Non-linear modulation of a boundary layer induced by vortex generators

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A novel approach, based on Non-Linear analysis of the flow, is proposed to study the modification of flat-plate boundary layer by bluff-body Vortex Generators (VGs). Small cylinders in a flat-plate boundary layer are used to create a set of counter-rotating streamwise vortices (CRSVs) which modify the global properties of the boundary layer. From the harmonic modulation and the global mode (or zeroth mode), one can define some characteristic parameters which can be used to quantify the efficiency of a given set of VGs. A first parametric study for a given Reynolds number shows a clear dependance of these parameters on the spacing between the VGs. Some of these parameters should be useful to choose, for instance, the right parameters to control a separated flow.

I. Introduction

Vortex generators are well known as efficient tools for the control of flow separation. They are commonly used in various industrial application e.g. in aeronautics, to enhance airplane lift force in near stall situations, in automotive aerodynamics to reduce the drag of a vehicle or in chemical industry to increase the efficiency of static mixers. Nevertheless, the question of the choice and design of the proper VG for a given application is still an open problem.

As a matter of fact, many different perturbations of the boundary layer can create streamwise vortices, but the efficiency of the system will depend on many parameters. For instance, one can choose to use jet vortex generators which will imply the choice of the jet diameter and velocity together with the forcing frequency if we deal with pulsed jets. In the case of solid blade vortex generators, it will imply the choice of the VG geometry (dimensions of the blade) together with their spatial organization which will depend on the objective of the control and wether one want to create co or counter-rotating streamwise vortices.

One can also use small bluff-bodies, like small cylinders, to create streamwise vortices. It has been shown that it can postpone the transition to turbulence in a flat-plate boundary layer. Another experiment showed that vertical blades can be used to control the separation over a 3D bluff-body. In every cases, it will be a difficult task to choose the longitudinal position of the VGs relative to the separation line. In fact, most of the time using VGs implies many choices that can be analyzed only through long and tedious parametric studies.

In order to facilitate the search of the proper parameters, it is essential to understand how the VG interact with a boundary layer and why the resulting perturbations will be efficient to control, for instance, a separated flow. Even if the global effect of vortex generators has been extensively discussed, the mechanism of vortex generation and interaction with the boundary layer still need further investigation. This is the objective of the present study.

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The following paper is organized as follows. In the first section, the experimental set-up and the vortex generators are defined. In the second section, the principle of boundary layer decomposition is presented. In the third and final section, the decomposition is applied to a parametric evolution of the set of vortex generators to show the dependance of some global physical parameters to the spacing between the vortex generators.

II. Experimental Set-Up

The experiments are carried out in a low-speed water tunnel made of plexiglas to allow optical measurement from any direction including downstream. The flow is driven by gravity using a water reservoir kept to a constant height. Stabilization of the flow is obtained by divergent and convergent sections separated by a honeycomb. The rectangular cross-section of the tunnel is 10\,cm high and 15\,cm wide. The test-section is 80\,cm long.

Figure 1. (a) Description of the vortex generators geometry and of the axis. (b) Description of the experimental set-up for PIV measurements. The velocity fields are measured in horizontal planes, downstream the VGs. The 3D field of \((u, w)\) velocity components can then be reconstructed.

The mean freestream velocity \(U_\infty\) ranges between 0.5\,cm\,to 20\,cm\,s\(^{-1}\). The typical boundary layer thickness at the beginning of the test section is about 1\,cm. The four vortex generators are located 10\,cm downstream from the beginning of the test section. As shown in figure 1 (a), the origin is taken on the lower wall at the symmetry point of the VGs line. The VGs are small cylinders mounted on the wall so that horse-shoe vortices are created at their bases before turning into counter-rotating streamwise vortices.

The cylinders are defined by their diameter \(d = 8\,mm\) and their height \(h = 6\,mm\). The spacing between the VGs, or wavelength \(\lambda\), ranges between 2 to 4 diameters. One can define a Reynolds number based on VGs height \(h\) and freestream velocity \(U_\infty\), \(Re_h = \frac{h U_\infty}{\nu}\).

