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Spatial Evolution of Görtler Instability in a Curved Duct of High Curvature

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The Görtler instability is investigated experimentally in a water channel of high curvature. Two different kinds of experiments are presented. The first one corresponds to an experiment where the flow rate increases and where we study the instability in a given longitudinal position along the concave wall. We find a law for the full nonlinear behavior as a correlation for the velocity perturbation as a function of the Görtler number. In the second set of experiments, we follow the spatial evolution of the instability along the concave wall for a given flow rate. We observe the saturation due to the nonlinearities and to the diminution of the thickness of the boundary layer. We show the importance of the initial conditions and of the wavelength in the evolution of the instability.

I. Introduction

THE flow in curved ducts has been, and still is, investigated largely because of its fundamental interest as well as its importance in technological realizations such as heat exchangers, airfoils, turbine blades, or any part with curved surfaces. The boundary-layer flow on a concave surface can be unstable. This instability is known as the Görtler instability. It is the consequence of the imbalance between the centrifugal forces and the radial pressure gradient. It produces longitudinal rolls in counter-rotating pairs aligned in the direction of the main flow.[1–5]

On turbine blades for instance, the instability perturbs the film cooling system. This system consists of injections of colder fluid on the surface of the blade with small jets coming from the inside of the blade. On the pressure side, which corresponds to the inner portion of the blade where the surface is concave, the Görtler instability can dramatically modify the heat exchange. Also, the rotation of the blades, which adds Coriolis effects, can disturb the basic flow as well as the instability development.[6,7]

Meanwhile the Görtler vortices are the natural prototype of longitudinal organized structures arising in flows. Indeed, in the field of heat transfer, many improvements were obtained with airfoils, pins, or other vortex generators, to induce artificially longitudinal vortices in the flow. The strongly nonlinear velocity field induced with theses vortex generators produces extended regions of down wash flow towards the wall, enhancing the heat and/or mass transfer.

The Görtler instability belongs to the class of centrifugal instabilities such as the Taylor–Couette instability between two concentric cylinders in rotation or the Dean instability of a Poiseuille flow within a curved channel.[8,9] Simultaneously, it also belongs to the class of instabilities in open systems. This double character generates all of the richness (as well as the complexity) of this system.

The control parameter of this instability is the Görtler number G. Its expression is

$$G = Re(\delta/R)^{\frac{1}{2}} = (U\delta/\nu)(\delta/R)^{\frac{1}{2}}$$

where U is a characteristic velocity of the flow, δ is the thickness of the boundary layer, ν is the kinematic velocity, and R is the radius of curvature of the concave wall. This Görtler number can be interpreted as the ratio between the inertial and centrifugal effects to the viscous effects.

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Aspect of Centrifugal Instability

Centrifugal instabilities appear when the curvature of the streamlines could induce longitudinal rolls. The criterion that gives a necessary condition for a centrifugal instability is the inviscid Rayleigh's criterion 8: a flow in which the Rayleigh's discriminate is negative is potentially unstable, where the Rayleigh's discriminate is written as

$$\phi(r) = \frac{1}{r^3}\frac{\mathrm{d}}{\mathrm{d}r}(rU)^2$$

In the case of a small curvature, when the characteristic length of the flow d is very small compared with the radius of curvature, the Rayleigh's discriminate can be written as

$$\phi(r) \propto U\frac{\mathrm{d}U}{\mathrm{d}r}$$

A boundary-layer flow is stable on a convex wall and potentially unstable on a concave one.

Aspect of Open Flow

The Görtler instability presents an important difference with the Taylor–Couette instability and at a lower level with the Dean instability. Indeed, in the Taylor–Couette problem the streamlines are closed on themselves and the instability appears in a confined system. Thus, a permanent feedback exists and the rolls are autoregenerated by themselves. This regeneration can explain the absolute character of the Taylor–Couette instability. In addition, the gap between the two walls remains constant. For given rotation speeds of the cylinders, the Taylor number

$$\mathrm{Ta} = (Ud/\nu)(d/R)^{\frac{1}{2}}$$

is equal to a constant value that does not change either in time or in space. When the rolls are formed, they remain at a constant amplitude in the flow, at least until a second threshold where, for example, they can oscillate. This system is spatially homogeneous. The Dean instability is not an instability in a physically closed system in the sense where the flow does not shape a loop. However, we can observe that the evolution of the thickness of the unstable region remains constant in time as well as in space (an established Poiseuille flow does not evolve along a flow). Thus, the Dean number

$$\mathrm{De} = (Ud/\nu)(d/R)^{\frac{1}{2}}$$

remains constant for a given flow in a given geometry. Nevertheless, the amplitude of the rolls that appear after the inlet of the curved section evolves along the flow in a different way to the Taylor–Couette problem, as in a typical problem of spatial instability with nonlinear saturation. The perturbations in this system are spatially inhomogeneous, even if the basic flow is spatially homogeneous.