Jamming vs. Glass transitions

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Lyon 2008
TWO SETS OF THEORETICAL IDEAS FOR AMORPHOUS SYSTEMS:

Dynamic and thermodynamic glass transition

and

‘Jamming point’ of (frictionless) granular matter

They should meet: e.g. an amorphous colloid is both!
J-Point

Procedure: increase the radius of the spheres gradually, infinitesimal overlaps are removed through repulsion. Continue until the pressure is infinite: this is the J-Point.

(O’Hern et al., Lubachevsky-Stillinger,...)

the actual volume fraction so reached is very close to the one quoted as Random Close Packing

More generally, finite $P$: propose a random displacement of particles, accept the move with Monte Carlo rule with weight $e^{-NP_{r_{min}}}$
Landscape conjugated to pressure

radius

number of clauses, number of links, ...
Landscape conjugated to pressure

radius

number of clauses, number of links, ...
Landscape conjugated to pressure

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Landscape conjugated to pressure

radius

number of clauses, number of links, ...
Landscape conjugated to pressure

classical pseudoenergy

radius

number of clauses, number of links, ...
The system so reached is in mechanic equilibrium. It can be:

**HYPOSTATIC**
less constraints than degrees of freedom

**HYPERSTATIC**
more constraints than degrees of freedom

**ISOSTATIC**
just as many constraints as degrees of freedom
The system so reached is in mechanic equilibrium.
It can be:

**HYPOSTATIC** ... a table with two legs touching the floor

**HYPERSTATIC** ... a table with four legs touching the floor

**ISOSTATIC** ... a table with three legs touching the floor
Unless something is finely tuned, a system of spherical particles evolves until reaching an isostatic situation...

...just as a table on a rough floor generically has three legs touching.
But a globally isostatic system is very sensitive to perturbations. Breaking one contact already makes it unstable.

Has zero modes, diverging susceptibilities and lengths.
Fragility of an isostatic system (Combe and Roux)

response is non-smooth and nonlinear
Vibration modes with small overlaps (Silbert et al.)

Vibration modes for hard particles (Brito and Wyart)
Isostaticity by itself implies diverging susceptibilities, diverging lengths, and in general a critical point.

Is this a characterisation of the glass phase?
A completely different set of ideas:

The standard folklore of ‘ideal’ (thermodynamic) glass phase
The glassy phase arises because the liquid runs out of entropy (Kauzmann, Adam-Gibbs, Random First Order).
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The glassy phase arises because the liquid runs out of entropy (Kauzmann, Adam-Gibbs, Random First Order).
If the glass phase is an ordinary solid phase (like a crystal or a quasicrystal):

the knowledge of the density distribution on one region should help us predict the density distribution far away.
In order to compare both sets of ideas we need a model for which both can be applied simultaneously.
A model of particles that:

has spatial features like isostaticity, vacancies and instertitials, force chains,...

AND

a thermodynamic glass phase
(Some) finite dimensional features and mean-field $\frac{Z}{N} \to 0$

$$E = \sum_{ij \text{ in the graph}} V(x_i - x_j)$$
Analytic solution (Mézard, Parisi, Tarzia and Zamponi) closely related to graph-coloring

A configuration
The model has a usual ideal glass transition and a dynamic transition at lower pressure.
At large pressures we find that all jammed configurations are amorphous and isostatic: they have the same properties as the finite-dimensional J-point(s).
Vibration modes with small overlaps

g(\omega) model.

g(\omega) Silbert et al.

\begin{align*}
\omega &= 0, 0.5, 1, 1.5, 2, 2.5, 3 \\
D(\omega) \\
\delta z &= 10^{-3}, 10^{-2}, 10^{-1}, 10^0 \\
\omega^* &= 10^{-2}, 10^{-1}, 10^0
\end{align*}
Vibration modes for hard particles

Particle trajectories
The model at different pressures

n.b.: at finite pressure criticality disappears!

Finite dimensional (taken from Brito and Wyart)
Density of modes for the locally immobilised particles (3-core) (Normalised by $P$)
(Unnormalized) density of modes for the rattlers.
A low frequency non-acoustic mode
An acoustic mode
Disappearance of criticality at finite pressure

Gap versus pressure
The J-points(s)
The J-points(s)
The J-points(s)

Deeper J-points have the same properties if NO crystallisation
J Point(s). Isostaticity-related vs. glassy lengths.
J Point(s). Isostaticity-related vs. glassy lengths.

A set of J–points, all amorphous, isostatic, and with diverging lengths.
J Point(s). Isostaticity-related vs. glassy lengths.
Fragility of frictionless granular matter

- Because of isostaticity, the system responds non-linearly to small stresses, as in finite dimensions (see Combe and Roux, Brito and Wyart).

- It does so in bursts ’using the soft modes’

- However, the system may sustain extensive stress, only flowing through thermal activation, just as one expects of a solid. (this does happen in the present model, in real life it depends on whether a solid glass phase exists)
An extreme example:
Pair correlation with and without ‘rattlers’.
We get soft modes just as in a true amorphous system
... and yet there is no doubt that at finite pressures the nature of the solid phase is quite indifferent to all this structure...

which leads us to think that there is a similar relation between ‘microstructure’ at the bottom of a glassy state, and the nature of the state ‘in the large’
CONCLUSIONS

- The criticality of the ‘J-point(s)’ near infinite pressure is not the glass transition, but a consequence.
- The fragility properties that are related to this criticallity seem to apply in a regime of parameters different from what one expects for the glass phase.
- The question of whether a glassy thermodynamic phase exists in finite dimensions – and what exactly is its nature – remains open.
Hard sphere packings

J-point procedure

Ideal glass state??

φ

random loose packing

mode-coupling

crystal

random close packing
PACKING

A given configuration
PACKING

Inflate slightly
PACKING

Displace particles to resatisfy
GRAPH COLORING

A well colored graph
GRAPH COLORING

add one link
Modify nearby configurations to resatisfy
The Spin-Glass landscape
(just for reference)
Random First-Order Landscapes

\[ E \]

\[ \text{E}_{\text{tap}} \]
\[ \text{E}_d \]
\[ \text{E}_f \]
\[ \text{E}_K \]
\[ \text{E}_o \]

\[ \text{threshold} \]

\[ \text{paramagnet} \]
Random First-Order Landscapes
Random First-Order Landscapes

A QUENCH

threshold

paramagnet

$E_{\text{tap}}$

$E_d$

$E_J$

$E_K$

$E_o$

$E$
Random First-Order Landscapes

internal temperature of states  (NOT an effective temperature!)

E

\( E_{\text{tap}} \)

\( E_d \)

\( E_f \)

\( E_k \)

\( E_o \)

T

threshold

paramagnet

(E internal temperature of states (NOT an effective temperature!))
Random First-Order Landscapes

\[
\Phi
\]

\[
\frac{1}{P}
\]

\[
\phi_{\text{tap}}
\]
\[
\phi_{\text{d}}
\]
\[
\phi_{J}
\]
\[
\phi_{K}
\]
\[
\phi_{o}
\]

threshold

paramagnet

internal pressure of states
Random First-Order Landscapes

RELAXATION OF PRESSURE

\( \Phi \)

\( \phi_{\text{tap}} \)
\( \phi_{\text{d}} \)
\( \phi_{\text{J}} \)
\( \phi_{\text{K}} \)
\( \phi_{\text{o}} \)

threshold
paramagnet

1/P
Random First-Order Landscapes

in general many ‘transparent states’ (cf. an old bet)

\[ \Sigma \]