Horizontal velocity fields are measured using Particle Image Velocimetry (PIV) in horizontal \((x, z)\) planes for different heights \(y\) (figure 1 (b)). The two components \((u_x(x, y, z), u_z(x, y, z))\) in every \(y = \text{const}\) PIV planes are stacked together so that a three dimensional representation of the velocity field can be rebuilt in a rectangular box approximately 12\,d long, 12\,d wide and 5\,h high.

A bi-cavity YAG pulsed laser is used for the PIV with a nominal energy of 170\,mJ. The laser sheet is less than 1\,mm thick. The image resolution is 1200 \(\times\) 1600 with a 12 bits dynamic range and a sampling frequency \(f = 15\,Hz\). Using this set up between 300 and 600 couples of images are recorded for each \(y = \text{const}\) plane. The \((x \times z)\) dimensions for every image are 100 \(\times\) 70\,mm\(^2\). Although different image processing techniques are used to obtain the vector fields and improve their quality, no data interpolation was done. The final vector fields correspond to the real measurements.

For every vector field we check the convergence of the temporal statistics by looking at the cumulative average and standard deviation of reference vectors located in high fluctuation areas. Deviation of the flowrate measured by an electromagnetic flowmeter is less than 1% during the acquisition process. All
III. Boundary layer decomposition

The modulation of the time averaged boundary layer can be decomposed in different contributions.

\[ U(x, y, z) = U_{base}(x, y) + u_L(x, y, z) + u_{nL}(x, y, z), \]  

(1)

Where \( U_{base} \) is the flow obtained without VGs, \( u_L \) is the harmonic modulation with the wavelength equal to the spacing between the two VGs and \( u_{nL} \) is the sum of the non-harmonic perturbations.

A. The harmonic perturbation.

The harmonic perturbation can be obtained by performing a Fourier transform of the boundary layer and extracting the wavelength and modulus of the mode which is the closest to the spacing between the VGs. By these means we can define \( E_1(x) \) and \( \lambda_1(x) \) which are respectively the modulus and wavelength of the harmonic mode so that \( u_L(x, y, z) = u_1^* \exp\left(\frac{2\pi z}{\lambda_1}\right) \) and \( E_1(x) = u_1^* \).

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length” $L_R$ where the modulus of the harmonic perturbation decays to zero before growing up again (figure 3(a)). This is due to the crossing of a boundary between two regions, one where the modulation is due to the recirculation areas and the high speed areas between the CVGS, and the other one further downstream where the modulation is due to the action of the counter-rotating streamwise vortices.

### B. The zeroth mode.

The nonlinear perturbations can be written as:

$$u_{nL}(x, y, z) = \sum_{k \geq 0, k \neq 1} u^*_k(x, y) \exp \left( ik \frac{2\pi}{\lambda(x)} z \right),$$

(2)

If we make a space average along the transverse direction every exponential term decays to zero at the exclusion of the zeroth mode. This mode is the expression of the mean flow modification induced by the nonlinear effects of the VGs. In our experiment we can obtain this zeroth mode by removing the base flow obtained without CVGs and removing all other perturbations by averaging along several wavelength.

$$U(x, y, z) = U_{\text{base}}(x, y) + u^*_0(x, y) \exp \left( i \frac{2\pi}{\lambda_1} z \right) + \sum_{k \geq 0, k \neq 1} u^*_k(x, y) \exp \left( ik \frac{2\pi}{\lambda(x)} z \right),$$

(3)

$$\langle U(x, y) \rangle_z = U_{\text{base}}(x, y) + u^*_0(x, y),$$

(4)

![Figure 4](image-url)

**Figure 4.** (a) Successive vertical profiles of the zeroth mode $u^*_0$ along the streamwise direction for a given spacing $\lambda = 3d$ and a given Reynolds number $Re_h = 300$. One can notice the inversion of the profile past $x/d = 10$. (b) Streamwise evolution of the integral over $y$ of the $u^*_0(x, y)$ profiles. It allows the definition of the inversion length $L_{inv}$ and the growth rate $\beta_0$ of $I_0(x)$.

