Jamming vs. Glass transitions

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TWO WAYS TO JAM A SYSTEM

▶ Fast an dramatic to $P \sim \infty$
▶ Slow and subtle to finite $P$

Both yield diverging timescales caused by underlying diverging lengths of cooperative motion (as seen in $\chi_4$)
TWO SETS OF THEORETICAL IDEAS FOR AMORPHOUS SYSTEMS:

Dynamic and thermodynamic glass transition

and

‘Jamming point’ of (frictionless) granular matter

They should meet: e.g. an amorphous colloid is both!
Procedure: increase the radius of the spheres gradually, infinitesimal overlaps are removed through repulsion. Continue until the pressure is infinite: this is the J-Point. (O’Hern et al., Lubachevsky-Stillinger,...)

the actual volume fraction so reached is very close to the one quoted as Random Close Packing

More generally, finite \( P \): propose a random displacement of particles, accept the move with Monte Carlo rule with weight \( e^{-NP r_{min}} \)
The system so reached is in mechanic equilibrium. It can be:

**HYPOSTATIC**
less constraints than degrees of freedom

**HYPERSTATIC**
more constraints than degrees of freedom

**ISOSTATIC**
just as many constraints as degrees of freedom
The system so reached is in mechanic equilibrium. It can be:

**HYPOSTATIC**
... a table with two legs touching the floor

**HYPERSTATIC**
... a table with four legs touching the floor

**ISOSTATIC**
... a table with three legs touching the floor
Unless something is finely tuned, a system of spherical particles evolves until reaching an isostatic situation...

...just as a table on a rough floor generically has three legs touching.
But a globally isostatic system is very sensitive to perturbations. Breaking one contact already makes it unstable.

Has zero modes, diverging susceptibilities and lengths.
Fragility of an isostatic system (Combe and Roux)

response is non-smooth and nonlinear
Vibration modes with small overlaps (Silbert et al.)

Vibration modes for hard particles (Brito and Wyart)
Isostaticity by itself implies diverging susceptibilities, diverging lengths, and in general a critical point.

Is this a characterisation of the glass phase?
A completely different set of ideas:

The standard folklore of ‘ideal’ (thermodynamic) glass phase
The glassy phase arises because the liquid runs out of entropy (Kauzmann, Adam-Gibbs, Random First Order).
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If the glass phase is an ordinary solid phase (like a crystal or a quasicrystal):

the knowledge of the density distribution on one region should help us predict the density distribution far away.
In order to compare both sets of ideas we need a model for which both can be applied simultaneously.
A model of particles that:

has spatial features like isostaticity, vacancies and instertitials, force chains,...

AND

a thermodynamic glass phase
(Some) finite dimensional features and mean-field
\[ Z/N \rightarrow 0 \]

\[ E = \sum_{ij \text{ in the graph}} V(x_i - x_j) \]
We find that all target configurations are amorphous and isostatic: they have the same properties as the finite-dimensional J-point(s).
Vibration modes with small overlaps

$g(\omega)$ model.

$g(\omega)$ Silbert et al.
Vibration modes for hard particles

The model at different pressures

n.b.: at finite pressure criticality dissappears!

Finite dimensional (taken from Brito and Wyart)
Glass transition
The J-points(s)
The J-points(s)
The J-points(s)

deeper J–points have the same properties if NO crystallisation
J Point(s). Isostaticity-related vs. glassy lengths.
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A set of J–points, all amorphous, isostatic, and with diverging lengths.
J Point(s). Isostaticity-related vs. glassy lengths.
CONCLUSIONS

- The relation between J-point criticality and the ideal glass transition is made explicit in a family of mean-field models that has both and can be shown to be distinct.

- Another example is a slightly polydisperse crystal, which has a crystalline order and J-point criticality.

- Jamming-related lengths can be easily made to be 100 interparticle separations in a few collision times in granular matter while it takes millions of collision times to grow to 6 in a supercooled liquid → clearly a different kind of length.

- The question of whether a glassy thermodynamic phase exists in finite dimensions – and what exactly is its nature – remains open.