilizing effect of the transient of saturation of the sediment transport. The saturation length $L_{\text{sat}}$ associated with this relaxation effect is an essential feature for the understanding of the instability. A second stabilizing effect is the bedform slope which drags the particles towards troughs.

For aeolian dunes, the scales of the problem are well separated. The initial dune wavelength is, as a first approximation, proportional to the saturation length, times the hydrodynamical factor $B/A$ encoding the phase advance of the bed shear stress. The saturation length results from grain inertia and is proportional to the drag length $(\rho_p/\rho_f)d$. Weaker effects arise from the slope parameter $S$. For subaqueous ripples, the conclusion is less clear-cut. Most of the experimental and field data lie in the transition between the laminar and turbulent regimes, where the bed shear stress is more sensitive to the turbulence modelling. However, there is direct evidence that ripples form by linear instability of a flat bed, and their initial wavelength, in the range 100–800 $d$, is consistent with $L_{\text{sat}}$ in the range 10–30 $d$. The rapid coarsening of the pattern makes it difficult for the interpretation of many data from the literature.

As for open issues, the laminar–turbulent transitional regime, in which most of the subaqueous ripples lie, remains to be understood. In addition, the saturation length $L_{\text{sat}}$ for bedload has to be measured and studied, as well as its large increase at the transition from bedload to suspension. Another current direction of research in this field is to investigate different nonlinear aspects of

**Figure 12.** (a) Stability diagram for bars of mode $m$: isocurves of the growth rate $\sigma$ of sand bars for a transverse wavenumber $k_y = m\pi/W$ and a saturation length $L_{\text{sat}} = 2D$. The graph is inspired from that shown in Seminara [68], but computed from the model developed by Andreotti et al. [28]. The dotted line corresponding to vanishing speed $c = 0$ and crosses the marginal stability curve (null growth rate $\sigma = 0$) at the so-called resonant conditions (point $r$). The thick line corresponds to the maximum growth rate and crosses the marginal stability curve (null growth rate) at the critical point (point $c$). (b) Diagram showing the different bedform regions in the plane $(W/D, L_{\text{sat}}/D)$. In the bottom-right dark-grey region, the bed is stable. In the left middle-grey region, ripples ($m = 0$) are the most unstable bedforms. In the remaining up-right part of the plane, higher modes ($m > 0$) are dominant: the dashed–dotted line separates alternate bars ($m = 1$) from multiple bars ($m > 1$). These curves have been computed from the model developed by Andreotti et al. [28], using $u_*/u_{\text{th}} = 2$, $F = 0.1$ and $D/z_0 = 10^2$. Adapted from Chiodi et al. [69].
ripple and dune formation. A typical example is that of star dunes, which have been tackled by means of numerical simulations only recently [71], and involve the nonlinear description of hydrodynamical response to such a complex relief as well as the description of dune interactions (collisions, splitting).

**Acknowledgements.** We thank our main collaborators on this subject: F. Charru, F. Chiodi, O. Durán, H. Elbelrhiti, A. Fourrière, B. Murray, O. Pouliquen and G. Wiggs. We are particularly grateful to F. Charru for recent stimulating discussions. Some of the field data presented here have been obtained in agreement with the Parc Naturel Régional des Landes de Gascogne.

**Funding statement.** We thank ANR Zephyr grant no. ERCStG_8 for funding.

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