We can plot the successive profiles of $u^*_0(x, y)$ and also the integral $I_0(x) = \int_0^\infty u^*_0(x, y) dy$ along the longitudinal direction (figure 4(a) and (b)). Accordingly to what have been done with the harmonic perturbation one can define a characteristic length $L_{inv}$ which is the distance defining the frontier between the region where the outflow profile is dominant ($x < L_{inv}$) and the region where the inflow profile is dominant ($x > L_{inv}$). In the outflow region the mean boundary layer is decelerated while in the inflow region the mean boundary layer is accelerated. We can also define a growth rate $\beta_0$ associated to the longitudinal growth of the zeroth mode.

### IV. Influence of the spacing on the non-linear properties of the boundary layer

We show on figure 5 contours of longitudinal velocity in a horizontal plane $y = 3mm$ above the flat plate. In every cases the modulations induced by the recirculation areas and the CRSVs are present so that we can define a reorganization length $L_R$. But we can also see an evolution in the interaction between the wake of each CVGs. For $\lambda_0 = 2d$ there is a strong interaction and the flow sees the set of CVGs as an obstacle leading to a strong acceleration of the flow around the VGs. This is a situation of near blockage of the
Figure 5. Contours of longitudinal velocity $U_x$ in the horizontal plane $(x, z)$ $y = 3\text{mm}$ above the wall for different spacings and for a given Reynolds number $Re = 300$. One can clearly see the spanwise modulation of the velocity field downstream the VGs.

Figure 6. Influence of the spacing between the VGs on the reorganization length $L_R$ (a) and the spatial evolution of the integral of the zeroth mode profiles $I_0(x)$ (b). From the spatial evolution of $I_0$, one can plot the spatial growth rate of the zeroth mode (c) and the inversion length (d) as a function of the spacing.
boundary layer. On the contrary, when \( \lambda_0 = 4d \) there is nearly no interactions between the wakes of the CVGs. In between, the counter-rotating streamwise vortices (CRSVs) created by the VGs can interact.

The dependance of the characteristics of the modified boundary layer on the spacing of the VGs is illustrated on figure 6. The figure 6(a) shows a clear dependance of the reorganization length \( L_{\text{reorg}} \) on the spacing. One can notice that \( L_{\text{reorg}} \) is maximum for \( \lambda_0 = 4d \), when there is no interaction between the VGs. It corresponds to the natural recirculation length downstream an isolated cylinder. \( L_{\text{reorg}} \) decreases nearly linearly when the spacing decreases, showing the strong effect of the interaction between VGs on the structure of the flow downstream of the VGs.

On figure 6(b) the spatial evolution of \( L_o(x) \) is plotted as a function of the streamwise coordinate. One can see that in all cases the effect of the streamwise vortices is to accelerate the mean boundary layer profile. It allows the computation of the spatial growth rate (figure 6(c)) and the inversion length (figure 6(d)) as a function of the spacing. From the two figures, one can see that the \( \lambda_0 = 3d \) spacing is quite remarkable as it corresponds to a maximum in zeroth mode growth rate and a minimum inversion length. From this results, one can conclude that this configuration is the most efficient at this Reynolds number to accelerate the boundary layer profile with the VGs. In a perspective of separation control, one should use this spacing to delay the separation. The VGs should also be located no closer than 10d upstream the separation line to be efficient.

V. Conclusion

In this study the perturbation of a boundary layer growing over a flat plate downstream a line of four cylindrical vortex generators is analyzed for a given Reynolds number. Using two-component PIV measurements, the linear and non-linear perturbations of the longitudinal velocity field is calculated in a 3D domain downstream the VGs. The spatial evolution of the harmonic perturbation and of the global mode (zero mode) is used to propose new physical criteria to characterize the influence of the VGs. Two characteristic lengths are defined: a reorganization length \( L_{\text{reorg}} \) defining the location where recirculation bubble downstream the VGs cancels out, and an inversion length \( L_{\text{inv}} \) where the mean boundary layer which was decelerated becomes accelerated. The spatial growth rate \( \beta_o \) of the zero mode seems to be also a good parameter to characterize the efficiency of the VGs. From a parametric study, it is clearly demonstrated that one can find a critical spacing for which \( L_{\text{inv}} \) is minimum and \( \beta_o \) is maximum. This study should be extended to larger Reynolds number and applied to the control of separated flows.

References

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